

AN IMPROVED FORMULATION FOR EXTENDING THE  
GTD USING THE MOMENT METHOD\*

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In 1975 two techniques were published which combined the method of moments (MM) and the geometrical theory of diffraction (GTD). One technique [1] extended the moment method through the use of the GTD while the second[2] used the moment method to solve for unknown diffraction coefficients thereby extending the GTD. We wish to consider the latter here.

One problem area that existed with the original solution of Burnside et. al. was associated with a field incident along or nearly along one wall of a wedge structure. This paper will show an improved series representation for the diffracted current that is sufficient at all incidence angles[3]. In addition, this improved formulation is insensitive to the location of the match points in the GTD region whereas the original formulation exhibited a sensitivity to the match point locations.

The improved formulation is then applied to the problem of bistatic scattering by a triangular pyramid[4]. The GTD-MM solution obtained is approximate and presently neglects the scattering by the four tips. Nevertheless results are obtained that compare very well with experimental data as will be indicated in the text below. (The field due to tip diffraction may be added in post facto in this case.) This is believed to be the first use of the GTD-MM technique in treating a truly 3-dimensional geometry.

The GTD-MM technique is a procedure that enables one to solve for the diffraction coefficient if the general form of the current away from the source of diffraction (i.e., the GTD region) is known. The current near the source of diffraction is treated as a complete unknown by using the method of moments (i.e., the MM region, see Fig. 1). We have overcome the difficulties mentioned in the second paragraph for the current in the GTD region by using a series of three terms based on approximate expressions for the Fresnel integral. Thus, the diffracted current  $J^d$  in our solution is proportional to

$$J^d \propto \sum_{-1}^{+1} D^{(n)} \frac{e^{-jk\rho}}{(\sqrt{\rho})^{-n}} \quad (1)$$

and the total current is given by

$$\bar{J} = \bar{J}^i + \bar{J}^r + \bar{J}^d \quad (2)$$

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where  $\bar{J}^i$  and  $\bar{J}^r$  are the currents associated with the incident and reflected fields respectively in the GTD region.

The advantage of the formulation here[3] over previous work[2] is that the use of the three term series in Equation (1) will permit one to obtain the correct currents for all incidence angles whereas the previous formulation required special treatment of those cases where the incident ray was near grazing upon one of the faces of the wedge. While the three term expansion requires the use of three match points on each face of the wedge in the GTD region instead of just one (or two in the case of grazing incidence) as in the previous formulation, this is a small price to pay for a solution that works over all incidence angles and is insensitive to the match point locations in the MM region as well.

Numerical results have been obtained for a variety of cases using the formulation in Equation (1). A typical result is shown in Fig. 2 which gives the current magnitude and phase along the x-wall for the critical grazing incidence case. The exact solution is given by the dashed line and the agreement is seen to be excellent.

Next we will use the method to treat a 3-dimensional geometry, the three sided pyramid (see Fig. 3). The reason for treating this particular geometry is to demonstrate the applicability of the technique to 3-dimensional geometries. As such, it represents a first step in applying the GTD-moment method technique to 3-dimensional problems[4].

For the problem of scattering by a pyramid as treated here each face of the pyramid is composed of two regions, a GTD region and a moment method region near the edges. The current distribution in the GTD region is found by solving the 3-dimensional wedge diffraction problem once for each wedge, or a total of six times. Next, the current near the edge of each wedge is found using the moment method. However, the currents obtained on the faces of the pyramid do not include that current caused by tip diffraction. The effect of this current on the far zone scattered field may be added post facto using a formula by Keller, et al[5]. While the effect of tip diffraction is not large, its inclusion does bring the theoretical results into closer agreement with measurements. In the work done here, only first order diffraction is considered and this is found to be adequate for determining the far zone field.

In all examples which will be presented we used a pyramid with the edges 1, 2, 3 of length  $9.144\lambda$  making an angle with the z-axis of  $15^\circ$ . At first the surface current was computed. In this current there is no diffracted current from the tips. The scattered field was computed two times, one without the tip diffraction and one with. By using Keller's formula the results as we will see are in closer agreement with the measurements than without the tip diffraction.

In Fig. 4 we plotted the magnitude of the surface current of the pyramid for an incident plane wave normal to edge 4 and with the magnetic field parallel to this edge. In Fig. 5 we show the bistatic radar cross section. Figure 5 shows the bistatic cross section for the case of rotation about the z axis, with the transmitted signal horizontally polarized. As we can see by including the tip diffraction we have results that are closer to the measurements although the theoretical results are a little down from the measurements

probably because second order diffraction is ignored. For the problem of finding the near zone field, our analysis shows that we must take into account diffracted rays of more than the first order.

#### REFERENCES

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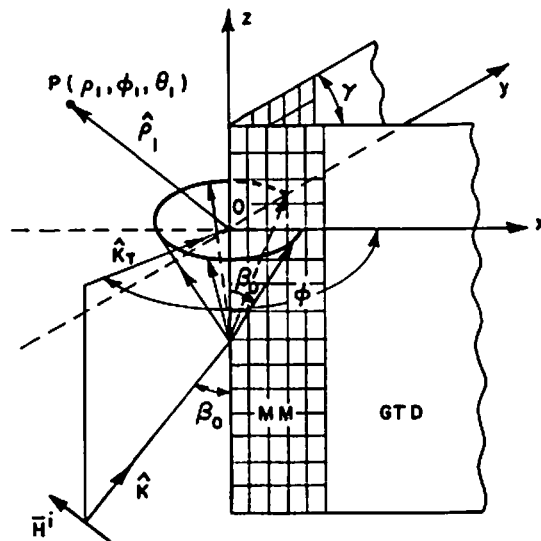


Fig. 1. Geometry for GTD-MM treatment of wedge.

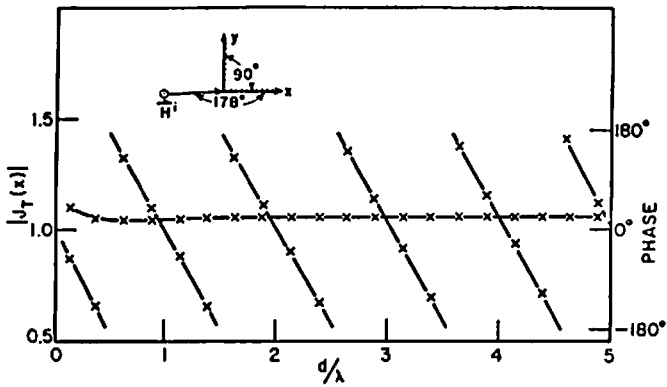


Fig. 2. GTD-MM result for grazing incidence.

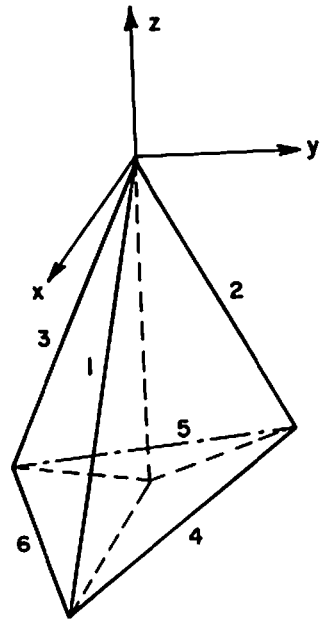


Fig. 3. Triangular pyramid with numbered edges.

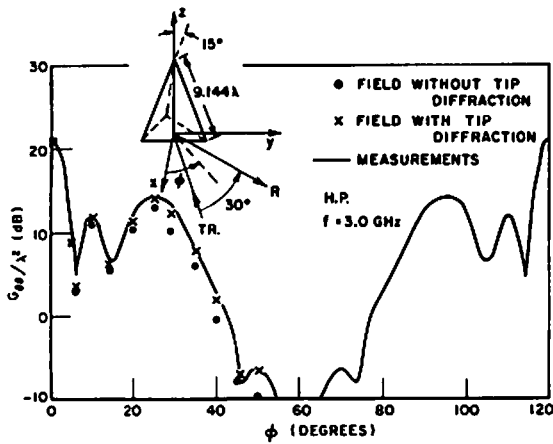


Fig. 5. Bistatic RCS for  $\phi$  polarization.

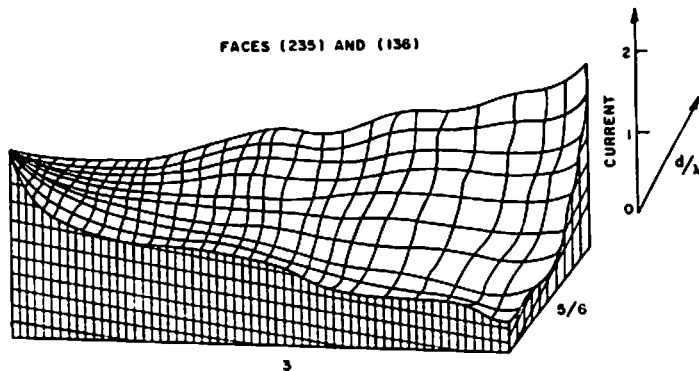


Fig. 4. Total current on pyramid face.