Analysis of Electromagnetic Pulse Responses by a Conducting Cylinder with Inhomogeneous Dielectric Coating

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***Abstract* – Electromagnetic wave scattering by a conducting cylinder with inhomogeneous dielectric coating is analyzed for application of radar systems to nondestructive testing of reinforced concrete structures. Numerical results of pulse responses indicate that degree of corrosion of the reinforcing bars can be evaluated by checking the waveform distortion of the pulse responses.**

***Index Terms* — radar, nondestructive testing (NDT), nondestructive evaluation (NDE), concrete structures.**

1. Introduction

Radar is one of the most useful tools for nondestructive testing (NDT) applications and it has great potential for corrosion inspection of reinforced concrete structures. When we apply the radar sensors to the accurate evaluation of the corrosion state of the reinforced concrete structure, sufficient knowledge of scattering characteristics of radar pulses by the corroded reinforcing bars is required. In this study, we analytically solve a problem of electromagnetic (EM) wave scattering by a conducting cylinder with inhomogeneous dielectric coating to reveal the scattering characteristics of the EM pulse by the corroded reinforcing bar.

2. Formulation of the Problem

Figure 1 shows the geometry of the problem. The conducting cylinder with inhomogeneous coating located in a lossy dielectric is illuminated by the plane *E*-wave. This is a simple model of a corroded reinforced concrete structure. The inhomogeneous coating layer represents the corrosion layer due to rusting of the reinforcing bar, and surrounding lossy dielectric is the healthy concrete (i.e. concrete without any corrosion). In order to express the inhomogeneous layer, we introduce the multilayer division method that is often used for modeling of inhomogeneous medium [1]. By applying the method to the problem, we can approximate the inhomogeneous layer with (*N*-1) thin layers of piecewise homogeneous layers as shown in Fig. 1 (b). The complex permittivity in each layer is expressed as

$$\hat{\varepsilon}_i = \varepsilon_i + \sigma_i/(j\omega), \quad (i = 1, 2, \dots, N) \tag{1}$$

where $\varepsilon_i, \mu_i (= \mu_0)$, and σ_i are the permittivity, permeability, and conductivity in layer *i*, respectively. An incident plane wave is given by

$$E_z^{inc} = e^{jkx} = \sum_{n=-\infty}^{\infty} J_n(k_N\rho)\, e^{jn\phi} \tag{2}$$

where $J_n(\cdot)$ is the Bessel function of order n. Taking account of the radiation condition, total electric field in each region can be expressed as follows:

Region i $(i = 1, 2, \dots, N-1)$

$$E_z^i(\rho,\phi) = \sum_{n=-\infty}^{\infty} \left(A_n^i J_n(k_i\rho) + B_n^i H_n^{(2)}(k_i\rho) \right) e^{jn\phi} \tag{3}$$

Region N

$$E_z^N(\rho,\phi) = \sum_{n=-\infty}^{\infty} \left(j^n J_n(k_N\rho) + C_n H_n^{(2)}(k_N\rho) \right) e^{jn\phi} \tag{4}$$

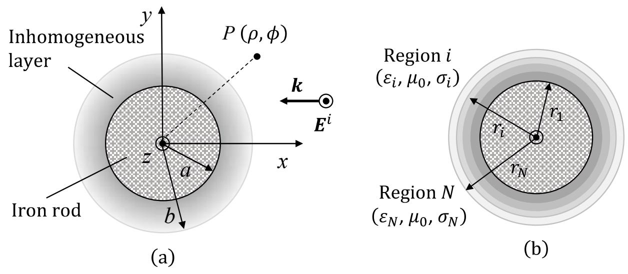
where $H_n(\cdot)$ is the Hankel function, A_n^i, B_n^i, C_n are unknown expansion coefficients to be determined, and k_i is the wave-number in Region i :

$$k_i = \omega\sqrt{\hat{\varepsilon}_i\, \mu_0} \tag{5}$$

By using the boundary condition of EM fields at the interfaces of the layers, we can derive a simultaneous equation to determine the unknown expansion coefficients expressed as

$$\mathrm{M}\boldsymbol{C} = \boldsymbol{f} \tag{6}$$

where $\boldsymbol{C}$ is a vector whose elements are unknown coefficients (A_n^i, B_n^i, C_n), and M and $\boldsymbol{f}$ are a known matrix and a known vector, respectively. By solving this equation we can determine the unknown coefficients, and consequently the electromagnetic field in each region are determined. Once the scattered field expression in frequency domain is obtained, pulse responses can be obtained by using the Fourier synthesis technique.



(a) (b)

Fig. 1. Geometry of the problem. (a) a conducting cylinder with inhomogeneous dielectric coating, and (b) its multilayer division model.

3. Numerical Calculation

As the simplest case, we here consider the corrosion is approximated by a single layer ($N = 2$). Diameters of the reinforced bar and the rust layer for some corrosion rates are tabulated in Table I [2]. As the corrosion increases, diameter of the reinforcing bar decreases for rusting and thickness of the corrosion layer increases. The relative permittivity and conductivity of the healthy concrete and rust layer are chosen as follows:

$$\varepsilon_{r1} = 10, \ \ \sigma_1 = 1.0\ [\mathrm{S/m}] \ : \ \text{rust layer}$$
$$\varepsilon_{r2} = 4, \ \ \ \sigma_2 = 0.1\ [\mathrm{S/m}] \ : \ \text{helthy concrete}$$

As an incident pulse, we adopt three times differentiated Gaussian pulse [3] expressed by

$$f_{TDG}(t) = -At\left(t^2 - \frac{3}{2\alpha}\right)e^{-\alpha t^2} \qquad (7)$$

where A is a constant for normalization and α is a constant that determine the pulse width and frequency bandwidth. The waveform of the pulse is shown in Fig. 2. The frequency range is about 2GHz to 10GHz and center frequency is about 4 GHz. Figure 3 shows backscattered pulse responses for different corrosion rates. As the corrosion of the concrete progresses, waveform deformation of the response increases. This result indicates that the degree of corrosion can be estimated by checking deformation of the response waveform.

4. Conclusion

We have analytically solved a problem of EM wave scattering by a conducting cylinder with inhomogeneous dielectric coating to reveal the scattering characteristics of the EM pulse by the corroded reinforcing bar. From the numerical results we have confirmed that the degree of corrosion can be estimated by checking the response waveform.

Table I Diameters of the reinforcing bar and the rust layer for different corrosion amounts [2].

Corrosion [%]	$2a$ [mm]	$2b$ [mm]
0	25.0	25.0
10	23.7	28.5
20	22.4	31.6
30	20.9	34.5

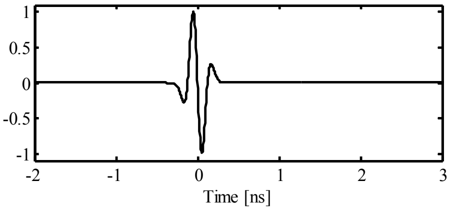


Fig. 2. Incident pulse used for numerical calculation. (Three-times differentiated Gaussian (TDG) pulse).

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References

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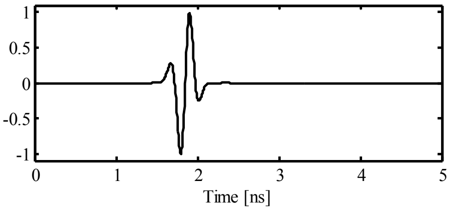
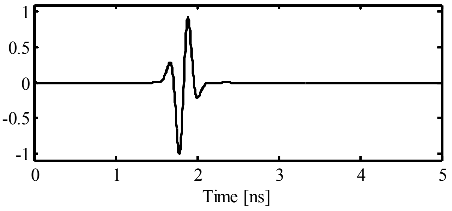
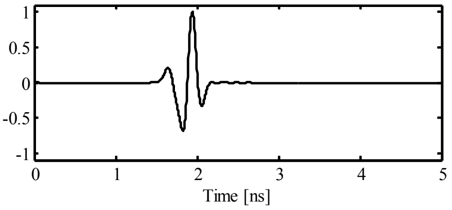


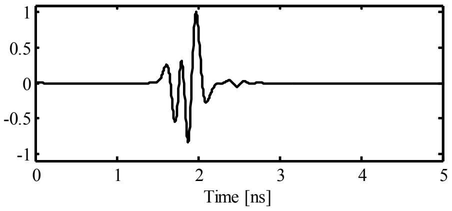
(a) Corrosion 0%



(b) Corrosion 10%



(c) Corrosion 20%



(d) Corrosion 30%

Fig. 3. Backscattered pulse responses from a corroded reinforcing bar for different corrosion rates listed in Table I.