ANALYSIS OF FREQUENCY-COMPENSATING ANTENNA ARRAYS

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1. INTRODUCTION - Frequency-independent antennas known at present 1 have low gain and angular resolution. To eliminate these defects the attempts were made to construct arrays of equally spaced identical frequency-independent antenna array elements. But all uniformly spaced antenna arrays (USAA) demonstrate strong dependence of directional properties on the signal frequency caused by periodic character of the array factor of USAA. When the frequency rises the USAA directivity increases at first and then falls sharply when the additional grating lobes arise in the real-angles region. Essential expansion of the operating-frequency range of the array can be achieved by nonuniform spacing of antenna array elements. The property of broadbandness is manifested in the most complete way by nonuniformly spaced arrays which we call the Frequency-Compensating Antenna Arrays (FCAA).

2. FORMULATION OF THE PROBLEM - The linear,planar and three-dimensional FCAA consisting of isotropic frequency-independent antenna array elements (elements, in the following) are considered. The amplitudes of the currents through the elements have the same values and their phases provide the necessary space orientation of the array beam. In the following we shall consider cophased arrays.

Elements of linear, planar or three-dimensional FCAA are located in the cross points of equally spaced grid (one-, two- or three-dimensional, respectively) as it is shown in the Fig. 1. The grid pitches in the directions of x,y,z coordinate axes are d_x, d_y, d_z correspondingly. Space location of the FCAA element with the number m is determined by the vector $(n_{mx}d_x, n_{my}d_y, n_{mz}d_z)$ or simply by three integers (n_{mx}, n_{my}, n_{mz}) where $0 \le n_{mx} \le N_x-1$, $0 \le n_{my} \le N_y-1$, $0 \le n_{mz} \le N_z-1$. If FCAA consists of M elements, its geometry may be described by the sequence of three-dimensional vectors with integer components $\{(n_{mx}, n_{my}, n_{mz})\}_{m=1}^{M}$.

The linear, planar or three-dimensional FCAA is constructed in the following way. The given number of elements is located in some cross points of equally spaced grid containing possibly the least number of cross points - $N_x \cdot N_y \cdot N_z$ (or the most number of elements is located in some cross points of equally spaced grid with required N_x, N_y and N_z), in such a way that the difference of coordinates of two arbitrary elements is unique and is not repeated for any other pair of elements from the same FCAA.

Applying the above principle, e.g., to the linear FCAA consisting of M elements the following two requirements on the integer sequence $\{n_{mx}\}_{m=1}^{M}$ can be formulated [2,3]:

1) every possible difference of its two terms $(n_{px} - n_{qx})$,

where $p,q=1,2,\ldots,M$ and $p \neq q$, occurs only once;

2) the required sequence has the least value of n_{Mx} among all sequences satisfying the preceeding demand 1).

For the synthesis of sequences necessary for constructing the FCAA the certain methods based on Galois field theory [4] may be used.

In the present paper the band and directional properties of FCAA are analyzed.

3. BAND AND DIRECTIONAL PROPERTIES OF FCAA - We have found that location of elements according to the above principle allows to suppress effectively additional grating lobes arising at high frequencies in the real-angles region of the radiation pattern of array with equally spaced elements. Under multiple frequency change the FCAA ensures the uniqueness of the main lobe in the space, constantcy of the peak sidelobe level and directivity. But the FCAA are not to be identified with frequency-independent antennas. In contradiction to the latter the f o r m of the radiation pattern of the former depends on the frequency: when the frequency rises the pattern contracts in the angle and the main-lobe width diminishes.

FCAA Radiation Pattern - The complex normalized radiation pattern of the FCAA is determined by the following expression:

$$F(\mathcal{H}_x, \mathcal{H}_y, \mathcal{H}_z) = \frac{1}{M} \cdot \sum_{m=1}^{M} \exp\left[ik(d_x n_{mx} \mathcal{H}_x + d_y n_{my} \mathcal{H}_y + d_z n_{mz} \mathcal{H}_z\right) , \quad (1)$$

where M is the number of FCAA elements, $k=2\pi/\lambda$ is the wave-number, $\mathcal{H}_x = \sin\theta\cos\varphi$, $\mathcal{H}_y = \sin\theta\sin\varphi$, $\mathcal{H}_z = \cos\theta$, θ and φ are the angles of the spherical coordinate system (see Fig. 1,c). The values of the function (1) on the sphere $\mathcal{H}_x^2 + \mathcal{H}_y^2 + \mathcal{H}_z^2 = 1$ correspond to the real-angles region.

The function (1) is the periodic one which periods in $\mathcal{H}_x$, $\mathcal{H}_y$, $\mathcal{H}_z$ are $T_x=2\pi/(kd_x)$, $T_y=2\pi/(kd_y)$, $T_z=2\pi/(kd_z)$ respectively. The maximal frequency satisfying the condition of uniqueness of the main lobe can be determined using the expression

$$k_{max} \approx 2\pi / \max(d_x, d_y, d_z), \quad (2)$$

which is the same one as for the USAA with the same grid.

Analysis of the module of the expression (1) and calculated results show that $|F(\mathcal{H}_x, \mathcal{H}_y, \mathcal{H}_z)|$ is characterized by approximately uniform and relatively low sidelobes, and their peak level can be described approximately by

$$U \approx \sqrt{C/M} , \quad (3)$$

where $C=2$ for linear FCAA [3] and $C=4$ for planar or three-dimensional one. The typical normalized amplitude patterns $|F(\theta, \varphi)|$ (section by the plane $\varphi=0°$) of the linear and planar FCAA are given in the Fig. 2. For other fixed $\varphi \neq 0°$ the sections of the function $|F(\theta, \varphi)|$ have the similar form in the case of planar (as well as three-dimensional) FCAA.

The main lobe width θ_0 of the FCAA radiation pattern, as well as for USAA, is determined by the dimension of its aperture, i.e. by N_x, N_y, N_z (Fig. 1). It permits the FCAA with small number M of elements to achieve the same angular resolution as the USAA consisting of almost M^2 elements. If $M>50$ ...

...100 then, according to equation (3), U for FCAA has lower value than for the filled uniform antenna array.

FCAA Directivity - The approximate formulae for FCAA directivity D are found allowing to estimate analytically the dependence of the directivity on the frequency and the number of elements:

1) for linear FCAA [3]

$$D \approx M^2/[M-1+\pi/(kd_x)] ; \quad (4)$$

2) for planar FCAA

$$D \approx M^2/[M-1+2\pi/(k^2 d_x d_y)] ; \quad (5)$$

3) for three-dimensional FCAA

$$D \approx M^2/[M-1+2\pi^2/(k^3 d_x d_y d_z)] . \quad (6)$$

Calculated directivity of FCAA depending on the relative frequency $f = k/k_{max}$ are given in the Fig. 3 together with corresponding curves for USAA with the same number of elements located in cross points of the same uniformly spaced grid.

FCAA Range Factor - In the broad frequency band the FCAA directivity remains practically constant and equal to M. The formulae (4)-(6) allow to determine the FCAA range factor K which is equal to the ratio of maximal frequency determined from the equation (2) ($D(k_{max}) \approx M$) to the minimal frequency for which the directivity diminishes to $D(k_{min}) = M\delta$, $\delta<1$:

1) for linear FCAA

$$K \approx 2\,[M(1-\delta)/\delta+1] ;$$

2) for planar FCAA

$$K \approx \sqrt{2\pi\,[M(1-\delta)/\delta+1]} \text{ , when } d_x=d_y ;$$

3) for three-dimensional FCAA

$$K \approx \sqrt[3]{2\pi\,[M(1-\delta)/\delta+1]} \text{ , when } d_x=d_y=2d_z .$$

Thus, K grows proportionally to the number of elements M to the power of 1/n, where n=1,2,3 for linear, planar and three-dimensional FCAA, respectively. For the USAA K does not depend practically on M and is equal approximately to 2.0 ... 2.5 .

4. CONCLUSION - By their nature the FCAA are strongly thinned arrays. Even at the relatively small number of elements (no more than 100) the main lobe width can be equal to some parts of degree at the maximal frequency. The high angular resolution, uniqueness of the pattern main lobe, constant directivity in the broad band of electromagnetic spectrum allow to use FCAA for various purposes, and foremost as a radio telescope antenna.

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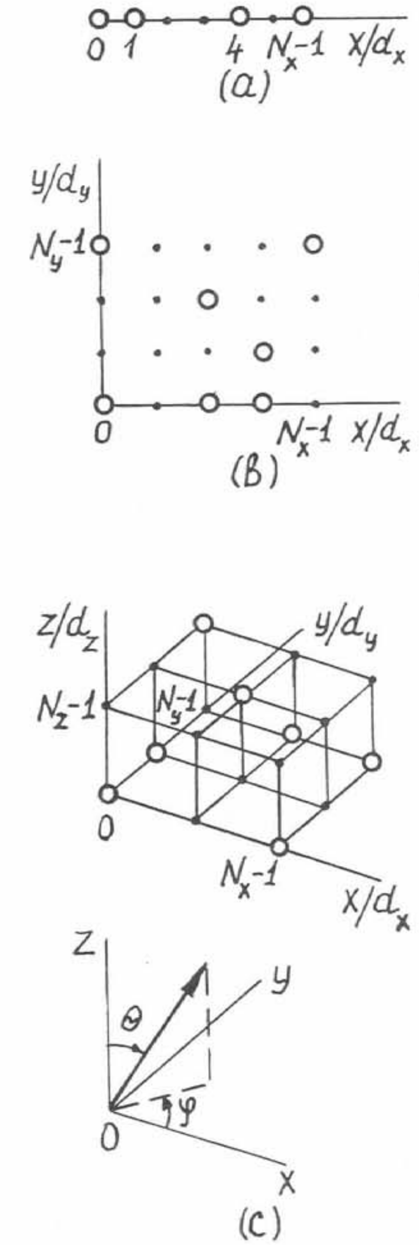

Fig.1. Frequency-Compensating Antenna Arrays:
(a) linear, M=4
(b) planar, M=7
(c) three-dimensional, M=7.

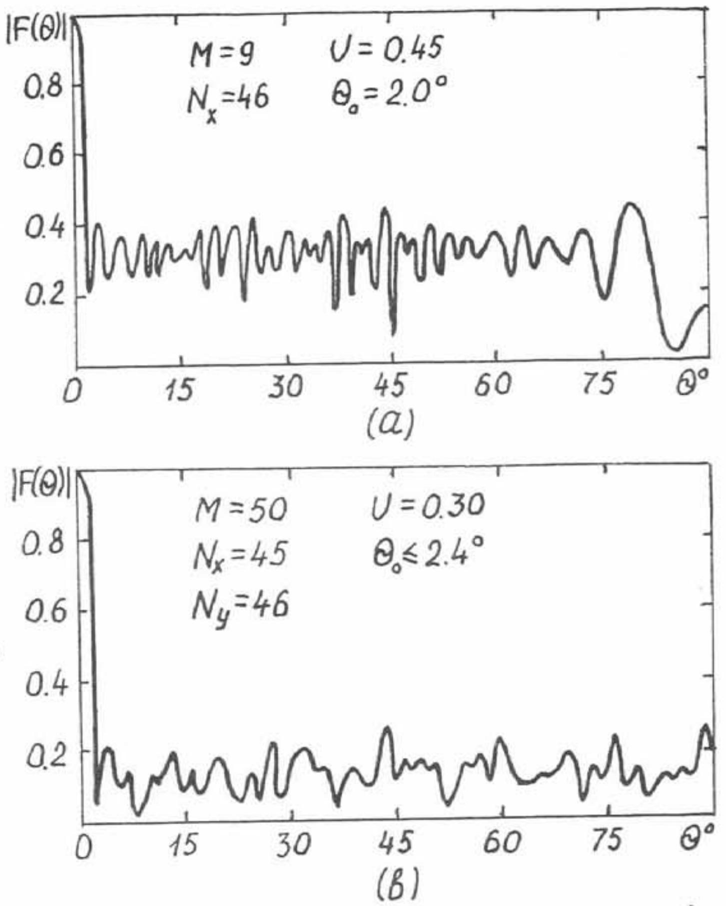

Fig.2. Calculated radiation pattern in ZOX-plane for (a) linear FCAA, (b) planar FCAA.

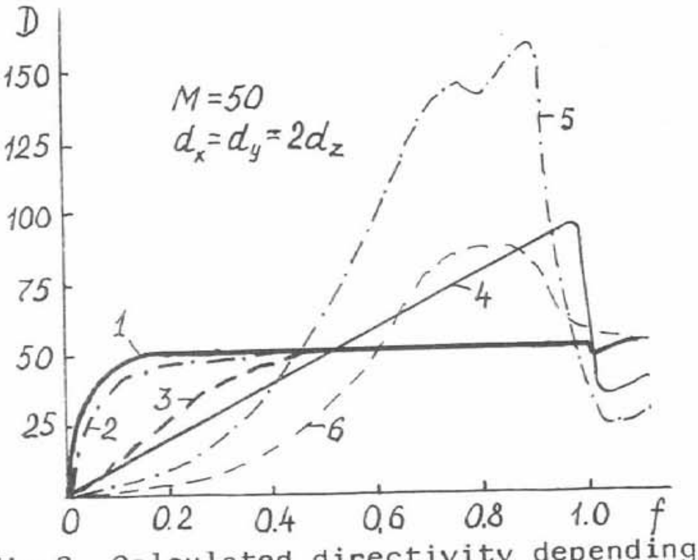

Fig.3. Calculated directivity depending on frequency for linear, planar and three-dimensional FCAA (1, 2 and 3) and USAA (4, 5 and 6).