B-8-1 EIGENFUNCTION OF HERMITIAN ITERATED OPERATOR AND ITS APPLICATION TO NUMERICAL ANALYSIS

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Introduction

Eigenfunctions have widely been used in electromagnetic boundary value problems when the boundaries coincide with coordinate surfaces in which the Helmholtz equation is separable. Garbacz[1] has shown that similar characteristic functions (modes) can be defined for bodies of arbitrary shape, and Harrington and Mautz have systematized the theory from a different viewpoint [2], and have developed a computational method[3]. Their characteristic functions constitute an orthonormal set in Hilbert space on the infinitely distant surface, but does not on the surface of the body itself.

This paper presents a set of eigenfunctions orthogonal and complete on the conducting body. They are eigenfunctions of an Hermitian iterated operator , which before the iteration gives the tangential electric field on the surface due to the surface current. The characteristic equation can be simplified to that of a real operator not iterated when the electromagnetic reciprocity holds. Our theory gives a good computational accuracy even when the body does not radiate, in which case the theory of Harrington et al. loses the applicability. Illustrative examples are given to the numerical analyses of a dipole and of a rectangular cylinder.

Characteristic equation

The boundary value problem is formulated as follows.

$$z(\bar{J}) = -\bar{E}^{i}, \qquad (1)$$

where z denotes the integral operator giving the tangential electric field on the surface of a perfectly conducting body S due to the current $\bar{J}$ on S, and $\bar{E}^{i}$ denotes the tangential component of the incident electric field. Concidering sets of functions $\{\bar{J}_n\},\{\bar{E}_n\}$ satisfying $z(\bar{J}_n)=\bar{E}_n$, and defining the inner product by the surface integral on S, we have

$$(\bar{E}_m,\bar{E}_n) = (z\bar{J}_m,z\bar{J}_n) = (\bar{J}_m,z^{*}z\bar{J}_n) \triangleq (\bar{J}_m,\kappa\bar{J}_n). \qquad (2)$$

Adopting a set of eigenfunctions of κ as $\{\bar{J}_n\}$, we have

$$\kappa(\bar{J}_n) = \lambda_n^{2}\bar{J}_n \qquad (3), \qquad (\bar{J}_m,\bar{J}_n) = \delta_{mn}. \qquad (4)$$

From (2),(3) and (4), $$(\bar{E}_m,\bar{E}_n) = \lambda_n^{2}\delta_{mn} \qquad (5),$$

where the eigenvalues of κ are nonnegative and hence expressed as λ_n^{2} for later convenience. Operating z on both sides of (3), we get the following equation considering the property $\kappa^{*}=zz^{*}$.

$$\kappa^{*}(\bar{E}_n) = \lambda_n^{2}\bar{E}_n. \qquad (6)$$

κ and κ^{*} are Hermitian operators, and their singularities are lower than that of z. Accordingly, $\bar{J}_n$ and $\bar{E}_n$ constitute orthogonal and complete sets on S, and lead to the solution of (1) in the following form whose strong convergence is gauranteed.

$$\bar{J} = -\sum_n \frac{(\bar{E}_n, \bar{E}^i)}{\lambda_n^2} \bar{J}_n . \qquad (7)$$

When the medium is isotropic, z is a complex and symmetric operator, which results in $z^c = z^*$. Therefore from (3) and (6), we have $\bar{E}_n = \alpha \bar{J}_n^c$, with α being a constant. From (4) and (5), we have $|\alpha|^2 = \lambda_n^2$. As the phase of α is arbitrary, it is convenient to specify α to be $j\lambda_n$. Then we have

$$z(\bar{J}_n) = j\lambda_n \bar{J}_n^{\,c}. \qquad (8)$$

Eq.(8) is equivalent to (3), and gives us a complete and orthonormal set on S. A shift of phase in λ_n by 2θ gives a shift of phase in $\bar{J}_n$ by θ. Decomposing z and $\bar{J}_n$ into the real and imaginary parts as $z = R + jX$ $\bar{J}_n = \bar{J}_n' + j\bar{J}_n''$, (8) is rewritten in the characteristic equation of a real and symmetric operator as follows.

$$\begin{pmatrix} X & R \\ R & -X \end{pmatrix} \begin{pmatrix} \bar{J}_n' \\ \bar{J}_n'' \end{pmatrix} = \lambda_n \begin{pmatrix} \bar{J}_n' \\ \bar{J}_n'' \end{pmatrix} . \qquad (9)$$

Dipole antenna

Eq.(8) has been transformed into a matrix equation by the moment method [4]. Dividing the antenna length l into $(2N-1)$ segments, and defining the generalized impedance matrix Z, we can solve (9) by available computer subroutines. For the center-fed dipole, the order of the matrix can be lowered to N by taking into account the structual symmetry, and then the order of (9) becomes 2N. Fig.1 shows the convergence of $\{\lambda_n\}$ with N for $kl=2\pi$, $\Omega=10$ (k:the free space phase constant), where eigenvalues of lower eight modes are shown. Subdivision with N=32 is seen to give satisfactory approximation to the integral equation (8). The change of $\{\lambda_n\}$ vs. kl is shown in Fig.2 where N=32. There appears a mode with exceptionally small eigenvalue when $kl=\pi$ and $kl=3\pi$. Fig.3 shows the monotonic convergence of the input admittance with the number of modes included in the calculation, which assures that there is no problem in the calculation of higher modes with little radiated power.

Rectangular cylinder

Similar computational procedures have been accomplished for a rectangular cylinder whose side lengths are a and b with $b/a=9/4$. Both TM and TE modes are treated, and the order of matrices are lowered by a factor of four by considering four kinds of symmetricity. Fig.4 shows the convergence of $\{\lambda_n\}$ with the subdivided length Δc for TM mode and (++++) symmetry as illustrated in the figure. The lower modes have larger eigenvalues in this case, and converge if $\Delta c/\lambda<0.1$. Figs 5 and 6 show $\{\lambda_n\}$ vs. kb for the symmetry illustrated in each figure. Some modes have zero eigenvalues for specific kb, which give the dispersion relation of TM_{mn} and TE_{mn} modes indicated below the abscissa. These values agree with the exact value within 0.51% error for TM modes and 0.24% error for TE modes. No extraneous root is observed.

References

[1] R.J.Garbacz, R.H.Turpin : IEEE Trans. AP-19,348(1971).
[2] R.F.Harrington,J.R.Mautz : ibid,AP-19,622(1971).
[3] R.F.Harrington,J.R.Mautz : ibid,AP-19,629(1971).
[4] R.F.Harrington : Field Computation by Momemt Methods (The Macmillan Company, New York, 1968).

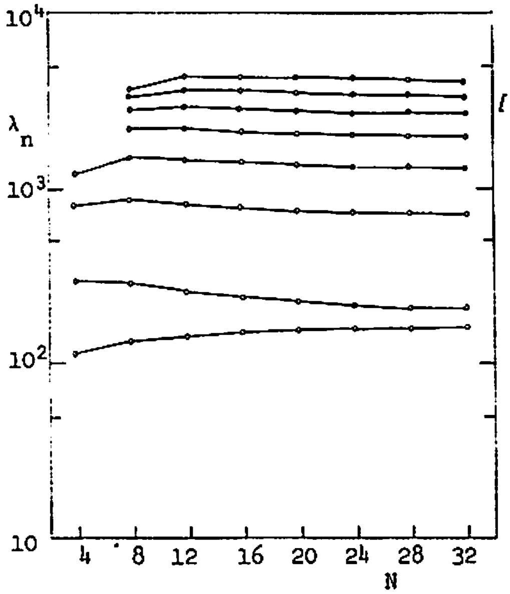

Fig.1 Convergence of λ_n with N. ($kl=2\pi,\Omega=10$)

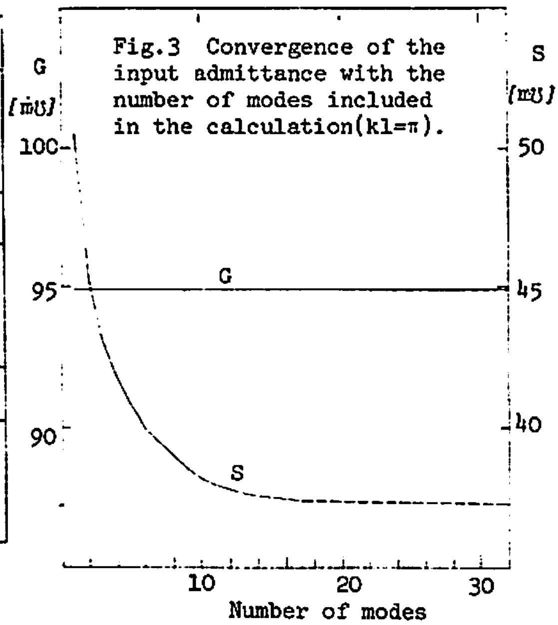

Fig.3 Convergence of the input admittance with the number of modes included in the calculation($kl=\pi$).

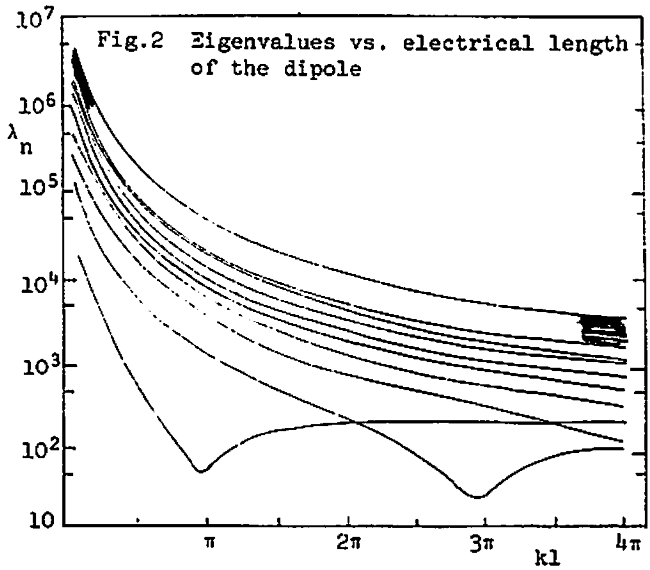

Fig.2 Eigenvalues vs. electrical length of the dipole

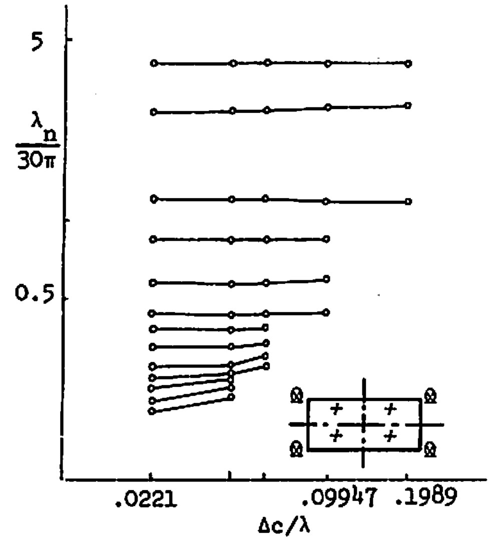

Fig.4 Convergence on $\{\lambda_n\}$ with subdivided length Δc(TM mode,kb=5).

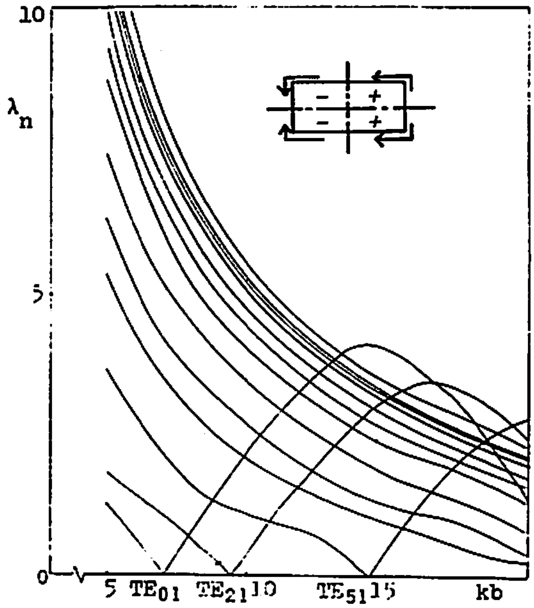

Fig.6 Eigenvalues vs. kb(TE mode).

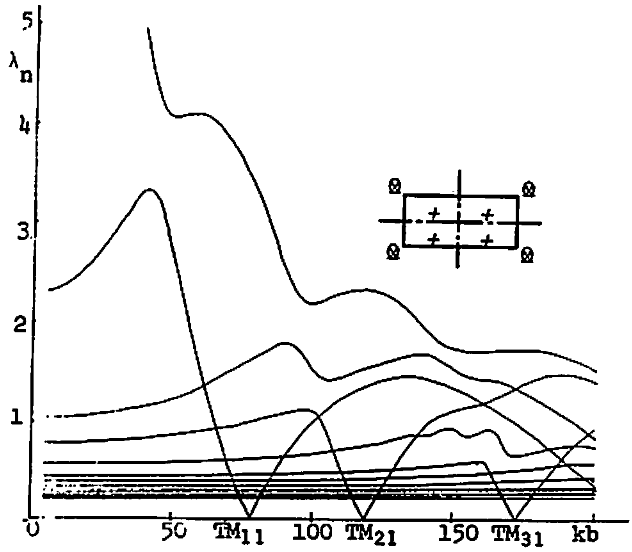

Fig.5 Eigenvalues vs. kb(TM mode).