SPATIAL SPECTRAL ESTIMATION USING MULTIPLE BEAM ANTENNAS $^{\star+}$

Joseph T. Mayhan Lincoln Laboratory Massachusetts Institute of Technology

A general class of spectral estimation algorithms have been recently developed aimed at improving the degree of resolution achievable amongst closely spaced spectral components. The traditional application of spectral resolution in the frequency domain obtained from limited samples of data in the time domain has been extended to the linear array of spatial sensors, resulting in improved angular resolution (i.e., much less than λ/D , where D is the array length) of the angular source distribution between two or more source wavefronts incident on the array aperture. Extensions of these techniques to two-dimensional angular estimation have been sparse, and even in these cases have been primarily developed for the array configuration.

The specific application in this paper considers the case where sources of unknown locations, to be determined, exist over an angular field of view (FOV) which is large relative to the half-power beamwidth characterized by the antenna aperture diameter of interest. It is well known that, if HPBW denotes the half-power beamwidth of the maximum directivity beam realizable from the given aperture size, then angular estimates of the source positions over the FOV to within HPBW/10 angular tolerance is feasible, depending on the source signal-noise ratio and measurement component imperfections present in the estimation processor. For our application, the desired resolution and FOV of interest is such that M non-overlapping beams, where M >> 1, of beamwidth HPBW are required to unambiguously cover the FOV in question. Consequently if a two dimensional array configuration is employed, a minimum of M elements are generally required to unambiguously estimate the source scenario. Of course array thinning techniques could be employed to minimize the number of elements required, but for the case considered, even the number of thinned elements would still lead to large processing requirements. As an example, consider a FOV defined by a 36° cone angle relative to the array normal. If source resolution on the order of 0.4° , say, is desired, then an aperture characterized by HPBW $\approx 4^\circ$ is required. This leads to an aperture diameter D, on the order of 60λ ; filling, or even thinning, such an aperture with array elements results in prohibitive processing requirements.

This paper considers the processing simplifications which result in using a multiple beam antenna (MBA) as the spatial sensor for spectral estimation. The MBA configuration consists of an aperture (usually either a reflector or lens) illuminated by a collection of feeds located in its focal plane, followed by a switch network for selecting the outputs of any desired feed-port. Since each feed port, when excited, illuminates the full aperture, a set of narrow, fixed position "spot" beams is produced by selectively exciting the various feeds, where the EPBW of the individual beams is inversely proportional to the aperture diameter, D/λ , expressed in wavelengths. Optimum coverage over a circular area is achieved by arranging the feeds in a hexagonal lattice with the aperture diameter, focal length, number of feeds and feed spacing selected so as to cover the desired FOV. As an example, an aperture diameter of 68λ using a 217 beam feed array provides circular coverage over the 36° cone angle mentioned above with one degree beams.

The two traditional estimation techniques are commonly known as the MEM (Maximum Entropy Method) and the MLM (Maximum Likelihood Method). Two refinements to these have been labeled according to the acronyms MUSIC

(Multiple Signal Classification) and PEGS (Principal Eigenvector – Gram Schmidt). The essential feature of these techniques is readily understood graphically by noting that the two-dimensional spatial spectrum estimate S(u,v) is given by

$$S(u,v) = \frac{1}{\left| \sum_{k=1}^{K} w_k g_k(u,v) \right|^2}$$
(1)

where u and v denote the two dimensional pattern variables

$$u = \frac{\pi D}{\lambda} \sin \theta \cos \phi$$
,

$$v = \frac{\pi D}{\lambda} \sin \theta \sin \phi$$
,

where D = the aperture diameter, λ = the wavelength, θ, ϕ = the pattern angles and K denotes the number of output ports to be processed. The w_k are an appropriately selected set of weight variables unique to each algorithm, and the $g_k(u,v)$ are the response functions for each antenna port (i.e., the radiation pattern). When viewed in terms of antenna nulling, S(u,v) is the inverse of the radiation power pattern placing nulls on the interference sources. Hence the zeros (nulls) of the denominator in Eq. (1) correspond to source location peaks. As is well known, the set $\{w_k\}$ placing nulls on J sources is not unique, accounting for the differences in the various estimation techniques. It should be emphasized that Eq. (1) is a computed function, not measured, based on a knowledge of the $g_k(u,v)$ and as such is an "open-loop" estimate and subject to component and modeling errors.

The architecture proposed here utilizes an M-beam MBA, so that the $g_k(u,v)$ in Eq. (1) are simply the radiation patterns (amplitude and phase) for each beam port. If we denote by J the number of sources present over the FOV, then we will develop an algorithm for which the processing requirements for the MBA depend only on J and not on M. Consequently increasing M and D, (resulting in enhanced resolution) although requiring a larger beam switching network, has little impact on the computational load on the processor.

Development

For the following development, we assume the radiation pattern characteristics of each beam port output are designed so that each beam pattern has, say,less than 40 dB sidelobes outside the first null regions. As a consequence, the hexagonal beam placement results in a maximum of three beams coupled to a given interference source. Define \underline{P} to be the M x l vector characterizing the long-term average of the power present at each beam port output. Then \underline{P} will take the form

$$\underline{P} = \left[\left| \overline{E_{m}} \right|^{2} \right] = \text{col} \left[0, 0, X, X, \dots, X, \dots, 0 \right]$$
 (2)

where E_m denotes the complex output of the mth port (assumed narrowband) the X's denote a power level measurement above the sidelobe threshold, and entries 40 dB below maximum are approximated as zero. Clearly \underline{P} can have at most K \leq 3J non-zero entries. Conventional estimates of the \overline{w}_k in Eq. (1) could be derived from the M x M correlation matrix R given by

$$\underline{\underline{\mathbf{E}}}_{\mathbf{k},\mathbf{q}} = \underline{\underline{\mathbf{E}}_{\mathbf{k}}} \underline{\underline{\mathbf{E}}_{\mathbf{q}}} \star \tag{3}$$

However, processing R in the form of Eq. (3) leads to prohibitive processing requirements due to the large number of beam-ports. Note, however, that due to the disjoint properties of non-adjacent beams, R contains at most K x K entries above some arbitrary threshold. Hence it is useful to transform R into a form where all computations can be performed in the K x K subspace, independent of M.

Define T to be the transformation according to

$$P' = T \cdot P = \operatorname{col} \left[X, X, \dots, X, 0, 0, \dots, 0 \right]$$
 (4)

where all non-zero entries in P are isolated in the first K rows. Similarly define

$$E' = \underline{T} \cdot E \quad . \tag{5}$$

Then the matrix R', defined as E' E'+, must take the form

where the K x K subspace is now evident. Hence only K x K cross-correlations need be measured, and the required K ports to be cross-correlated are characterized by the matrix T. The cross-correlations are performed by a switchable time-shared correlator, and the K switch settings required from the M:l controllable switches are thus controlled by T. Note that now only a fixed M:l switch network is required so that any feed port need be routed to only a single output. Thus the switch design is conventional and straightforward. Thus the spectral estimation algorithms only require computations in the K x K subspace.

It is readily shown that T is a matrix of only K non-zero entries, all of which are unity. Although T^{-1} does not exist, transformation back to "weight space", after performing computations in the K \times K subspace, is given by

$$\underline{\underline{w}} = \underline{\underline{T}}^+ \cdot \underline{\underline{w}}' \tag{7}$$

where the \underline{w}' are the K x K subspace weight controls. As a result, the sum in the denominator of Eq. (1) is only performed over the K beams containing power output above the threshold.

Discussion

Space limitations prohibit detailed development of further processing simplifications. One can show, however that the K x K subspace can be further decomposed into a block diagonal form of approximately J overlapped sub-matrices. A method of processing these sub-matrices will be presented which tends to remove ambiguities inherent in the estimate when sources overconstrain a given area, and further minimize the computational load on the processor. Numerical examples resulting from a simulation program will then be discussed.

^{*}This work has been sponsored by the Department of the Air Force.

[&]quot;The U.S. Government assumes no responsibility for the material presented."