BEAMFORMING UNDER DIRECTIONAL AND SPATIAL DERIVATIVE CONSTRAINTS

M H ER
School of Electrical and Electronic Engineering
Nanyang Technological Institute
Nanyang Avenue, Singapore 2263

1. INTRODUCTION

Optimum beamforming with multiple linear constraints [1-6] is now a well known technique in array processing. In the simplest case, a single constraint is imposed [2] namely unity gain response in the beam steer direction; the weight vectors are then calculated by minimizing the beamformer mean output power subject to this constraint.

In general, in the design of beamforming systems, the choice of the phase center coordinate origin is simply a matter of notational convenience and the resulting structure of the designed system is independent of this location. Surprisingly, this is not the case when constraint involving the derivatives of the beampattern are employed [7].

This paper presents a technique based on a power response approach for deriving a new set of constraints for controlling the beampattern spatial derivatives. The new set of constraints ensure that the array beampattern is independent of the choice of the phase center coordinate origin. It is shown that for the first-order case, the constraint is linear, but for the second-order case, the constraint is quadrative The paper also presents a technique for approximating the effect of the quadratic constraint by two linear constraints. It is shown that using these two set of linear constraints, the dependency of the beampattern on the choice of the phase center coordinate origin can not be totally eliminated, but the effect has been very much reduced.

2. DERIVATION OF A NEW SET OF CONSTRAINTS BASED ON POWER RESPONSE APPROACH

The power response of a beamformer to a plane wavefront of unity amplitude arriving from direction θ is given by

$$\rho(f_o, \theta) = \underline{W}^H \underline{S}_o \underline{S}_o^H \underline{W} \tag{1}$$

where $\underline{W}$ is the N-dimensional complex weight vector given by $\underline{W} = [w_1, w_2, \ldots, w_N]^T$ and $\underline{S}_o$ is the N-dimensional space vector given by $\underline{S}_o = [\exp(j2\pi f_o \tau_1), \ldots, \exp(j2\pi f_o \tau_N)]^T$, where $\{\tau_i, i = 1, 2, \ldots, N\}$ are the propagation delays between the plane wavefront and the antenna elements with respect to some reference point. The superscript H denotes complex conjugate transpose.

In the vicinity of the look direction θ_o, $\rho(f_o, \theta)$ can be expanded as a Taylor series as follows

$$\rho(f_o, \theta) = \rho(f_o, \theta_o) + \Delta\theta \left.\frac{\partial\rho}{\partial\theta}\right|_{\theta=\theta_o} + \frac{\Delta\theta^2}{2!}\left.\frac{\partial^2\rho}{\partial\theta^2}\right|_{\theta=\theta_o} + \ldots \tag{2}$$

It is clear that $\rho(f_o, \theta)$ can be forced to be equal to $\rho(f_o, \theta_o)$ in a maximum flat response sense by setting

$$\left.\frac{\partial \rho}{\partial \theta}\right|_{\theta = \theta_o} = 0 \ , \quad \left.\frac{\partial^2 \rho}{\partial \theta^2}\right|_{\theta = \theta_o} = 0 \ , \quad \ldots \tag{3}$$

Let us now consider the first-order and the second order cases as follows.

For First-Order-Case, it follows from (1) that

$$\frac{\partial \rho}{\partial \theta} = \underline{W}^H \, \underline{S}_1 \, \underline{S}_o^H \, \underline{W} + \underline{W}^H \, \underline{S}_o \, \underline{S}_1^H \, \underline{W} \tag{4}$$

where $\underline{S}_1$ is the first derivative of $\underline{S}_o$ with respect to θ, evaluated at the look direction θ_o. Since $\underline{W}^H \, \underline{S}_o = 1$, it is clear from (4) that the necessary and sufficient condition for $\frac{\partial \rho}{\partial \theta} = 0$ is that

$$\text{Re} \, [\underline{W}^H \, \underline{S}_1] = 0 \tag{5}$$

For Second-Order-Case, it follows from (4) that

$$\frac{\partial^2 \rho}{\partial \theta^2} = \underline{W}^H \, \underline{S}_2 \, \underline{S}_o^H \, \underline{W} + 2\underline{W}^H \, \underline{S}_1 \, \underline{S}_1^H \, \underline{W} + \underline{W}^H \, \underline{S}_o \, \underline{S}_2^H \, \underline{W} \tag{6}$$

Again, since $\underline{W}^H \, \underline{S}_o = 1$ and $\text{Re} \, [\underline{W}^H \, \underline{S}_1] = 0$, it is clear from (6) that the necessary and sufficient condition for $\frac{\partial^2 \rho}{\partial \theta^2} = 0$ is that

$$\text{Re} \, [\underline{W}^H \, \underline{S}_2] + \{I_m \, [\underline{W}^H \, \underline{S}_2]\}^2 = 0 \tag{7}$$

3. APPROXIMATION OF THE EFFECT OF QUADRATIC CONSTRAINT WITH LINEAR CONSTRAINTS

The difficulty with the quadratic constraint defined by (7) is that the constrained optimization problem becomes more difficult to solve. This section presents a technique for approximating the effect of quadratic constraint with two set of linear constraints.

Using real notation, the constraint given by (7) can be expressed as

$$\underline{W}_r^T \, Q \, \underline{W}_r - \underline{P}^T \, \underline{W}_r = 0 \tag{8}$$

where $Q \triangleq \underline{D}_{i_1} \underline{D}_{i_1}^T$, $\underline{P} = \frac{1}{2\pi f_o} \underline{D}_{r_2}$ with $\underline{D}_{i_1}$ and $\underline{D}_{r_2}$ given by $\underline{D}_{i_1} = [\underline{c}^T \Lambda, \ \underline{s}^T \Lambda]^T$, $\underline{D}_{r_2} = \left[2\pi f_o \ \underline{c}^T \Lambda^2 + \underline{s}^T \frac{\partial \Lambda}{\partial \theta} \ , \ 2\pi f_o \ \underline{s}^T \Lambda^2 - \underline{c}^T \frac{\partial \Lambda}{\partial \theta}\right]^T$ where $\underline{c}$ and $\underline{s}$ are the N-dimensional vectors given by, $c = [\cos (2\pi f_o \ \tau_1), \ \ldots \ , \cos (2\pi f_o \ \tau_N)]^T$, $\underline{s} = [\sin (2\pi f_o \ \tau_1) \ , \ \ldots \ , \sin (2\pi f_o \ \tau_N)]^T$ and Λ, Λ^2 and $\frac{\partial \Lambda}{\partial \theta}$ are the N x N dimensional diagonal matrices defined by $\Lambda = \text{diag}\left[\frac{\partial \tau_1}{\partial \theta} \ , \ \ldots \ , \frac{\partial \tau_N}{\partial \theta}\right]$, Λ^2

$$= \text{diag}\left[\left[\frac{\partial\tau_1}{\partial\theta}\right]^2, \ \ldots \ , \left[\frac{\partial\tau_N}{\partial\theta}\right]^2 \right] , \ \frac{\partial A}{\partial\theta} = \text{diag}\left[\frac{\partial^2\tau_1}{\partial\theta^2} , \ \ldots \ , \frac{\partial^2\tau_N}{\partial\theta^2} \right] ,$$

respectively.

It can be shown that the quadratic constraint given by (8) can be approximated by two linear constraints given by

$$\underline{D}_{i_1}^T \, \underline{W}_r = \gamma \quad , \quad \underline{D}_{r_2}^T \, \underline{W}_r = 2\pi f_o \, \gamma^2 \tag{9}$$

where γ is a constant given by $\gamma = \sum_{i=1}^{N} \left[\frac{\partial\tau_i}{\partial\theta}\right]^3 \bigg/ \sum_{i=1}^{N} \left[\frac{\partial\tau_i}{\partial\theta}\right]^2$.

4. NUMERICAL EXAMPLES

To demonstrate the performance characteristics of the narrowhead beamformer with the new set of linear constraints as formulated above, computer studies involving the double-ring circular array shown in Figure 1 have been carried out. The interring spacing was set at 0.25 λ_0.

Figures 2 and 3 show the polar responses using the conventional set of derivative constraints [4, 5] for the case where the phase center location is chosen at the center of gravity of the array and at the first element of the array, respectively. The source scenario was assumed to consist of a 90^o desired signal of power 0 dB and a 180^o interference of power 0 dB. Uncorrelated noise at level of -30 dB was added. It can be seen that for the zero order case, the performance of the processor is not affected by the choice of phase center coordinate origin as expected. But for the first and second order cases, the performances are remarkably affected by the choice of phase center coordinate origin.

Figure 4 shows the polar responses using those set of linear constraints derived in this paper for the case where the phase center location is chosen at the first element of the array. The polar responses for the case where the phase center location is chosen at the center of gravity of the array are same as that in Figure 2. It can be seen from Figure 4 that for the zero and first order cases, the performances of the beamformers are not affected by the choice of the phase center location. For the second order case, the dependency of the beampattern on the choice of the phase center location has been very much referred.

5 CONCLUSION

The paper has presented a technique based on a power response approach for deriving a new set of constraints for controlling the beampattern spatial derivatives. The new set of constraints ensure that the array beampattern is independent of the choice of the phase center coordinate origin. It is shown that for the first-order case, the constraint is linear, but for the second-order case, the constraint is quadratic. The paper then presented a technique for approximating the effect of the quadratic constraint by two linear constraints. It is shown that using these two set of linear constraints, the dependency of the beampattern on the choice of the phase center coordinate origin can not be totally eliminated, but the effect has been very much reduced.

6. REFERENCES

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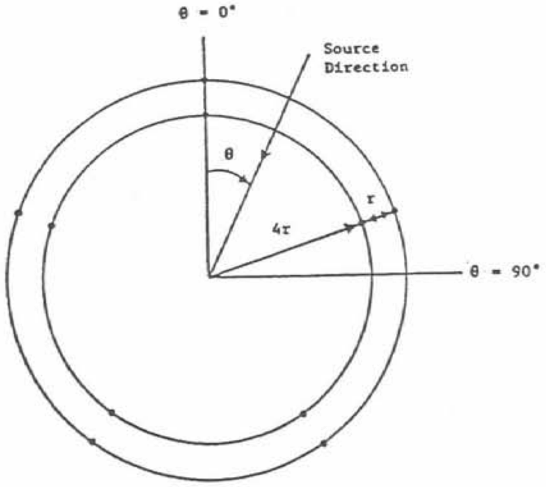

Figure 1

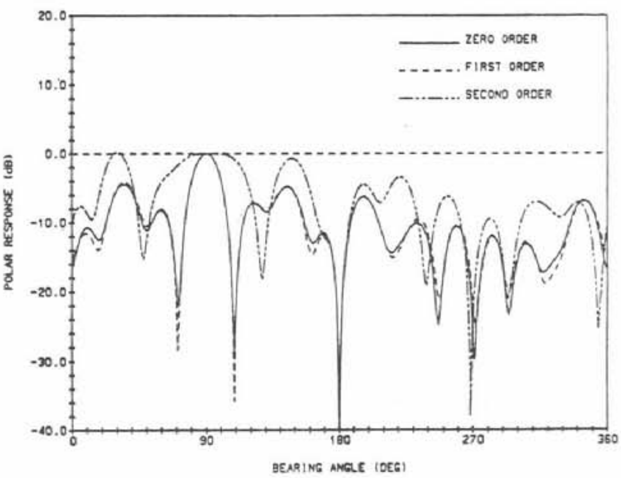

Figure 2

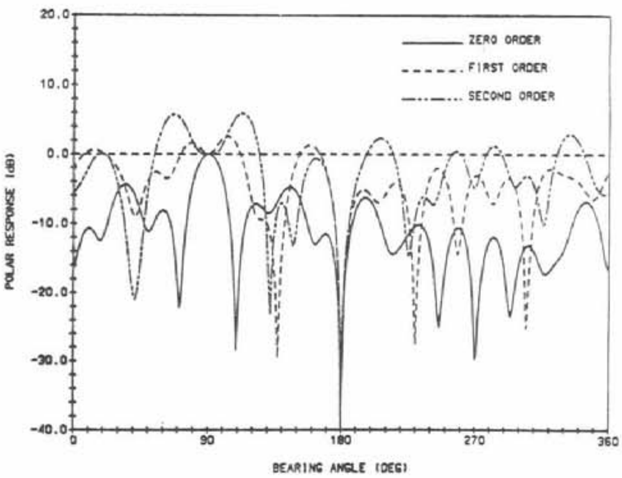

Figure 3

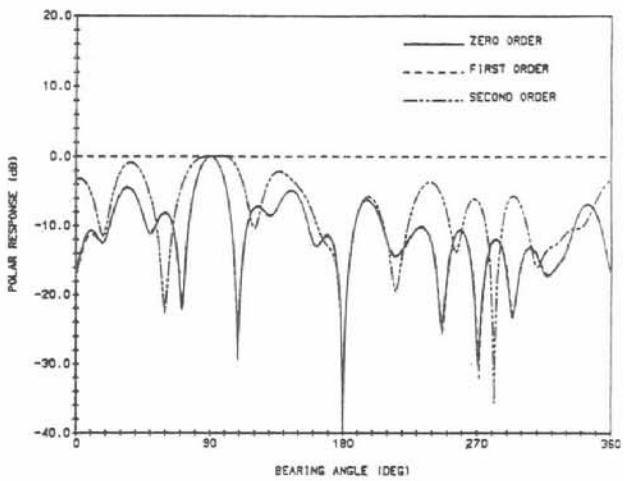

Figure 4