NULL PLACEMENT IN ARRAY ANTENNA PATTERNS

H. Steyskal, R. Haupt, R. Shore
Electromagnetic Sciences Division
Rome Air Development Center
Hanscom AFB, MA 01731, USA

1. INTRODUCTION

The increasing pollution of the electromagnetic environment makes careful antenna pattern sidelobe control imperative, and therefore synthesis methods, which provide the additional capability of placing one or more nulls in the pattern at specified directions, are attracting interest. In an array antenna the pattern control can be achieved in various forms: control of both amplitude and phase of each array element, control of element phase only, and controls only at a select subset of elements or at the subarray level. We have studied these approaches and a summary of our results is presented below.

A linear array of 2N + 1 isotropic equispaced elements is considered, which has the pattern

$$f(u) = \sum_{-N}^{N} a_n e^{jnkdu} \tag{1}$$

where a_n, k, d, u denote the element excitation, the wave number, the interelement spacing and the sine of the angle θ from broadside, resp. The method for pattern null synthesis starts from a given original pattern $f_o(u)$, with desired main beam and average sidelobe level, corresponding to given original element coefficients $\{a_{on}\}$. These coefficients are then perturbed such that the perturbed pattern has nulls at the desired directions. We limit ourselves here to the important special case of real and symmmetric coefficients a_{on} and hence to real symmetric patterns f_o.

2. PATTERN NULLING WITH FULL AMPLITUDE/PHASE CONTROL

For the case with full amplitude and phase control we define the desired pattern f(u) mathematically by the criterion

$$\begin{cases} f(u_m) = 0 & m = 1,2,\ldots, M \qquad (2a) \\ \sum_n c_n |a_n - a_{on}|^2 = \text{minimum} & \qquad (2b) \end{cases}$$

Eq. (2a) ensures that the synthesized pattern has nulls at the desired directions u_m and (2b) ensures that the array excitation is only minimally perturbed. The real positive, symmetric weight coefficients c_n add flexibility to the criterion. The solution for the perturbed element coefficients is [1]

$$a_n = a_{on} - \sum_{m=1}^{M} \gamma_m (1/c_n) e^{-jnkdu_m} \tag{3}$$

where the beam coefficients γ_m are determined from an Mth order system of linear equations. Hence the perturbed pattern is

$$f(u) = f_o(u) - \sum_{m=1}^{M} \gamma_m \sum_n (1/c_n) e^{jnkd(u-u_m)} \tag{4}$$

Note that in (4) each term in the sum over m represents one cancellation beam with amplitude coefficient γ_m and the direction u_m of the desired null. The null synthesis problem thus has only the dimensionality M, rather than the dimensionality 2N+1 of element space. The weight coefficients c_n determine the shape of the cancellation beams, $c_n = 1$ leads to sinc-beams, $c_n = 1/a_{on}$ leads to beams which are replicas of the original pattern f_o. A Chebyshev pattern with four closely spaced nulls, so as to create a low sidelobe sector, is shown in Fig. 1.

Normally for an array, a point source with large bandwidth will appear to be smeared out over a wide angular pattern sector. Thus a pattern trough, as in Fig. 1, is tantamount to a wideband pattern null. Based on the above synthesis method, we have derived an estimate [2], Fig. 2, for the number of nulls required to produce any desired wideband sidelobe suppression. This useful number is indicative of how many degrees of freedom a conventional array with full amplitude/phase control must allocate to attain a specific nulling performance.

3. PATTERN NULLING WITH CONTROL OF PHASE ONLY

In the case where only phase perturbations of the original excitation are allowed, in criterion (2) we replace a_n by $a_{on}\exp j\phi_n$, where $\{\phi_n\}$ are the desired phase perturbations. This results in a nonlinear problem which does not necessarily have a solution. However, when a solution exists, the phase perturbations can be written in the form

$$\phi_n = \text{phase}\left[a_{on} - \sum_{m=1}^{M} (\gamma_m/c_n) e^{-jnkdu_m}\right] \qquad (5)$$

which in structure is remarkably similar to (3) above. Eq. (5) shows that again the problem is of dimensionality M only. In addition it can be shown [3] that the phase perturbations are odd symmetric $\phi_{-n} = -\phi_n$, and hence the γ_m are real. When discussing the resulting patterns it is proper to distinguish the cases of small and of large phase perturbations.

3a. SMALL PHASE PERTURBATIONS

Placing a relatively small number of nulls ($M << 2N+1$) in a region of low sidelobes constitutes a relatively modest pattern perturbation and therefore the associated phase perturbations will also be small. In this case, via a two term Taylor expansion of the phase terms $\exp j\phi_n$, the problem can be linearized and solved analytically, [1], [4].

The resultant pattern can again be viewed as the original pattern on which M cancellation beams have been superimposed. This time however, each cancellation beam consists of a beam pair, where the two members have opposite sign and are directed at u_m and $-u_m$, resp. The coefficients c_n determine the shape of each member in the same way as before. A pattern with one imposed null at $\theta = 15.23°$ and the corresponding cancellation beam is shown in Fig. 3. Note the sidelobe level increase at $\theta = -15.23°$ caused by the second member of the cancellation beam pair.

This approximate null synthesis method is satisfactory for most situations. However, it can not realize two symmetric nulls due to the odd symmetry of the cancellation beams.

3b. LARGE PHASE PERTURBATIONS

Large phase perturbations are required when a null is imposed in the main beam vicinity, when two nulls are imposed symmetrically at $u = \pm u_j$, when multiple nulls are imposed within a relatively narrow angular sector, or when the number M of imposed nulls increases beyond $M << N$. In these cases we have solved the phase-only null synthesis problem numerically [5].

One example of a pattern with two symmetrically imposed nulls is shown in Fig. 4. Note that, curiously, the symmetry of the original pattern has been lost. Another example is shown in Fig. 5, where four nulls have been symmetrically imposed over the first sidelobe of a sinc-pattern so as to create a wideband null. The cost is both main beam gain and increased sidelobe level.

4. NULLING WITH LIMITED NUMBER OF DEGREES OF FREEDOM

Frequently the number of desired nulls M is much smaller than the N-1 nulls inherent in an N-element array, and then synthesis methods with a reduced number of degrees of freedom become of interest. Two approaches are to implement complex amplitude control at either selected elements in the array aperture or at the subarray level. For these cases we have again applied a least mean square criterion similar to (2) and solved the problem analytically [6].

Figure 6 shows the result of imposing nulls on the fifth and eighth sidelobe in the pattern of a 20-element array with complex amplitude control at elements 1,2,19 and 20. The original pattern, which corresponds to a sampled 30 dB Taylor distribution, is surprisingly little perturbed in this case. The control location at the array edge appears to be optimal.

Next, the array was divided into four subarrays, five elements each, and the same pattern nulls were imposed at the subarray level. The resultant pattern, shown in Fig. 7, is considerably more perturbed than the one above. The explanation is that the imposed nulls lie in the sidelobe region of the subarray patterns, whose main beams are aligned with the original main beam direction. The desired pattern cancellation is therefore affected with subarray sidelobes, which leads to subarray weights of large magnitude. Consequently, in the directions of the main beams and the grating lobes of the subarrays, large perturbations of the original pattern occur.

We conclude that for nulls in the sidelobe region, nulling with selected elements in the aperture is the preferred approach.

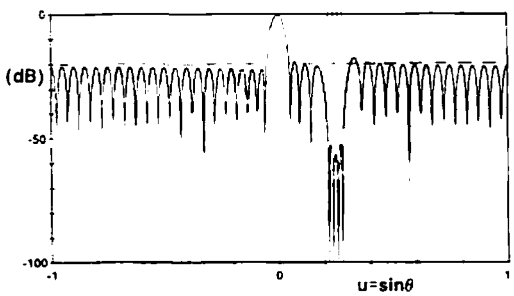

Fig.1. Perturbed 20dB-Chebyshev pattern w. four nulls imposed at u=0.22,0.24,0.26 0.28. Sidelobe cancellation is 32 dB. $2N+1=41$, $d=\lambda/2$, $c_n=1$.

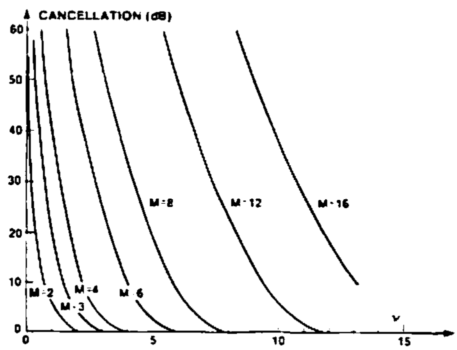

Fig.2. Sidelobe cancellation versus number of equispaced pattern nulls M and desired number of cancelled sidelobes ν.

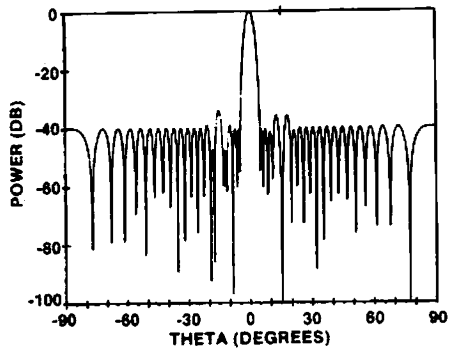

Fig.3a. Perturbed 40dB-Chebyshev pattern w. one null imposed at 15.23° w. phase-only Perturbations. $2N+1=41, d=\lambda/2, c_n=1/a_{on}^2$.

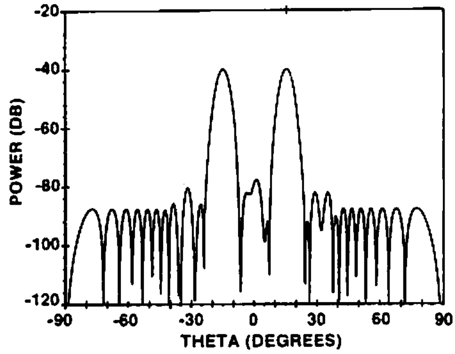

Fig.3b. Cancellation pattern.

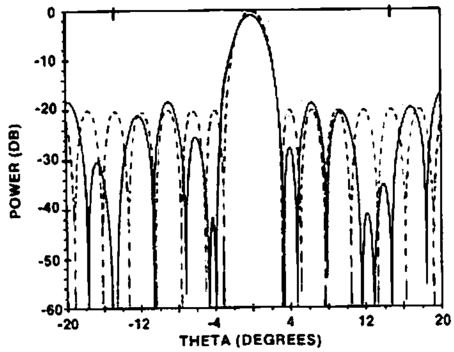

Fig.4. Original 20dB-Chebyshev pattern (--) and perturbed pattern (—) w. nulls imposed at ±14.7°. $2N+1=41$, $d=\lambda/2$, $c_n=1$.

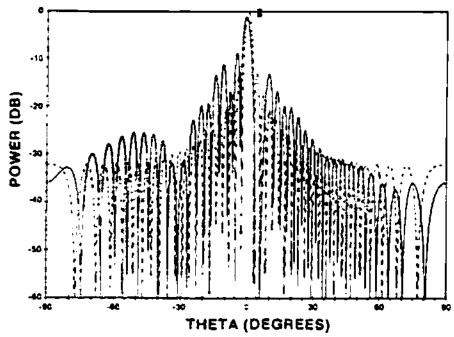

Fig.5. Original sinc-pattern (--) and perturbed pattern (—) w. nulls imposed at 4.0, 4.6, 5.2 and 5.8°. $2N+1=41$, $d=\lambda/2$, $c_n=1$.

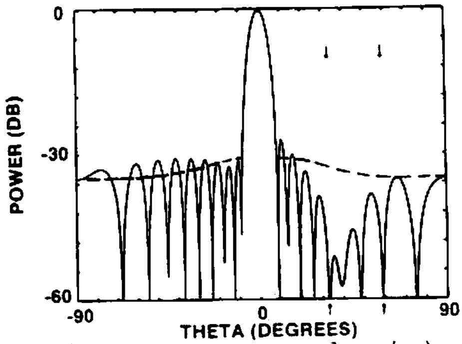

Fig.6. Original pattern envelope (--) and perturbed pattern (—) w. nulls imposed at 33 and 58°, using 4 elements w. variable weights.

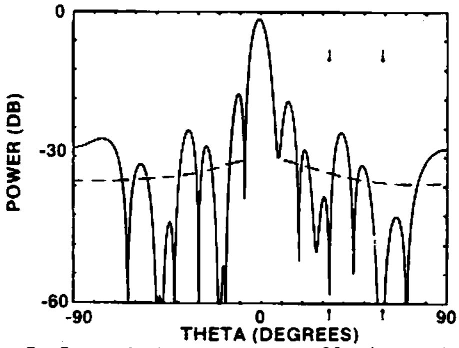

Fig.7. Perturbed pattern w. nulls imposed at 33 and 58°, using 4 subarrays w. variable weights.

5. REFERENCES

[1] R. Shore, H. Steyskal, RADC-TR-82-32, Feb. 1982.

[2] H. Steyskal, IEEE Trans. AP-31, 159-163, 1983.

[3] R. Shore, IEEE Trans. AP-32, 528-530, 1984.

[4] H. Steyskal, IEEE Trans. AP-31, 163-166, 1983.

[5] R. Shore, RADC-TR-83-124, May 1983.

[6] R. Haupt, RADC-TR-83-114, Apr. 1983.