DESIGNING LINEAR ANTENNA ARRAY WITH BROAD SPATIAL NULLS

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1. INTRODUCTION

Array antennas constitute one of the most versatile classes of radiators due to their capacity for beam shaping, beam steering, and high gain. One of the most basic problems in array design is to determine the radiation pattern of an array. Synthesis techniques for determining weights which result in a desired pattern response for a given array have been available for more than 40 years [1]. The great majority of this work has focused on methods which result in patterns having prescribed mainlobe width and reduced sidelobe levels.

In recent years there has been a considerable interest in designing antenna array with broad null sectors [2 - 8]. The need for a broad spatial null often arises when the direction of arrival of the unwanted interference may vary slightly with time or may not known exactly, and where a comparatively sharp null would require continuous steering for obtaining a reasonable value for the signal-to-noise ratio.

In this paper, a technique for synthesising a linear antenna array pattern with prescribed broad nulls is presented. The array pattern synthesis problem is formulated as a least-square null constrained optimization problem. Numerical techniques are also developed for reducing the computational complexity of determining the optimal weight vector. Numerical results are presented to illustrate the performance achievable.

2. PROBLEM FORMULATION AND SOLUTION

Consider a linear array of N isotropic antenna elements with uniform spacing. The antenna far-field pattern is described by

$$G(f_o,\theta) = \underline{W}^H \underline{S}(f_o,\theta) \tag{1}$$

where $\underline{W}$ is the N-dimensional complex weight vector given by $\underline{W} = [w_1, w_2, \ldots, w_N]^T$ and $\underline{S}(f_o,\theta)$ is the N-dimensional space vector given by $\underline{S}(f_o,\theta) = [\exp(j2\pi f_o\tau_1), \ldots, \exp(j2\pi f_o\tau_N)]^T$, where $\{\tau_i, i = 1, 2, \ldots, N\}$ are the propagation delays between the plane wavefront and the antenna elements with respect to some reference point. The superscripts H and T denote complex conjugate transpose and transpose, respectively.

The array pattern synthesis problem is formulated as the following least-square null constrained optimization problem:

$$\underset{\underline{W}}{\text{minimize}}\ (\underline{W} - \underline{W}_o)^H(\underline{W} - \underline{W}_o) \quad , \quad \text{subject to} \quad \underline{W}^H Q\, \underline{W} \le \xi \tag{2}$$

where $\underline{W}_o$ is a pre-determined vector obtained via say Chebyshev synthesis to achieve a specified sidelobe level and Q is the NxN dimensional Hermitian matrix given by

$$Q = \int_{\theta_o-\Delta\theta/2}^{\theta_o+\Delta\theta/2} \underline{S}(f_o,\theta)\underline{S}^H(f_o,\theta)d\theta \tag{3}$$

where $\Delta\theta$ defines the spatial region in the interference direction θ_o over which a broad null is to be formed. ξ (≥ 0) given in (2) defines the mean-square null depth over the spatial region of interest.

The optimization problem defined in (2) can be interpreted as finding a weight vector which is to be as close to $\underline{W}_o$ as possible subject to an integrated power constraint over the nulling sector.

Using the method of Lagrange multiplier, it can be shown that the optimal weight vector $\underline{W}$ which solves (2) is given by

$$\hat{\underline{W}} = (I + \alpha Q)^{-1}\underline{W}_o \tag{4}$$

where $\alpha \geq 0$ is the Lagrange multiplier for the quadratic constraint and is the solution of the following equation

$$\underline{W}_o^H (I + \alpha Q)^{-1}Q(I + \alpha Q)^{-1}\underline{W}_o = \xi \tag{5}$$

The computational complexity of determining α that satisfies (5) can be reduced significantly by using the matrix factorization method described next.

Since Q is a Hermitian matrix, it can be factorized as, $Q = \Gamma \Lambda \Gamma^H$, where $\Gamma = [\underline{E}_1, \underline{E}_2, \ldots, \underline{E}_N]$ is the NxN dimensional unitary matrix and $\Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_N]$ is the NxN dimensional diagonal matrix.

Using the factorization of Q, (5) can be expressed as

$$\underline{W}_o^H \Gamma(I + \alpha\Lambda)^{-1}\Lambda(I + \alpha\Lambda)^{-1}\Gamma^H \underline{W}_o = \xi \tag{6}$$

Any root finding method can be used to solve for α that satisfies (9). This was further found to be made very efficient by using the half interval method followed by the regula falsi algorithm [9], as it has shown excellent convergence properties.

3. APPROXIMATION OF QUADRATIC CONSTRAINT BY A SET OF LINEAR CONSTRAINTS

In the previous section, a numerical method based on a matirx factorization is used to solve for the optimal Lagrange multiplier, for reduction of the computational load. This section presents another approach to this by approximating the quadratic constraint defined in (2) by a set of linear constraints, also known as eigenvector constraints.

When Q has full rank or because, in practice, the eigenvalue evaluation will not yield exactly zero eigenvalues, one can impose n_o linear constraints of the form

$$\underline{W}^H\underline{E}_i = 0 \quad , \quad i = 1, 2, \ldots, n_o \tag{7}$$

where n_o is the smallest integer such that the percentage trace of the Q matrix defined by

$$\%tr \overset{\Delta}{=} \left(\sum_{i=1}^{n_o} \lambda_i \Big/ \sum_{i=1}^{N} \lambda_i \right) \times 100\% \tag{8}$$

is greater than or equal to some threshold value.

Note that (7) can be thought of as a set of constraints that yield a small value for $\underline{W}^H Q \underline{W}$. The value of n_o determines the number of degrees of freedom lost.

Replacing the quadratic constraint defined in (2) by the set of linear constraint defined in (7), and using the method of Lagrange multiplier, it can be shown that after some simplification the optimal weight vector is given by

$$\hat{\underline{W}} = [I - D\, D^H]\underline{W}_o \tag{9}$$

where D is the N x n_o dimensional matrix given by $D = [\underline{E}_1, \underline{E}_2, \ldots, \underline{E}_{n_o}]$

4. NUMERICAL RESULTS

To demonstrate the performance characteristics of the new design approach, computer studies involving a linear array having 20 equally spaced elements have been carried out. The interelement spacing is set at $0.5\lambda_o$. $\underline{W}_o$ in (2) is obtained by using Chebyshev synthesis to have a sidelobe level of -30 dB.

Figure 1 shows the polar response using the quadratic constraint approach. A controlled broad null at $\theta_o = 30^o$ with $\Delta\theta = 5^o$ (centred at 30^o) and mean-square null depth $\xi = 10^{-5}$ is designed. It is clearly shown that deep nulls are achieved over the spatial region of interest at the prescribed null position.

Figure 2 illustrates the polar response using eigenvector constraints. The design parametes are the same as that in Figure 1 except that the quadratic constraint is approximated by a set of eigenvector constraints. The value of n_o is chosen such that the percentage trace defined by (8) is $\geq$ 99.99%. It can be seen that perfect null depth of over 60 dB is achieved at the prescribed null position over the spatial region of interest.

5. CONCLUSION

This paper has presented a new technique for synthesising linear antenna array pattern with broad spatial nulls. The optimal weight vector is obtained by matching to a pre-determined vector in a least-square sense subject to an integrated power constraint over a nulling sector. Numerical techniques based on the matrix factorization method have been proposed for reducing the computational complexity of determining the optimal weight vector. Subsequently, a set of eigenvector constraints are used to approximate the effect of the quadratic constraint. Numerical results showed that the proposed techniques are very effective in the design of an antenna pattern, with a specified controlled null width and null depth.

6. REFERENCES

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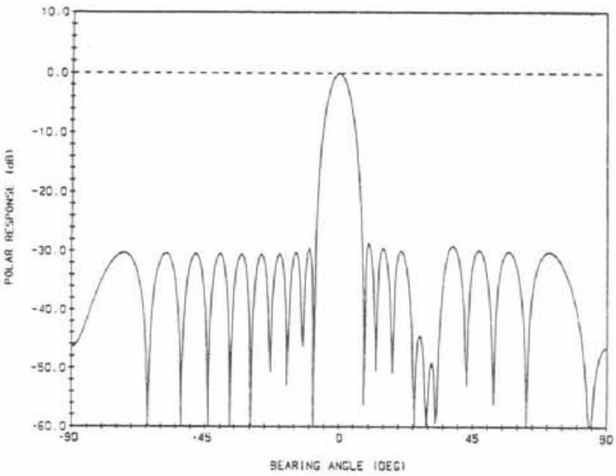

Figure 1

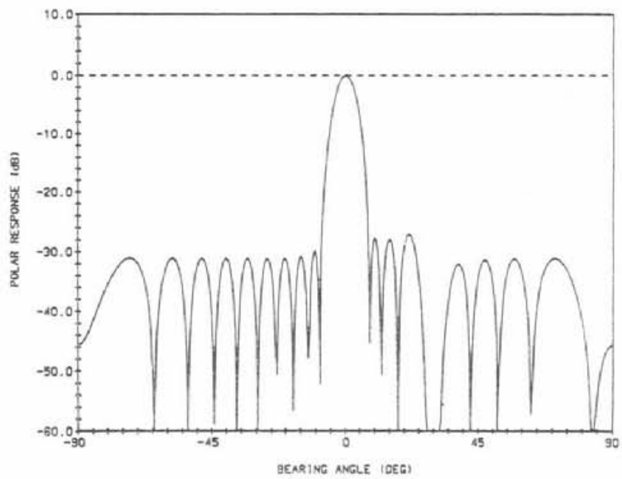

Figure 2