Solution of Electrically Large Scattering problems using the Characteristic Basis Function Method

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***Abstract* - In this paper, parallel multilevel characteristic basis function method (MLCBFM) has been hybridized with the randomized pseudo-skeleton approximation method (RSPA) and randomized singular value decomposition (rSVD) method, for the analysis of the scattering from electrically large rough surfaces. MLCBFM defines the Characteristic Basis Functions (CBFs) on a larger domain and thereby achieves a higher compression rate. The reduced matrix has a much smaller size compared to that of the original system, which enables us to use a direct solver, rather than an iterative one. The RSPA algorithm accelerates the generation of the reduced matrix while the rSVD expedites the generation of CBFs. The hybrid method is found to be both accurate and efficient, and we use it in this work to investigate the problem of bistatic scattering from Gaussian rough surfaces.**

Index Terms—*Characteristic basis Functions (CBFs), Rough Surface, Scattering, rSVD, RSPA.*

1. Introduction

In recent years, extensive research has been carried out with the goal of evaluating and understanding the phenomenon of rough surface scattering, which finds broad applications in areas such as microwave remote sensing and ground penetration radar. Although analytical methods [1,2] provide a fast solution to the problem at hand, they tend to become inaccurate when the roughness parameters exceed the domains of their validity. Numerical methods are capable of providing rigorous solutions without being limited by the degree of roughness [3] but they have to deal with the issue of high computational burden and memory requirement. Discretization of the surface integral equations for large rough surfaces result in a large and dense linear system of equations with millions of unknowns, which requires the use of advanced algorithms running on supercomputers.

The conventional CBFM is an iteration-free algorithm, which generates high-level basis functions, called the Characteristic Basis Functions (CBFs) that are tailored for the object under analysis. The application of the CBFM leads to a reduced matrix, which is much smaller than the conventional Method of Moments (MoM) matrix. This, in turn, enable us to derive a direct solution of the problem, avoiding convergence issues and numerical inefficiencies associated with iterative solvers in the context of multiple excitation scenarios. Furthermore, the CBFs generated by MLCBFM can be defined on larger-size subdomains with respect to the mono-level CBFM, enabling us to further reduce the size of the associated impedance matrix.

In this work, new enhancement techniques are investigated to further accelerate the solution of rough surface scattering. In particular, the randomized pseudo-skeleton approximation method (RSPA) is introduced here for the first time to accelerate the generation of the reduced matrix in the context of parallel MLCBFM. RSPA is very simple and efficient. The complexity of the algorithm is $O(r^3)$, where r is the rank of the matrix. Moreover, in order to further accelerate the generation of the CBFs, a randomized singular value decomposition (rSVD) is introduced. It has been demonstrated [5] that rSVD is capable of decomposing a rank-deficient matrix with dimension exceeding 300,000 in less than 10 seconds. Using a random projection method, the decomposition can be implemented in a lower-dimensional space, and therefore the singular vectors (CBFs) can be derived in a highly efficient way.

2. Proposed method

2.1 The MLCBFM

Fig.1 summarize all the steps involved in the parallel algorithm.

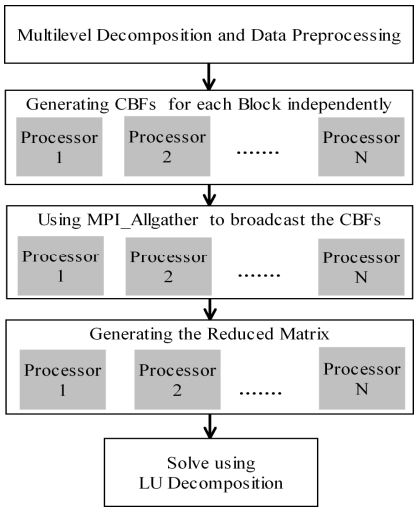

Fig.1. *Flowchart of parallel MLCBFM*

The generation of the CBFs and reduced submatrices for each block can be carried out independently and parallelized

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readily. The block sizes are deliberately chosen such that they are approximately the same, in order to achieve the desired load balance. A collective MPI broadcast function, MPI_Allgather, has to be employed to synchronize the data between processes. Since the dimensions of the CBFs and the reduced matrix are relatively small, the communication overhead is negligible. The parallel efficiency is shown in Fig.4 and is close to $1/N_p$ (N_p is the number of processes), which is the ideal case. After we generate the reduced matrix, all the remaining operations can be carried out sequentially by assigning a process to be the "master" or "root". The computational efficiency of the MLCBFM can be further improved by introducing new techniques aiming to accelerate the generation of the reduced matrix and the CBFs.

2.2 Acceleration of CBFs

The solutions for the induced currents derived for the number of sources (N_e) are arranged in columns to form a matrix $\mathbf{J}^{CBF}$ and apply the SVD, to derive the following factorized representation

$$\mathbf{J}^{CBF} = \mathbf{U\Sigma V} \quad (1)$$

where U and V are orthogonal matrices with dimensions of $N_i \times N_i$ and $N_e \times N_e$, respectively, and N_i is the number of unknowns. In this work, rSVD is introduced to accelerate this process [5], and the steps are summarized below:

1) Draw an $N_e \times k$ random matrix $\mathbf{\Omega}$.
2) Compute the $N_e \times k$ sample matrix $\mathbf{Y}=\mathbf{A\Omega}$.
3) Form an $N_e \times k$ matrix $\mathbf{Q}$ whose columns form an orthogonal basis for the columns of the matrix Y. Then $\mathbf{A}=\mathbf{QQ}^*\mathbf{A}$.
4) Form the $k \times N_e$ small matrix $\mathbf{B}=\mathbf{Q}^*\mathbf{A}$. Then $\mathbf{A}=\mathrm{Q}\mathbf{B}$.
5) Form the SVD of $\mathbf{B}$ (cheap since $\mathbf{B}$ is "small") $\mathbf{B}=\underline{\mathbf{U}}\mathbf{\Sigma V}$.
6) Form $\mathbf{U}=\mathrm{Q}\underline{\mathbf{U}}$.

2.3. Acceleration of reduced matrix generation

The generation of the reduced matrix entails the calculation of the off-diagonal submatrices corresponding to two separated blocks, which makes this process time-consuming, though the use of the adaptive cross approximation (ACA) algorithm has been employed in the past [6] to accelerate this process. In this work, a much more efficient algorithm, called RSPA [7], is introduced to achieve the same goal. We consider the submatrix $\mathbf{A}$ as an example, and summarize by the steps needed to implement the RSPA as follows:

1) Randomly select k (k>r) columns/rows to form matrices $\mathbf{C}$ and $\mathbf{R}$.
2) Form the intersection matrix $\underline{\mathbf{A}}$.
3) Find the rank r by performing SVD to $\underline{\mathbf{A}}$ (cheap since $\underline{\mathbf{A}}$ is "small"), and get its pseudo inverse $\mathbf{G}$.

The original matrix can be approximated by

$$\mathbf{A}=\mathbf{CGR} \quad (2)$$

3. Numerical Result

We consider a Gaussian rough surface with an RMS height of 0.1 λ, and a correlation length of 0.5λ, as an illustrative test example. As seen from Fig.2, the larger the dimension of the rough surface, the narrower is the main beam of the bistatic RCS plotted as a function of the observation angle. The incident angle for this simulation is θ=30°, φ=90°.

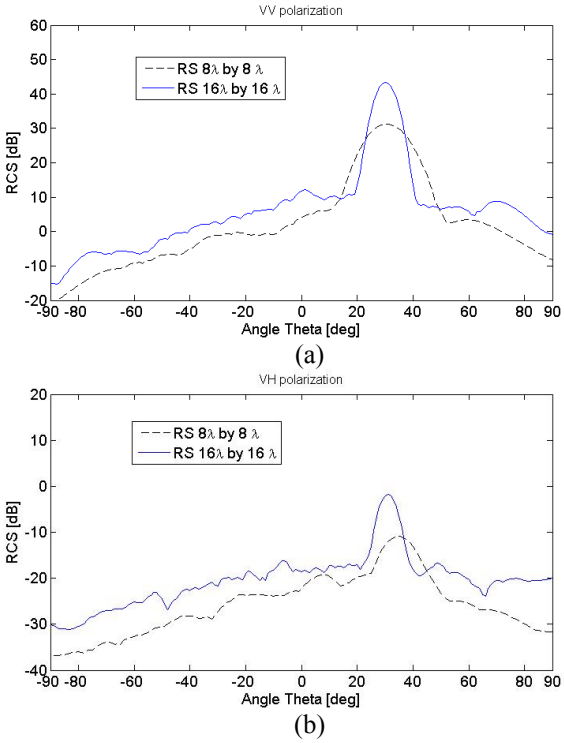

Fig.2. *Bistatic scattering from Rough surfaces*

References

[1] I.MFuks and A. G. Voronovich, "Wave diffraction by rough interfaces in an arbitrary plane-layered medium," Waves in Random Media, vol.10, pp. 253–272, 2000.

[2] N. Pinel, J. T. Johnson and C. Bourlier, "A Geometrical Optics Model of Three Dimensional Scattering From a Rough Layer With Two Rough Surfaces," Antennas and Propagation, IEEE Transactions on, vol. 58, pp. 809-816, 2010.

[3] M. El-Shenawee, "Polarimetric scattering from two-layered two-dimensional random rough surfaces with and without buried objects," Geoscience and Remote Sensing, IEEE Transactions on, vol. 42, pp. 67- 76, 2004.

[4] R. Mittra, G. Bianconi, C. Pelletti, K. Du, S. Genovesi, and A. Monorchio, "A computationally efficient technique for prototyping planar antennas and printed circuits for wireless applications", Proc. of the IEEE, vol. 100, no. 7, pp. 2122-2131, March 2012.

[5] J. Zhang, J. Erway, X. Hu, Q. Zhang, and R. Plemmons, "Randomized SVD Methods in Hyperspectral Imaging," Journal of Electrical and Computer Engineering, vol. 2012, pp. 1–15, 2012.

[6] C. Li, G. Bianconi, R. Mittra, and G. Zhu, "Parallelized Multilevel Characteristic Basis function Method (MLCBFM) combined with Adaptive Cross Approximation (ACA) for the analysis of the scattering from electrically large rough surfaces," in Antennas and Propagation Society International Symposium (APSURSI), 2014 IEEE, pp. 1626–1627, 2014.

[7] X. Zhu and W. Lin, "Randomised pseudo-skeleton approximation and its application in electromagnetics," Electronics Letters, vol. 47, no. 10, p. 1, 2011.