Back Scattering Cross Section Along the Axis of a Tubular Cylinder of Finite Length

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Introduction

The electromagnetic scattering from a tubular cylinder of finite length is studied. The cylinder is assumed to have an infinitesimally thin wall. In the cylindrical coordinate so scaled that the cylinder runs from z=-1 to z=1, two coupled integrodifferential equations for the sum of the inside and outside surface scattering current can be formulated [1] using the Stratton-Chu equations for the scattered electric fields [2]. The two components $K_z(\phi,z)$ and $K_\phi(\phi,z)$ of the sum surface scattering current can be represented as the following series:

$$K_z(\phi,z) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} e^{in\phi} \sum_{p=0}^{\infty} K_{zn}^{p} \sin(p+1)v \tag{1}$$

$$K_\phi(\phi,z) = \frac{1}{\pi \sin v} \sum_{n=-\infty}^{\infty} e^{in\phi} \sum_{p=0}^{\infty} K_{\phi n}^{p} \cos pv \tag{2}$$

where $v = \cos^{-1} z$. These expressions assure that the edge conditions at the ends of the cylinder are satisfied term by term. Furthermore, by multiplying the factor $\sin^3 v \sin(p+1)v$ to the equation with the z-component of the incident electric field as the known quantity and multiplying the factor $\sin v \sin(p+1)v$ to the other equation and then integrating both equations over the surface of the cylinder, a system of infinite number of linear equations are obtained. The coefficients of this set of linear equations are triple integrals. But analytic expressions in terms of double series sums for these coefficients are known [1] and can be evaluated accurately. The surface current components are obtained by truncating the representations, equations (1) and (2), and then inverting the system of linear equations. For any incident field, the scattered wave from the cylinder can thus be evaluated.

Experimental Results and Discussions

For a wave incident along the axis of the cylinders, back scattering cross sections between 10.1 to 15 GHz at 0.1 GHz steps have been obtained for the cylinders listed in Table 1 and plotted on Figures 1 and 2. Each cylinder has a length of 2h and an outer diameter of 2a. They are separated into two groups according to whether one has a length to diameter ratio (h/a) of 4 or 6, and their cross sections are plotted in Figures 1 and 2 respectively. The corresponding theoretical values are also plotted. Agreement between theory and experiment away from the cutoff frequency of the H_{11} circular waveguide mode (ka=1.8415) is apparent. The minima and maxima right above the cutoff ka value are shifted up by about 5%. Since the waveguide modes are determined completely by the inner diameter of a tubular cylinder, this deviation may be reduced by taking 2a as the inner diameter instead of the outer diameter. Studies in the effects of the wall thickness of a cylinder on its back scattering cross section are in progress.

The back scattering cross section decreases significantly when the frequency of the incident wave is increased beyond the cutoff of the H_{11} mode. This can be understood as due to the propagation of the incident wave through the interior of the cylinder. Measured back scattering cross sections of solid cylinders confirm this explanation as can be seen from the bottom graph of Figure 1.

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References

[1] Lee, H.-M. Double series expansion of the Green's function for a perfectly conducting tubular cylinder of finite length, Radio Science, 18(1), 48-56 (1983)

[2] Stratton, J.A. Electromagnetic Theory, section 8-14, McGraw-Hill, New York (1941)

Table 1 Dimensions of the Cylinders

	Length	Outer Diameter	Wall Thickness	Inner Diameter
Cylinder 1	1 1/2"	3/8"	.012"	.351"
Cylinder 2	2"	1/2"	.014"	.472"
Cylinder 3	2 1/2"	5/8"	.014"	.597"
Cylinder 4	3"	3/4"	.011"	.728"
Cylinder 5	2 1/4"	3/8"	.012"	.351"
Cylinder 6	3"	1/2"	.014"	.472"
Cylinder 7	3 3/4"	5/8"	.014"	.597"
Cylinder 8	4 1/2"	3/4"	.011"	.728"

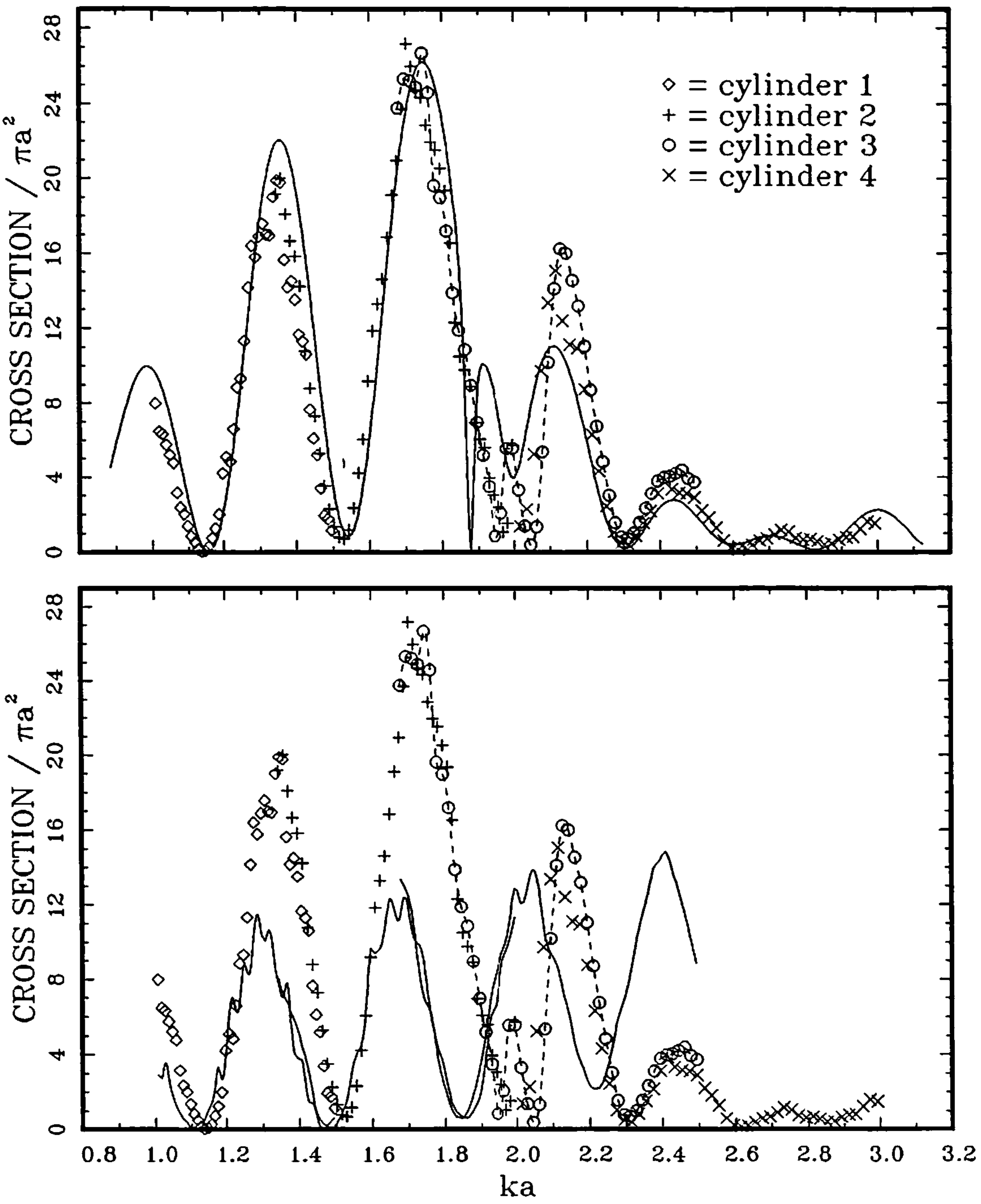

Fig. 1 Back scattering cross section along the axis of a tubular cylinder (h/a=4) together with theoretical predictions (top, solid curve) and with data from a solid cylinder (bottom, solid curves)

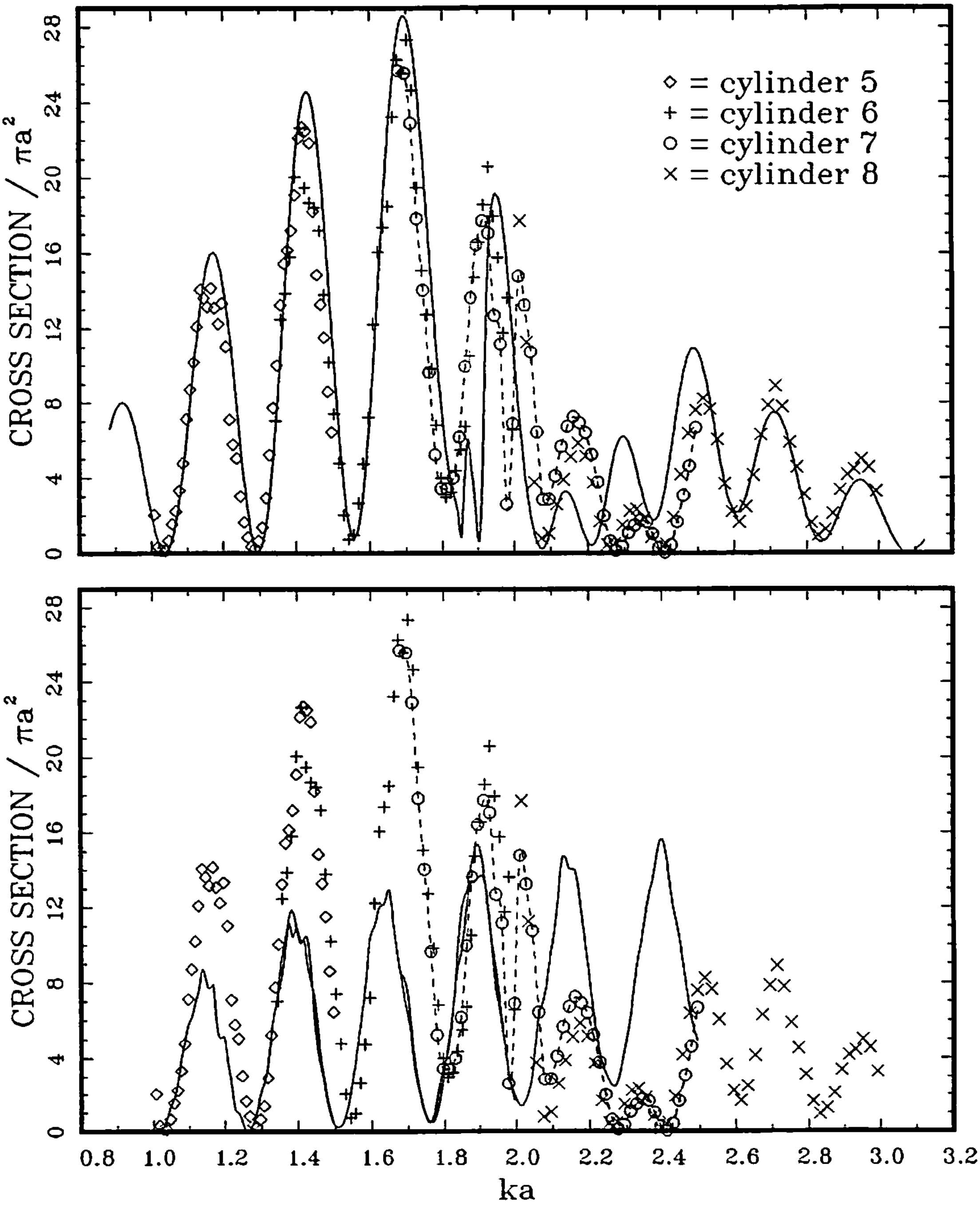

Fig. 2 Back scattering cross section along the axis of a tubular cylinder (h/a=6) together with theoretical predictions (top, solid curve) and with data from a solid cylinder (bottom, solid curves)