SCATTERING OF H-POLARIZED PLANE WAVES BY TWO PARALLEL CONDUCTING RECTANGULAR CYLINDERS

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The scattering of electromagnetic waves by cylindrical objects is of continuing interest[1],[2] as a fundamental problem in electromagnetic wave theory. In this paper, the scattering of H-polarized plane waves by two parallel conducting rectangular cylinders is investigated by using an analytical method[3] based on the Wiener-Hopf technique. This method is useful when the dimension of the cylinders is larger than the wavelength and the separation between the cylinders is not so large. The numerical results of the backscattering cross section are given as a function of the frequency. The time dependence $\exp(i\omega t)$ is assumed and suppressed.

The geometry of this problem is shown in Fig.1. Since the polarization is invariant for the diffraction from this structure, the electromagnetic field can be derived from the z-component of the magnetic vector as follows:

$$\begin{cases} E_x = \dfrac{1}{i\omega\varepsilon_0}\dfrac{\partial H_z}{\partial y}, \quad E_y = \dfrac{-1}{i\omega\varepsilon_0}\dfrac{\partial H_z}{\partial x} \\ E_z = H_x = H_y = 0 \end{cases} \tag{1}$$

where H_z is the solution of Eq.(2)

$$\begin{cases} \left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \kappa^2\right) H_z(x,y) = 0 \\ \kappa = \omega\sqrt{\varepsilon_0\mu_0}\ . \end{cases} \tag{2}$$

Let the incident wave (E^i, H^i) be given by

$$H^i = i_z \exp[-i\kappa(x\cos\theta + y\sin\theta)]. \tag{3}$$

The total field (E^t, H^t) is expressed as

$$(E^t, H^t) = (E^i, H^i) + (E^s, H^s) \tag{4}$$

where (E^s, H^s) is the scattered field.

For convenience, let us divide the scattered field into three parts as follows:

$$(E^s, H^s) = (E_1^s, H_1^s) + (E_2^s, H_2^s) + (E_3^s, H_3^s) \tag{5}$$

where each part is defined by the following relations,

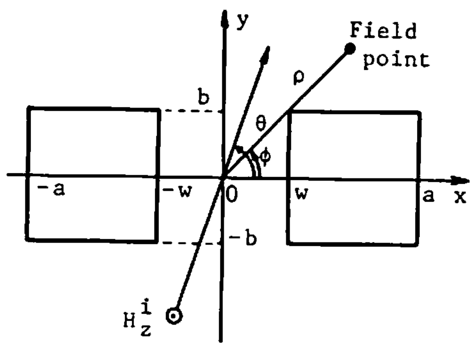

Fig.1. Geometry of two parallel rectangular cylinders.

(i) $|y|<b$

$$(E_1^s,H_1^s)=\begin{cases}(E^s,H^s) & x>a\\ -(E^i,H^i) & x<a\end{cases} \quad (6a)$$

$$(E_2^s,H_2^s)=\begin{cases}0 & x>-a\\ (E^i,H^i)+(E^s,H^s) & x<-a\end{cases} \quad (6b)$$

$$(E_3^s,H_3^s)=\begin{cases}0 & |x|>w\\ (E^t,H^t) & |x|<w\end{cases} \quad (6c)$$

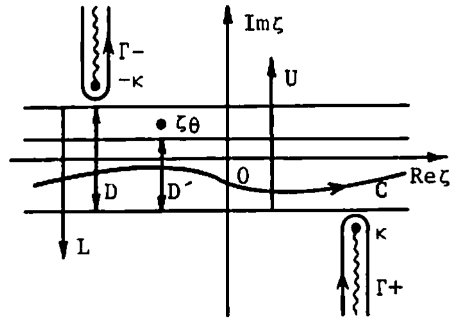

Fig.2. Resions U,L,D and D´ and integration contours C and Γ±.

(ii) $|y|>b$

$$\partial H_{z1}^s(x,\pm b\pm 0)/\partial y=\begin{cases}\partial H_z^s(x,\pm b)/\partial y & x>a\\ -\partial H_z^i(x,\pm b)/\partial y & x<a\end{cases} \quad (7a)$$

$$\partial H_{z2}^s(x,\pm b\pm 0)/\partial y=\begin{cases}0 & x>-a\\ \partial[H_z^i(x,\pm b)+H_z^s(x,\pm b)]/\partial y & x<-a\end{cases} \quad (7b)$$

$$\partial H_{z3}^s(x,\pm b\pm 0)/\partial y=\begin{cases}0 & |x|>w\\ \partial H_z^t(x,\pm b)/\partial y & |x|<w.\end{cases} \quad (7c)$$

It is evident from Eqs.(6) and (7) that $(E^t,H^t)=0$ inside the cylinders and that $E_x^t(x,\pm b)=0$ for $w<|x|<a$.

The Fourier transform and its inverse transform are defined by

$$f(\zeta)=\mathcal{F}[f(x)]=\int_{-\infty}^{\infty}f(x)e^{i\zeta x}dx \quad (8a)$$

$$f(x)=\mathcal{F}^{-1}[f(\zeta)]=\frac{1}{2\pi}\int_C f(\zeta)e^{-i\zeta x}d\zeta \quad (8b)$$

where C is a contour in region D´ as shown in Fig.2. A function that is regular in region U or L will henceforth be distinguished by the superscript "+" or "-".

Let the unknown functions $V^{\pm}(\zeta,\pm b)$ and $V_0(\zeta,\pm b)$ be defined as follows:

$$\kappa V^+(\zeta,\pm b)e^{i\zeta a}/(\zeta-\zeta_\theta)=\mathcal{F}[\partial H_{z1}^s(x,\pm b)/\partial y] \quad (9a)$$

$$\kappa V^-(\zeta,\pm b)e^{-i\zeta a}/(\zeta-\zeta_\theta)=\mathcal{F}[\partial H_{z2}^s(x,\pm b)/\partial y] \quad (9b)$$

$$\kappa V_0(\zeta,\pm b)=\mathcal{F}[\partial H_{z3}^s(\zeta,\pm b)/\partial y] \quad (9c)$$

where $\zeta_\theta=\kappa\cos\theta$.

From Eq.(9), the transform of $H_z^s(x,y)$ for $|y|>b$ can be represented by

$$H_z^s(\zeta,y)=\mp\frac{\kappa}{ik}\{[V^+(\zeta,\pm b)e^{i\zeta a}+V^-(\zeta,\pm b)e^{-i\zeta a}]/(\zeta-\zeta_\theta)+V_0(\zeta,\pm b)\}e^{\mp ik(y\mp b)} \quad (10)$$

where $k=\sqrt{\kappa^2-\zeta^2}$ ($\mathrm{Im}\,k<0$) and the upper sign refers to $y>b$, the lower to $y<-b$.

Consider the field in $|y|<b$. The current $J_y(\pm a,y)$ can be expanded in the Fourier series as

$$\begin{cases} J_y(\pm a,y)=\sum_n(-1)^{n-1}[\beta_{\frac{1}{2}cn}\sin b_{cn}y-\beta_{\frac{1}{2}sn}\cos b_{sn}y] & |y|<b \\ b_{cn}=(n-1/2)\pi/b\ (n=1,2,3,\ldots),\ b_{sn}=n\pi/b\ (n=0,1,2,\ldots). \end{cases} \tag{11}$$

From Eqs.(9) and (11), the transform of $H^s_{z1,2}(x,y)$ for $|y|<b$ is represented by

$$H^s_{z\frac{1}{2}}(\zeta,y)=\frac{\kappa}{2k}\left(V_c^{\pm}(\zeta)\frac{\sin ky}{\cos kb}-V_s^{\pm}(\zeta)\frac{\cos ky}{\sin kb}\right)\frac{e^{\pm i\zeta a}}{\zeta-\zeta_\theta}$$
$$-i\zeta\sum_n(-1)^{n-1}\left(\beta_{\frac{1}{2}cn}\frac{\sin b_{cn}y}{\zeta^2-k_{cn}^2}-\beta_{\frac{1}{2}sn}\frac{\cos b_{sn}y}{\zeta^2-k_{sn}^2}\right)e^{\pm i\zeta a} \tag{12}$$

where $k_{qn}^2=\kappa^2-b_{qn}^2$ $(q=c,s)$ and

$$V_c^{\pm}(\zeta)=V^{\pm}(\zeta,b)+V^{\pm}(\zeta,-b),\quad V_s^{\pm}(\zeta)=V^{\pm}(\zeta,b)-V^{\pm}(\zeta,-b). \tag{13}$$

Since $H^s_{z1}(\zeta,y)\exp(-i\zeta a)$ and $H^s_{z2}(\zeta,y)\exp(i\zeta a)$ must be regular in region U and L, respectively, except for $\zeta=\zeta_\theta$, we have

$$\beta_{\frac{1}{2}s0}=\frac{V_s^{\pm}(\mp\kappa)}{2i(\kappa\pm\zeta_\theta)b},\quad \beta_{\frac{1}{2}qn}=\frac{\kappa V_q^{\pm}(\mp k_{qn})}{ik_{qn}(k_{qn}\pm\zeta_\theta)b}\quad (q=c,s;\ n=1,2,3,\ldots). \tag{14}$$

The field $H^s_{z3}(x,y)$ can be expressed as

$$\begin{cases} H^s_{z3}(x,y)=\sum_{n=0}^{\infty}\Theta(x)[B_{cn}\sin h_ny+B_{sn}\cos h_ny]\cos w_n(x-w) \\ \Theta(x)=\begin{cases}1 & |x|<w\\ 0 & |x|>w\end{cases},\quad w_n=\frac{n\pi}{2w},\quad h_n^2=\kappa^2-w_n^2 \end{cases} \tag{15}$$

where the expansion coefficients $\{B_{cn}\},\{B_{sn}\}$ are unknown.

Taking the Fourier transform of $[H_z^t(x,\pm b-0)-H_z^t(x,\pm b+0)]$, we have

$$J_{xq}(\zeta)=-H_q(\zeta)\frac{V_q^+(\zeta)}{\zeta-\zeta_\theta}e^{i\zeta a}-H_q(\zeta)\frac{V_q^-(\zeta)}{\zeta-\zeta_\theta}e^{-i\zeta a}-\frac{\kappa}{ik}V_{0q}(\zeta)$$
$$+2i\zeta\sum_n\frac{\beta_{1qn}}{\zeta^2-k_{qn}^2}e^{i\zeta a}+2i\zeta\sum_n\frac{\beta_{2qn}}{\zeta^2-k_{qn}^2}e^{-i\zeta a}+\sum_{n=0}^{\infty}F_{qn}(\zeta) \tag{16}$$

where $q=c,s$ and

$$J_{xc}(\zeta)=J_x(\zeta,b)+J_x(\zeta,-b),\quad J_{xs}(\zeta)=J_x(\zeta,b)-J_x(\zeta,-b) \tag{17a}$$

$$H_c(\zeta)=[-\kappa/(k^2b)][ikb\,e^{ikb}/\cos kb],\quad H_s(\zeta)=[-\kappa/(k^2b)][kb\,e^{ikb}/\sin kb] \tag{17b}$$

$$F_{cn}(\zeta)=-2B_{cn}\sin h_nb\,f_n(\zeta),\quad F_{sn}(\zeta)=-2B_{sn}\cos h_nb\,f_n(\zeta) \tag{17c}$$

$$f_n(\zeta)=-i\zeta[e^{i\zeta w}-(-1)^ne^{-i\zeta w}]/(\zeta^2-w_n^2) \tag{17d}$$

$$V_{0c}(\zeta)=\sum_{n=0}^{\infty}B_{cn}(2h_n/\kappa)\cos h_nb\,f_n(\zeta),\quad V_{0s}(\zeta)=-\sum_{n=0}^{\infty}B_{sn}(2h_n/\kappa)\sin h_nb\,f_n(\zeta). \tag{17e}$$

Since the current $J_x(x,\pm b)=0$ for $|x|>a$ and $|x|<w$, $J_x(\zeta,\pm b)\exp(i\zeta a)$ are regular in region U and $J_x(\zeta,\pm b)\exp(-i\zeta a)$ in region L. Therefore, multiplying Eq.(16) by $\exp(\pm i\zeta a)$ leads to the simultaneous Wiener-Hopf equations which can be solved by the conventional factrization technique.

Multiplying $[H_z^t(x,\pm b-0)-H_z^t(x,\pm b+0)]$ by $\cos w_m(x-w)$ and integrating it over $|x|\leq w$ yield the simultaneous equations for $\{B_{qn}\}$:

$$\frac{1}{2\pi}\int_{\Gamma-}\frac{-\kappa}{ik}\frac{e^{ita}}{t-\zeta_\theta}V^+_{\substack{c\\s}}(t)f_m(-t)dt+\frac{1}{2\pi}\int_{\Gamma+}\frac{-\kappa}{ik}\frac{e^{-ita}}{t-\zeta_\theta}V^-_{\substack{c\\s}}(t)f_m(-t)dt$$

$$\pm\sum_{n=0}^{\infty}\frac{2h_n}{\kappa}B_{\substack{c\\s}n}I_{nm}\,{\cos \atop \sin}h_nb-2(1+\delta_{0m})wB_{\substack{c\\s}m}\,{\sin \atop \cos}h_mb\mp 2e^{i\kappa b\sin\theta}f_m(-\zeta_\theta)=0 \qquad (18)$$

for $m=0,1,2,\ldots$, where δ_{0m} is the Kronecker delta and

$$I_{nm}=\frac{1}{2\pi}\int_C\frac{-\kappa}{ik}f_n(t)f_m(-t)dt. \qquad (19)$$

Figure 3 shows the numerical results of the backscattering cross section σ defined by

$$\begin{cases}\sigma=\lim\limits_{\rho\to\infty}2\pi\rho\,|H_z^s(\rho,\theta-\pi)|^2/|H_z^i|^2\\ \rho=\sqrt{x^2+y^2}\ .\end{cases} \qquad (20)$$

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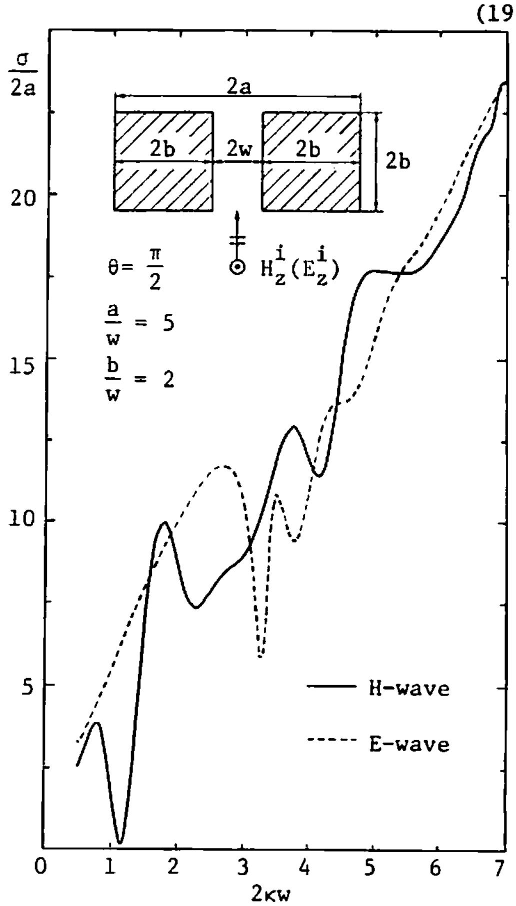

Fig.3. Normalized backscattering cross section $\sigma/(2a)$ of two parallel conducting square cylinders as a function of $2\kappa w$.