NONUNIFORM SAMPLING TECHNIQUES BASED ON MAXIMUM ENTROPY METHOD FOR ANTENNA APPLICATIONS

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1. Introduction

A two-dimensional nonuniform sampling technique based on matrix inversion algorithm was proposed by Yahya Rahmat-Samii and Rudolf Lap-Tung Cheung in the paper "Nonuniform Sampling Techniques for Antenna Applications", IEEE Transactions on Antennas and Propagation, vol. AP-35, No.3, March 1987, p.268. However this technique does not allow to obtain acceptable reconstruction quality in the case of improperly stipulated matrices in presence of errors. Maximum Entropy Method (MEM) is well known in the image reconstruction area as a very stable-to-noise algorithm . But classical MEM suitable for reconstruction of only real nonnegative distributions can not be used in the case considered because far-field patterns are described by complex functions. Therefore in this work Generalized form of MEM (GMEM) suitable for reconstruction of functions of any type recently proposed by A.T.Bajkova ("The Generalization of Maximum Entropy Method for Reconstruction of Complex Functions", Astronomical & Astrophysical Transactions, 1991, in press) is used. In this paper a brief description of the GMEM is given. Two effective approaches to reconstruction are considered. Numerical simulation results demonstrating stability to errors of GMEM in comparison to matrix inversion method are shown.

2. Generalized form of Maximum Entropy Method for reconstruction of complex functions

Classical MEM can be represented as the following optimization problem

$$\min \sum_i r_i \ln(r_i), \qquad (1)$$

$$\sum_i r_i a_i^k = A_k, \quad k=1,\dots,M \qquad (2)$$

$$r_i >= 0, \quad i=1,\dots,N, \qquad (3)$$

where a_i^k are constants, A_k are known data, r_i are unknowns.

In the case of complex unknowns it is proposed to represent them in the following way

$$r_i + jq_i = (x_i - y_i) + j(z_i - v_i), \qquad (4)$$

where

$$x_i,\ y_i,\ z_i,\ v_i >= 0 \qquad (5)$$

and minimize the following functional

$$\min \sum_i |r_i|\ln|r_i| + |q_i|\ln|q_i|. \qquad (6)$$

where $|*|$ is absolute value of $*$.

Let x_i, y_i, z_i and v_i meet the conditions

$$\begin{aligned} &\text{if } r_i>0 \text{ then } y_i\Rightarrow 0 \;\Rightarrow\; r_i\cong x_i,\\ &\text{if } r_i<0 \text{ then } x_i\Rightarrow 0 \;\Rightarrow\; r_i\cong -y_i,\\ &\text{if } q_i>0 \text{ then } v_i\Rightarrow 0 \;\Rightarrow\; q_i\cong z_i,\\ &\text{if } q_i<0 \text{ then } z_i\Rightarrow 0 \;\Rightarrow\; q_i\cong -v_i \end{aligned} \qquad (7)$$

in order to exclude solution ambiguity caused by the function of absolute value and representation (4).

If conditions (7) are satisfied expression (6) can be rewritten as

$$\min \sum_i x_i\ln(x_i)+y_i\ln(y_i)+z_i\ln(z_i)+v_i\ln(v_i). \qquad (8)$$

Let (8) be modified as

$$\min \sum_i x_i\ln(\alpha x_i)+y_i\ln(\alpha y_i)+z_i\ln(\alpha z_i)+v_i\ln(\alpha v_i), \qquad (9)$$

where α is a positive parameter which can be chosen to satisfy conditions (6).

Linear constraints derived from known data are

$$\sum_i(x_i-y_i)a_i^k - (z_i-v_i)b_i^k = A_k. \qquad (10)$$

Optimization problem (9),(10),(5) is similar to (1)-(3) and can be solved by using well known methods (for example, Lagrange method). Terminal solution is determined according to (4).

The specific feature of the solution is that x_i, y_i, z_i and v_i are connected by

$$x_iy_i = z_iv_i = \exp(-2-2\ln(\alpha)) = K(\alpha).$$

It is clear that conditions (7) are satisfied and ambiguity of solution is excluded when $K(\alpha)\Rightarrow 0$, i.e. α is of sufficiently large value.

Thus new Generalized form of MEM means special representation of the sought for sequence in form of (4) and solution of optimization problem (9),(10),(5) with the appropriate value of α.

3. Two approaches to reconstruction of far-field patterns

Let complex two-dimensional far-field pattern be sampled at a rate slightly higher than the Nyquist rate. It is required to determine the value of this complex distribution at any point. Such an interpolation problem can be easily solved in the case of uniform sample distribution by using Sinc interpolation functions. In the case of nonuniform distribution it is proposed to reconstruct far field values at uniformly distributed points from given nonuniformly distributed samples by GMEM and then solve the interpolation problem for uniform distribution. Let us consider two approaches to solving reconstruction problem. The first approach assumes reconstruction in the far-field pattern domain and the second one assumes reconstruction in the dual (Fourier) domain.

In the first case corresponding optimization problem may be written as follows (for two-dimensional sequences)

$$\min \sum_m \sum_l x_{ml}\ln(\alpha x_{ml})+y_{ml}\ln(\alpha y_{ml})+z_{ml}\ln(\alpha z_{ml})+v_{ml}\ln(\alpha v_{ml})$$

$$\sum_m \sum_l (x_{ml}-y_{ml})a_{ml}^{nk} = A_{nk}, \quad \sum_m \sum_l (z_{ml}-v_{ml})a_{ml}^{nk} = B_{nk},$$

$$x_{ml},\ y_{ml},\ z_{ml},\ v_{ml} >= 0,$$

$$r_{ml}+jq_{ml}=(x_{ml}-y_{ml}) + j(z_{ml}-v_{ml}),$$

where $r_{ml}+jq_{ml}$ are unknown uniformly distributed samples with coordinates $(\Delta_u m, \Delta_v l)$, where Δ_u and Δ_v are the spacings between adjacent uniform sampled points in the direction U and V of (U-V) far-field domain respectively, (l,m) are integers; $A_{nk}+jB_{nk}$ are known samples at the points with coordinates (α_n, α_k); $a_{ml}^{nk} = \sin(\Delta_u m-\alpha_n)\sin(\Delta_v l-\alpha_k)/((\Delta_u m-\alpha_n)(\Delta_v l-\alpha_k))$.

In the second case optimization problem is

$$\min \sum_m \sum_l x_{ml}\ln(\alpha x_{ml})+y_{ml}\ln(\alpha y_{ml})+z_{ml}\ln(\alpha z_{ml})+v_{ml}\ln(\alpha v_{ml})$$

$$\sum_m \sum_l (x_{ml}-y_{ml})a_{ml}^{nk} - (z_{ml}-v_{ml})b_{ml}^{nk} = A_{nk},$$

$$\sum_m \sum_l (x_{ml}-y_{ml})b_{ml}^{nk} + (z_{ml}-v_{ml})a_{ml}^{nk} = B_{nk},$$

$$x_{ml},\ y_{ml},\ z_{ml},\ v_{ml} >= 0,$$

$$r_{ml}+jq_{ml}=(x_{ml}-y_{ml}) + j(z_{ml}-v_{ml}),$$

where

$a_{ml}^{nk}=\cos(2\pi(m\alpha_n/\Delta_u N1+l\alpha_k/\Delta_v N2))$, $b_{ml}^{nk}=-\sin(2\pi(m\alpha_n/\Delta_u N1+l\alpha_k/\Delta_v N2))$, where N1xN2 is the size of map in pixels; A_{nk} and B_{nk} are real and imaginary parts of measured samples at point (α_n, α_k) respectively. Sought for uniform far-field pattern distribution is found by Fourier transformation of complex sequence $(r_{ml}+jq_{ml})$.

The second approach in comparison to the first one provides extrapolation ability. It means that in the second case the same reconstruction quality can be achieved by lesser number of measured samples.

4. Error studies by numerical simulation

In this section results of the error sensitivity study of both previous technique based on matrix inversion algorithm and presented one based on Generalized Maximum Entropy Method are shown. Nonuniformly sampled U-V grid of far-field pattern of elliptical aperture is depicted in Fig.1. The region of U-V coverage was chosen in order to reconstruct main lobe and two adjacent sidelobes of far field. The simulated random error bounds are ± 0.5 dB in sample magnitude and ±0.03 degree in angular position of samples. Corresponding matrix to be inverted, with elements depending only on sample position points, has condition number 31480. Obviously, the algorithm based on inversion of matrix with such a great condition number is very sensitive to errors. As is seen from Fig.2. this algorithm failed in the case of chosen level of errors. On the contrary, presented technique based on Generalized Maximum Entropy Method provides satisfactory reconstruction quality (see Fig.3).

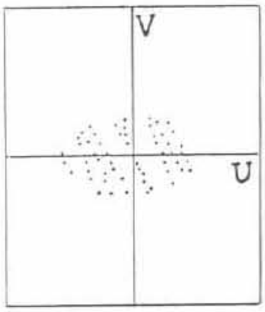

Fig.1.

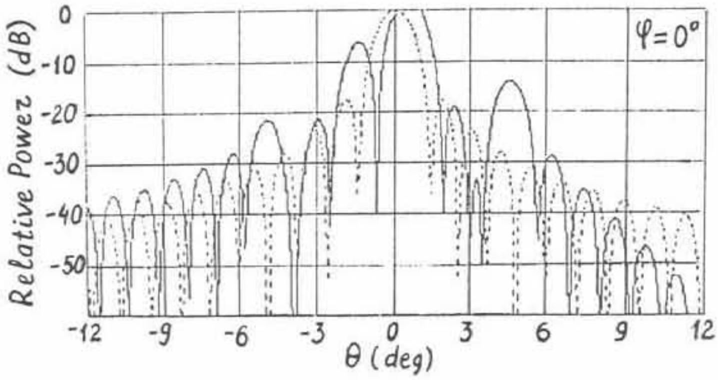

Fig.2.

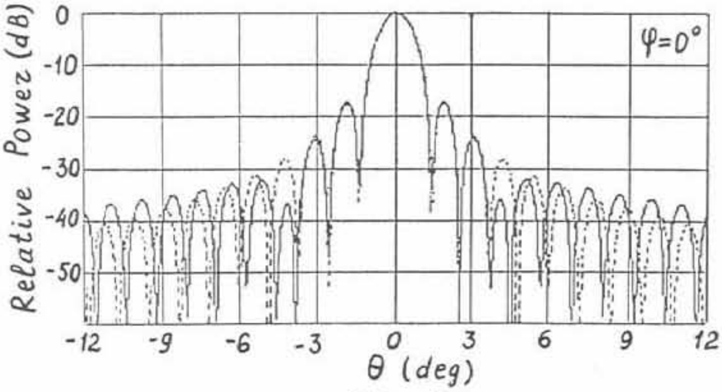

Fig.3.

- - - exact ——— reconstructed

5.Conclusion

In this paper new Generalized form of Maximum Entropy Method suitable for reconstruction of complex functions has been presented for restoring antenna far-field patterns from nonuniformly distributed samples. It has been shown, by numerical simulation, that presented technique is much more stable to measurement errors in comparison to previous one based on matrix inversion algorithm.

6.Acknowledgment

I would like to thank Professor Yahya Rahmat-Samii for help in formulation of the problem and generation of far-field patterns and discussion of results.