EFFECT OF INITIAL VALUES ON THE LMS ADAPTIVE ARRAY

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INTRODUCTION

Most adaptive arrays adopt feedback approach to attain the optimum weights. Especially, the gradient method is widely used. Generally, in control problems, the selection of the initial values is often a matter of great importance since it very likely affects the transient behavior of the system. Strangely, this problem has not attracted much interest in the field of the adaptive antennas. Some papers set identical weight values on all the elements of the array, some set zeros on all but one weights, some set the uniform magnitude on the weights whose phases are such that produce the main beam pointing to the desired signals, and so on.

This paper considers the case of least-mean-square(LMS) adaptive array operated on the steepest gradient approach [1],[2],[3], and demonstrates how dramatically the transient behavior of the system is affected by the initial weight values.

PRINCIPLE AND ALGORITHM

The guideline of the LMS adaptive array dictates minimization of the error between the actual array output signal and the reference signal. The reference signal is so-called a replica of the desired signal, and must be either given apriori, or learned by the system itself. When the system is adapted, the interference and thermal noise components are expected to be suppressed in the output so that the maximum signal-to-interference-plus-noise ratio(SINR) may be obtained.

In the present discussion, we assume that both the input desired signal and interference are narrow-banded and can be treated by complex expression. Then, we align the inputs and variable weights at the elements of the array and express them in the form of complex column vectors, **X** and **W**, respectively. Thus, the array output, y can be given by

$$y=\mathbf{X}^T\mathbf{W}^*=\mathbf{W}^+\mathbf{X} \tag{1}$$

where superscripts T, * and + denote the transpose, complex conjugate, and complex conjugate transpose, respectively. The optimum steady-state weight vector is given by

$$\mathbf{W}_{opt}=R_{xx}^{-1}\mathbf{r}_{xd} \tag{2}$$

where R_{xx} is the correlation matrix of the input signals and $\mathbf{r}_{xd}$ is the correlation vector between the input signals and the reference signal[1],[3]. They are defined by

$$R_{xx}=E[\mathbf{X}\mathbf{X}^+],\quad \mathbf{r}_{xd}=E[\mathbf{X}d^*] \tag{3}$$

where E[ ] denotes the expectation and d, the reference signal.

By feedback, the optimum weight vector, $\mathbf{W}_{opt}$ is asymptotically reached by the following vector differential equation[1],[3].

$$dW(t)/dt + qR_{xx}W(t) = qr_{xd} \tag{4}$$

or, in the integral form,

$$W(t) = q\int_0^t \{r_{xd} - R_{xx}W(t)\}dt + W(0) \tag{5}$$

where q is the feedback gain. The initial weight vector of our present interest is expressed as $W(0)$ in this context.

ANALYSIS OF PERFORMANCE

We assume that two monochromatic plane waves, the desired signal and interference, are incident on the array. Thermal noise of equal power is assumed to exist at each weight which is statistically independent of each other. Under this radio environment, Eq.(5) can be solved as follows[3].

$$W(t) = W_{opt} + \sum_{k=1}^{2}\{G_k^+W(0)/G_k^+G_k - G_k^+r_{xd}/(\lambda_k G_k^+G_k)\}G_k\exp(-q\lambda_k t) + \sum_{k=3}^{K}\{G_k^+W(0)/G_k^+G_k\}G_k\exp(-q\lambda_k t) \tag{6}$$

where K is the number of elements. λ_k's and G_k's(k=1,...,K) are the eigenvalues and eigenvectors of R_{xx}, respectively. Since R_{xx} is Hermitian, K eigenvectors G_k's are orthogonal with each other. And λ_k's are expressed as follows[3].

$$\begin{aligned}\lambda_1 &= P_n + K(P_s+P_i)/4 + K\sqrt{(P_s-P_i)^2+4P_sP_iA_{si}^2}/4\\ \lambda_2 &= P_n + K(P_s+P_i)/4 - K\sqrt{(P_s-P_i)^2+4P_sP_iA_{si}^2}/4\\ \lambda_k &= P_n \quad (k=3,\cdots,K)\end{aligned} \tag{7}$$

where P_s, P_i, P_n are the input powers of the desired signal, interference, and thermal noise, respectively, and A_{si} represents the normalized array factor. For a linear, equispaced array with the element spacing of a half wavelength, A_{si} is given by

$$A_{si} = \sin\{K\pi(\sin\theta_s - \sin\theta_i)/2\}/[K\sin\{\pi(\sin\theta_s - \sin\theta_i)/2\}] \tag{8}$$

where θ_s and θ_i denote the angles of the arrival of the desired signal and interference, respectively. For the case where the angular separation between θ_s and θ_i is larger than the beamwidth of the array factor of Eq.(8), and also P_n is much smaller than P_s and P_i, the time constants in the third term of Eq.(6) is much larger than the other two in the second term, as is easily seen by Eq.(7). Because of these terms, it takes a very long time for the weight vector to converge to the optimum state. If, however, a wise selection of the initial weight vector is made, so that it may be orthogonal with G_k's(k=3,...,K), the third term vanishes. An example of such weight is the uniform exitation with its mainbeam directed to the desired signal. Here, special attention should be paid that the above analysis is only valid when the degree of freedom of the system, i.e., the number of weights is enough to accomodate more than two eigenvalues. If not, the third term in Eq.(6) does not exist, and the present problem of convergence due to the noise-related time constants does not arise.

COMPUTER SIMULATION

Computer simulation is carried out on a 4-element, 2-tap linear array with isotropic elements and spacing of a half wavelength. The parameters are shown in Table 1. For simplicity, we assume that the exact reference signal is established in the system by some means. The simulation is worked on the sampling feedback system and the algorithm of Eq.(4) must be modified as

$$\mathbf{W}(m+1)=\mathbf{W}(m)+\mu e^{*}(m)\mathbf{X}(m) \quad (9)$$

$$e(m)=d(m)-y(m) \quad (10)$$

where the parenthesis (m) denotes the values at m-th iteration time, and μ is the step size connected with the feedback loop gain q in Eq.(4) by $\mu=qT_s$ where T_s is the sampling interval. "e" is the error, i.e. difference between the reference signal and the array output.

(1) We first consider the following three cases of initial weight vectors setting.

(a) $\mathbf{W}(0)=\mathbf{F}$, the uniform exitation vector with the mainbeam directed to the desired signal.

(b) $\mathbf{W}(0)=\mathbf{M}_1$, the vector with zero weights for all elements except the first one.

(c) $\mathbf{W}(0)=\mathbf{M}_2$, the vector with zero weights for all elements except the second one.

The magnitudes of above weight vectors are such that would produce the desired response in the absence of interference. Fig.1 shows the theoretical learning curves of SINR for the above three cases. By theory, all curves, (a),(b) and (c) must asymptotically reach the same value of SINR, but the rate of convergence in (b) for $\mathbf{M}_1$ and (c) for $\mathbf{M}_2$ are much slower than that in (a) for the uniform exitation vector F. In these situations, the system must suffer from low SINR for a long time before the adaptation completes. As mentioned in the previous section, this is caused by the third term of Eq.(6) which contains large time constants. In contrast, the trouble-making third term vanishes in the case (a) for $\mathbf{W}(0)=\mathbf{F}$, resulting in the fast convergence.

(2) Next, the cases are compared where the magnitudes of the initial weights are wrongly given. Modifying the case (a) in (1), we multiply F by the factor r which ranges from 0 to 10. The results of the learning curves of SINR are shown in Fig.2. The difference of the behaviors with respect to the values of r depends on the remaining first two exponential terms in Eq.(6). In these cases, their time constants are identical, but the magnitudes of the coefficients of those terms differ. The best transient performance is obtained when r =1.

CONCLUSION

This paper considered the effect of the initial values on the transient performance of the LMS adaptive array. It was found that the best choice is the uniform exitation weights with its mainbeam directed to the desired signal and also with good prediction on the magnitude of the incoming signal. Unfortunately, this requirement almost damages the best merit of the LMS system, that it does not need to know these information to attain the optimum weight.

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Table 1 Input data used in the computation

desired signal (S)	$\theta_S = 0°, P_S = 1$
interference (I)	$\theta_i = -50°, P_i = 100$
thermal noise (N)	$P_n = 0.01$
step size $\mu = 0.2/\mathrm{trace}(R_{xx}) = 0.495\times10^{-3}$	

θ ;angle of arrival

P ;power

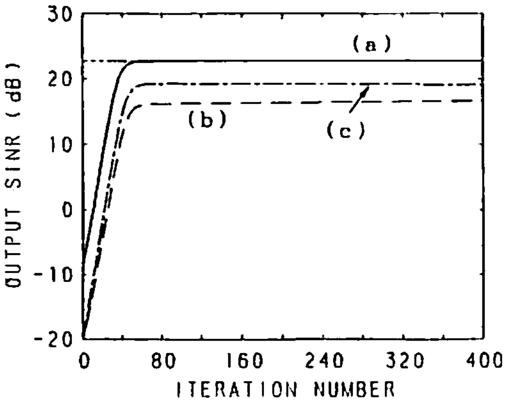

Fig.1

Learning curves of SINR for three initial weights: F, M_1, and M_2

(a) $W(0)=F$
(b) $W(0)=M_1$
(c) $W(0)=M_2$

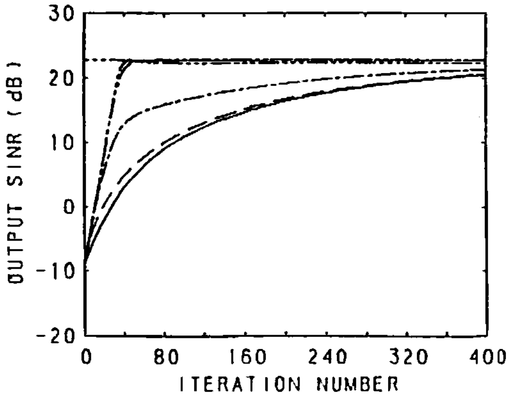

Fig.2

Learning curves of SINR for the various magnitudes of initial weights with uniform exitation: $W(0)=rF$

r= 0
r= 0.01
r= 0.1
r= 1.0
r=10.0