

# Scattering of Light by Periodic Array of Metal-Coated Nanocylinders on Dielectric Slab

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**Abstract** – Scattering of light by a periodic array of metal-coated nanocylinders located on a dielectric slab is analyzed by using a semi-analytical method based on a recursive algorithm combined with the lattice sums technique. The resonance phenomena observed in the spectral responses of the scattered field are numerically investigated.

**Index Terms** — Metal-coated nanocylinder, Periodic array, Light wave, Resonant scattering.

## 1. Introduction

With a rapid development of nanoscience and nanotechnology, the interaction of light with nanoscaled metallic objects remains as an important issue in recent years [1] because of its wide range applications to optical sensors, imaging, and integrated photonic devices. The literature on the interaction of light with nanoscaled objects is much broader. The studies on the interaction are organized in many different ways, being dependent on the dimensionality of the objects and exciting sources.

In this paper, we shall analyze the scattering of TE polarized plane wave by a periodic array of metal-coated nanocylinders located on a dielectric slab by using a semi-analytical method [2] based on the recursive algorithm combined with the lattice sums technique. In this approach, the reflection and transmission matrices of the array is firstly extracted and then a recursive formula is applied to derive the scattering matrix over the whole structure consisting of the grating layer and the dielectric slab. The spectral responses in the power transmission, reflection, and absorption coefficients are numerically investigated and their resonance behaviors are discussed

## 2. Formulation of the Problem

The cross section of a periodic array of nanocylinders supported on a dielectric slab and situated in a background medium is shown in Fig. 1. The array period is  $h$ , the thickness of the dielectric slab is  $d$ , and the nanocylinder is a metal-coated circular coaxial cylinder consisting of a dielectric core with radius  $r_2$  and a coating metal layer of thickness  $r_1 - r_2$ . The material constants of the background medium, dielectric slab, coating metal, and dielectric core are denoted by  $(\epsilon_B, \mu_0)$ ,  $(\epsilon_s, \mu_0)$ ,  $(\epsilon_M, \mu_0)$ , and  $(\epsilon_c, \mu_0)$ , respectively. The array is illuminated by a TE ( $H_z, E_x, E_y$ ) polarized plane wave of unit amplitude. The angle of

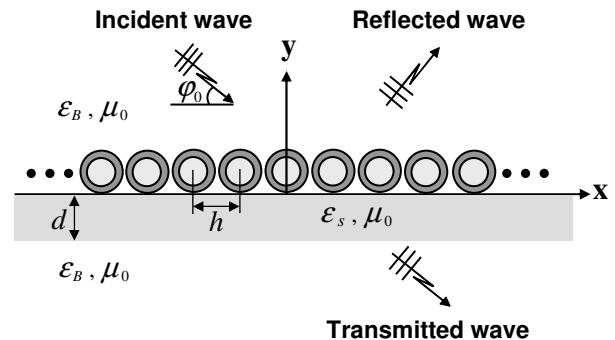


Fig. 1. Cross-sectional view of a periodic array of metal-coated nanocylinders supported on a dielectric slab and illuminated by a TE ( $E_x, E_y, H_z$ ) polarized plane wave.

incidence of the plane wave is  $\phi_0$ .

The scattering problem is formulated using the T-matrix and lattice sums techniques combined with the recursive formula for the generalized reflection and transmission matrices for a layered structure. Firstly, the T-matrix of the coaxial nanocylinder in isolation is calculated in closed form [3] by using the cylindrical wave expansion. Then the reflection and transmission matrices for the array of nanocylinders are calculated in semi-analytical form [2] by using the lattice sums technique. The matrices are expressed in terms of the space harmonics of the incident plane wave as the basis. The reflection and transmission matrices are concatenated to obtain the generalized reflection and transmission matrices for the layered structure. The structure shown in Fig. 1 consists of four layers formed by the upper background layer with  $y > 2r_1$ , the grating layer with  $0 \leq y \leq 2r_1$ , the slab region with  $-d \leq y < 0$ , and the lower background layer with  $y < -d$ .

## 3. Numerical Results and Discussion

Although a substantial number of numerical examples could be generated, we study here the normal incidence ( $\phi_0 = 90^\circ$ ) of the plane wave on the array of Ag-coated nanocylinders under the structural parameters:  $r_1 = 40\text{nm}$ ,  $r_2 = 20\text{nm}$ ,  $h = 300\text{nm}$ ,  $\epsilon_c / \epsilon_0 = 6.5$ ,  $d = 200\text{nm}$ , and  $\epsilon_s / \epsilon_0 = 2.5$ . The dispersive permittivity  $\epsilon_M(\lambda)$  of Ag is evaluated using Drude-Lorentz model with the fitting parameters.

Fig. 2 shows the power transmission ( $P_0^t$ ), reflection ( $P_0^r$ ), and absorption ( $P_0^a$ ) coefficients of the zeroth diffraction order as functions of the wavelength in the range

$250\text{nm} \leq \lambda \leq 1000\text{nm}$  where the background medium is assumed to be free space with  $n_B = \sqrt{\epsilon_B/\epsilon_0} = 1.0$ . These coefficients are obtained by truncating the diffraction orders by  $\pm 20$  and satisfy the energy conservation relation within the accuracy of  $10^{-8}$  order. We can see that  $P_0^t$  takes minima at four wavelengths as marked in the inset. The sharp resonant dip at  $\lambda = 300\text{nm}$  indicates Rayleigh anomalies which satisfy the relation  $\lambda = n_B h$  for the  $\pm 1$ st diffraction order under the normal incidence ( $\varphi_0 = 90^\circ$ ). A resonant dip at  $\lambda = 415\text{nm}$  represents the guided mode resonance which occurs when the scattered wave of the  $m$ -th diffraction order is in phase matching with the guided modes supported by the dielectric slab. The phase matching between the fundamental TE guided mode of the dielectric slab and the scattered wave of  $\pm 1$ st diffraction order is attained at around  $\lambda = 415\text{nm}$  for the normal incidence ( $\varphi_0 = 90^\circ$ ) case. Other two moderate resonant dips observed at around  $\lambda = 335\text{nm}$  and  $\lambda = 613\text{nm}$  correspond to the localized surface plasmon resonances. The shorter wavelength resonance is related to the plasmon localized on the outer surface of radius  $r_1$  and the longer wavelength resonance is related to the plasmon localized on the inner surface of radius  $r_2$ . Fig. 3 shows the near field distribution for the wavelength  $\lambda = 415\text{nm}$  at which the scattered waves of the  $\pm 1$ st diffraction orders are resonantly coupled to the forward and backward propagating guided modes in the dielectric slab.

The resonant behaviors of power transmission spectra ( $P_0^t$ ) are compared in Fig. 4 when the refractive index  $n_B$  of the background medium is changed. The Rayleigh anomalies are shifted toward the longer wavelengths through the relation  $\lambda = n_B h$  as  $n_B$  increases.  $\lambda = 390\text{nm}$ ,  $450\text{nm}$ , and  $510\text{nm}$  indicate Rayleigh wavelengths for  $n_B = 1.3$ ,  $1.5$ , and  $1.7$ , respectively. The guided mode resonances are also shifted toward longer wavelengths as  $n_B$  increases. This is because the effective refractive index for guided modes on the dielectric slab increases with  $n_B$ . The wavelengths  $436\text{nm}$  and  $456\text{nm}$  correspond to the guided mode resonances for  $n_B = 1.3$  and  $1.5$ , respectively. When  $n_B = 1.7$ , the dielectric slab does not support guided waves because  $\epsilon_s < \epsilon_B$  and the guided mode resonance is not observed in the transmission spectra. The wavelengths for surface plasmon resonances, which were observed at  $335\text{nm}$  and  $613\text{nm}$  for  $n_B = 1.0$ , do not show any noticeable changes when  $n_B$  is increased. But it is worth mentioning that the dip of transmission spectra at around  $\lambda = 613\text{nm}$  is significantly enhanced as  $n_B$  increases.

## References

- [1] J. Zhang and L. Zhang, "Nanostructures for surface plasmons," *Advances in Optics and Photonics*, Vol. 4, No. 2, pp. 157-321, 2012.
- [2] K. Yasumoto, H. Toyama, and T. Kushta, "Accurate analysis of two-dimensional electromagnetic scattering from multilayered periodic arrays of circular cylinders using lattice sums technique," *IEEE Trans. Antennas Propagat.*, Vol. 52, No. 10, pp. 2603-2611, 2004.
- [3] V. Jandieri, P. Meng, K. Yasumoto, and Y. Liu, "Scattering of light by gratings of metal-coated circular nanocylinders on a dielectric substrate," *J. Opt. Soc. Am. A*, Vol. 32, No. 7, pp. 1384-1389, 2015.

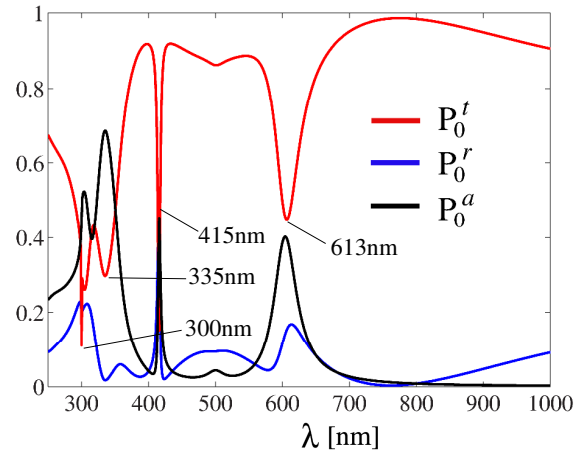


Fig. 2. Power transmission ( $P_0^t$ ), reflection ( $P_0^r$ ), and absorption ( $P_0^a$ ) coefficients of the zeroth diffraction order as functions of wavelength  $\lambda$  where  $r_1 = 40\text{nm}$ ,  $r_2 = 20\text{nm}$ ,  $h = 300\text{nm}$ ,  $\epsilon_c/\epsilon_0 = 6.5$ ,  $d = 200\text{nm}$ ,  $\epsilon_s/\epsilon_0 = 2.5$ , and the background medium is free space with  $n_B = \sqrt{\epsilon_B/\epsilon_0} = 1.0$ .

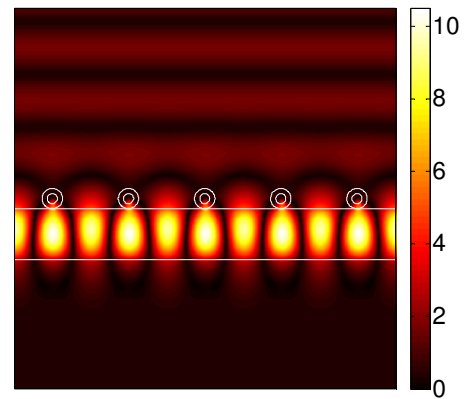


Fig. 3. Near field distribution of  $|H_z|$  for  $\lambda = 415\text{nm}$  at which the guided mode resonance is observed in Fig. 2.

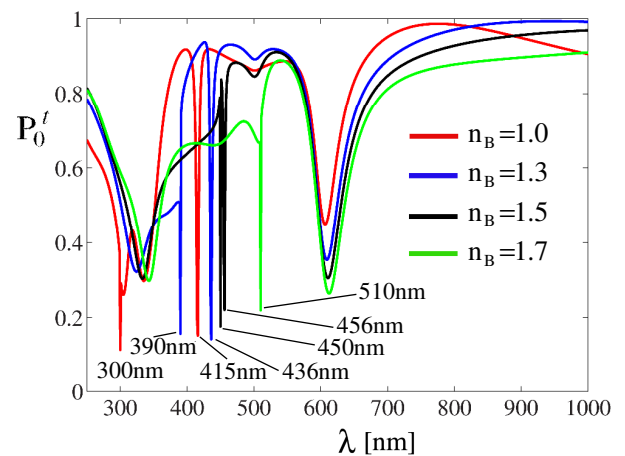


Fig. 4. Power transmission coefficient ( $P_0^t$ ) as functions of wavelength  $\lambda$  for four different background media with  $n_B = \sqrt{\epsilon_B/\epsilon_0}$ . Other structural parameters are the same as those given in Fig. 2.