Matrix Analysis of Nonreciprocal/Control Symmetrical Multi-Way Dividers/Combiners

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1. Introduction

Different types of dividers/combiners are used in microwave region. Relatively not long ago nonreciprocal dividers/combiners (ND/C) and control dividers/combiners (CD/C) with ferrites were suggested, which fulfill two functions simultaneously: dividing/combining and isolating, or dividing/combining and adjustable phase shifting. The use of the divider/adjustable phase shifter allows to reduce the number of phase shifters of phased array antennas.

All of the ND/C and CD/C (N/CD/C) exhibit one or another type of geometrical symmetry. It is possible to simplify the problem of their analysis using the color group approach [1]. The theory of symmetry allows us to reduce the number of independent parameters of the matrices [S], [Z] and [Y] considerably and then to analyse these matrices using the requirement of unitarity. The purpose of this work is analysis of two types of symmetrical ferrite N/CD/C.

2. Multi-way 3-D N/CD/C

Consider multiport 3-D structure (n-way N/CD/C) with ring dc magnetic field produced by a current line (Fig.1a). The structure has geometrical symmetry of a regular pyramid. The ports are connected to the points 1, 2, ..., n+1. In the regime of dividing the port n+1 is input one, and the ports 1,2,3...n are output ones. In the regime of combining the ports 1, 2, 3, ...n are input ones and n+1 is output port.

If there are planes of symmetry going through the n+1 port, we have the magnetic group C_{nv}, and the commutation relation are the following ones [1]:

$$[R]_{\sigma_v}[S] = [S][R]_{\sigma_v} \text{ and } [R]_{C_n}[S] = [S][R]_{C_n},$$

where [R]- the matrices of symmetry, corresponding to generators of the group. As a result of calculation, we may find the scattering matrix:

$$[S] = \left|\begin{array}{ccccc|c} S_{11} & S_{12} & S_{12} & . & . & S_{1,n+1} \\ S_{12} & S_{11} & S_{12} & . & . & S_{1,n+1} \\ S_{12} & S_{12} & S_{11} & . & . & . \\ . & . & . & . & . & . \\ : & : & : & : & : & S_{1,n+1} \\ S_{n+1,1} & S_{n+1,1} & . & . & S_{n+1,1} & S_{n+1,n+1} \end{array}\right|$$

where the matrix inside the dotted lines is the matrix of reciprocal n-port with the symmetry of regular polygon.

For the sake of clarity, apply to the case with n=4, i. e. to a 3-way N/CD/C. The matrix [S] has now 5 independent complex parameters. The condition of unitarity leads to the following result:

$|S_{11}| = (2 \pm |S_{44}|_{min})/3$.

This relation is shown in Fig.1b. The possible $|S_{11}|$ and $|S_{44}|$ are restricted to a region given by the shaded area. The minimum of $|S_{11}|$ is 1/3 with $|S_{12}|$ =2/3 and $|S_{41}|$ =0.

The transferring of power from the port 4 into ports 1, 2, 3 is defined by the equation $|S_{14}|^2 = -3|S_{11}|^2 + 4|S_{11}| - 1$. This equation is plotted in Fig.1c. Realizable values of $|S_{14}|^2$ are in the shaded area. If the port 4 is matched perfectly, $|S_{14}|^2=|S_{41}|^2 = 1/3$, $|S_{12}|^2 = 1/9$, $|S_{11}| = 2/3$. Under this condition, we have $S_{11} = -2S_{12}$. Hence with the port 4 matched, the phase relation $\varphi_{11}=\varphi_{12} \pm \pi$ is satisfied, where φ_{ij} are the phase angles of S_{ij}.

3. Multi-way planar NDC

Let us consider several multiport planar structures on the bases of disc Y-circulator with $E_{\pm 210}$ modes (Fig.2a). The standing wave pattern of the modes $E_{\pm 210}$ has four zeros.

In order to design ND/C, several additional ports may be connected to the points a, b, c (Fig.2a, dotted lines). These devices differ from those described in Section 2 as for example, circulators differ from isolators, since in the regime of dividing reflected from mismatched loads in the output ports waves do not go back into the input port, but go into isolated one.

Here we analyze 3-way ND/C (Fig.2b). In the regime of dividing the input port is port 1 and output ones are 2, 3, and 4 (the continuous arrows on the

Fig.2b) with the power ratios $P_2 : P_3 : P_4 = d_2 : d_3 : 1$. In the regime of summing ports 2, 3 and 4 are input ports, port 5 is output one (the dotted arrows on the Fig.2b), and port 1 is isolated. The scattering matrix elements should meet the following requirements:

$S_{25}=S_{35}=S_{45}=0$, $S_{12}=S_{13}=S_{14}=0$, $|S_{21}|=\sqrt{d_2/A}$, $|S_{31}|=\sqrt{d_3/A}$, $|S_{41}|=\sqrt{1/A}$,
where $A=d_2+d_3+1$.

In order to find the scattering matrix of the ND/C, we use the unitarity condition and the necessary requirements for the regimes of dividing and combining. After some algebraic transformations we come to the following matrix:

$$[S]=\begin{vmatrix} 0 & 0 & 0 & 0 & \exp(j\varphi_{15}) \\ \sqrt{d_2/A}\exp(j\varphi_{21}) & 0 & \sqrt{1/A}\exp(j\varphi_{23}) & \sqrt{d_3/A}\exp(j\varphi_{24}) & 0 \\ \sqrt{d_3/A}\exp(j\varphi_{31}) & \sqrt{1/A}\exp(j\varphi_{32}) & 0 & \sqrt{d_2/A}\exp(j\varphi_{34}) & 0 \\ \sqrt{1/A}\exp(j\varphi_{41}) & \sqrt{d_3/A}\exp(j\varphi_{42}) & \sqrt{d_2/A}\exp(j\varphi_{43}) & 0 & 0 \\ 0 & \sqrt{d_2/A}\exp(j\varphi_{52}) & \sqrt{d_3/A}\exp(j\varphi_{53}) & \sqrt{1/A}\exp(j\varphi_{54}) & 0 \end{vmatrix} \quad (1)$$

This matrix has been deduced without any requirements of symmetry. If the device has antiplane of symmetry (the plane a-a on Fig.2b), for the elements of [S], some relations may be written which reduce the number of independent parameters: $S_{12}=S_{45}$, $S_{21}=S_{54}$, $S_{23}=S_{34}$, $S_{43}=S_{32}$, and so on.

Analysis of the matrix (1) for lossless ND/C with $d_2=d_3=1$ shows that maximum of isolation between output ports in the regime of dividing is 5.2 dB.

Preliminary experiments with a microstrip 3-way nonreciprocal divider using $E_{\pm 210}$ disc resonator gave the following results in centimeter wave region. In the frequency band 5% isolation between output ports and input port is more than 12 dB, insertion losses less than 1 dB, irregularity of division less than 1 dB, isolation between output ports more than 7 dB. No attempts have been made to improve the bandwidth and the parameters of the device yet.

On the bases of this analysis, new N/CD/C may be designed.

References

[1] V.A. Dmitriyev, "Color Group Approach to Matrix Description of Symmetrical Waveguide Junctions and Coupled Lines with Gyrotropic Media," IEEE Trans. Microwave Theory Techn., vol. MTT-43, Dec. 1995.

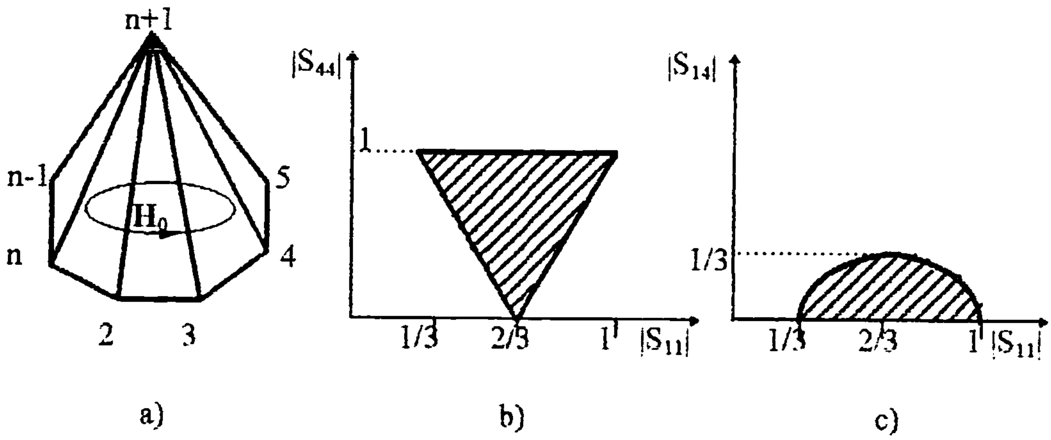
n+1
n-1
5
H_0
n
4
2
3
a)
$|S_{44}|$
1
1/3
2/3
1
$|S_{11}|$
b)
$|S_{14}|$
1/3
1/3
2/3
1
$|S_{11}|$
c)

Fig.1

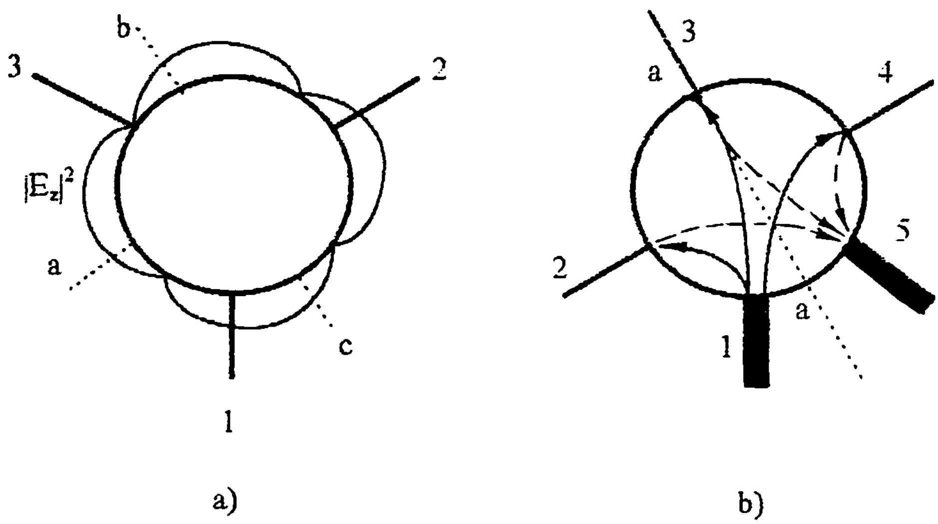
b
3
2
$|E_z|^2$
a
c
1
a)
3
a
4
5
2
a
1
b)

Fig.2