# TRANSIENT ANALYSIS OF DIFFERENTIAL INTERCONNECTS BY SPICE-LIKE CIRCUITS ACCOUNTING FOR FREQUENCY-DEPENDENT LOSSES

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**Abstract** – The propagation of high-speed differential signals on interconnects is simulated by means of SPICE-like equivalent circuits. Both conductor and dielectric losses are accounted for, along with the effects of dispersion. The suitability and the accuracy of the proposed models for the extraction of typical information required for the design of interconnect, e.g. the eye-diagram and the scattering parameters, is investigated.

**Key words**: Interconnects, microstrip lines, differential signalling, SPICE circuits, frequency-dependent losses, effective permittivity.

## 1. Introduction

The full-wave numerical analysis of complex configurations, at the on-board or on-chip level, appears as the only reliable way of taking all distributed effects, e.g. couplings and radiation, into account. However every adjustment, either geometrical and physical, which commonly occurs during the design procedure, would require a new full-wave simulation, resulting in a overwhelming computational effort and in a lengthy process often requiring some manual operations of trained and expert personnel. Moreover the simulation of a complete interconnect path could require more computer resources than those available.

In this framework, the extraction of equivalent lumped SPICE-like models to approximate the electromagnetic behaviour of interconnects gain a paramount importance in the preliminary design stage for their friendly and efficient use.

The main goal of this paper is to propose improved equivalent models of differential interconnects for digital signal transmission accounting for both conductive and dielectric losses, along with frequency dispersion. In the wake of previous studies [1], a careful analysis has been conducted aimed at assessing the range of validity of the approximate approach, with special attention to its accuracy and suitability.

Critical aspects in the simulation concern the accurate modelling of the influence of both dielectric and conductive losses on crucial parameters either preliminary, like the effective dielectric constant and the characteristic impedance, intermediate, like the scattering parameters, or tightly linked to the final use and performance of the interconnect, like the eye-diagram.

## 2. Characterization of Differential Interconnects

The evaluation of the per unit length (p.u.l.) parameters of differential interconnects is traditionally performed by numerical methods such as the finite element method or the method of moments [2]. Nevertheless, the calculation by numerical methods of the interconnect p.u.l. impedance and admittance, for a wide frequency range, would require a great computational effort. To overcome this problem, an efficient mixed analytical/numerical approach is adopted and presented in the following.

### 2.1 Definition of p.u.l. Impedance

The p.u.l. impedance of the differential interconnect shown in Fig. 1 is defined as:

$$Z'(\omega) = Z'_i(\omega) + j\omega L'_e \tag{1}$$

where $Z'_i(\omega)$ is the frequency-dependent internal impedance accounting for skin effect, and $L'_e$ is the external inductance. The internal impedance $Z'(\omega)$ can be derived by approximate analytical expressions [2], while the external inductance $L'_e$ is calculated by the procedure proposed in [3].

### 2.2 Definition of p.u.l. Admittance

The p.u.l. admittance of the differential interconnect is defined as:

$$Y'(\omega) = G'(\omega) + j\omega C'(\omega) \tag{2}$$

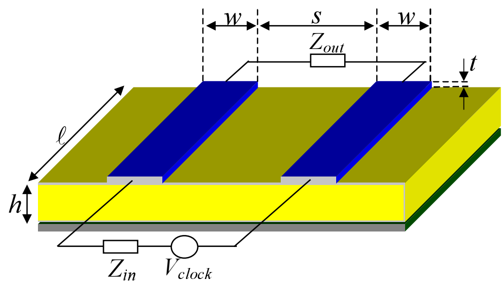



*Fig. 1 – Configuration of a typical differential interconnect.*

where $G'(\omega)$ and $C'(\omega)$ are the differential frequency-dependent conductance and capacitance, respectively, which permit to account for dielectric losses. The p.u.l. capacitance $C'(\omega)$ is given by

$$C'(\omega) = \sqrt{\varepsilon_{eff}(\omega)} / \left[c_0 Z_0^{odd}(\omega)\right] \quad (3)$$

where $\varepsilon_{eff}(\omega)$ is the so-called effective relative dielectric constant, key parameter in any analytical study due to its influence on several design parameters, e.g. the characteristic impedance and wave propagation constant, and $Z_0^{odd}(\omega)$ is the frequency-dependent differential characteristic impedance which can be computed as in [4]. The effective permittivity of the dielectric substrate can be obtained by means of the Kobayashi expression modified as in [1]

$$\varepsilon_{eff}(\omega) = \varepsilon_r(\omega) - \frac{\varepsilon_r(\omega) - \varepsilon_{eff}(0)}{1 + (f/f_{50})^m} \quad (4)$$

which models the dispersive behaviour of the structure due to the rise of modes of higher order at the increasing frequencies, when the wavelength becomes comparable to the cross-section dimensions. The analytical expressions for m and $f_{50}$ in the odd-mode case have been proposed in [1] and are not reported here for the sake of conciseness. The dielectric constant $\varepsilon_r(\omega)$ has been computed adopting the Debye model, whose validity has been demonstrated by extensive experimental measurements on typical commercial materials. A single-pole form is considered, since it is generally accurate enough for practical purposes. The values of the model – i.e. the zero-frequency relative permittivity $\varepsilon_s$, the relative permittivity at infinite frequency $\varepsilon_\infty$, and the pole relaxation time $\tau$ – can be derived from measured data available in the data-sheets of commercial products.

The equivalent transversal conductance $G'(\omega)$ is readily computed as [1]

$$G'(\omega) \cong \frac{2C'(\omega)10^{-\alpha_d(\omega)/20}}{\sqrt{L'_e C'(\omega)}} \quad (5)$$

where $\alpha_d(\omega)$ is the attenuation constant of the odd mode in dB per unit length which depends on the loss tangent $\tan[\delta(\omega)] = \varepsilon''(\omega)/\varepsilon'(\omega)$ of the dielectric substrate [1].

## 3. SPICE Like Circuit Models of Differential Interconnects

Two different techniques to account for frequency-dependent losses are compared. The frequency dependent p.u.l. parameters are used to build up SPICE like circuits suitable for transient analysis. The first circuit is based on the use of the lossy TL model available in the SPICE library. The second equivalent circuit is based on the Π-type representation of the differential interconnect; the modellization of the various admittances by simple RLC circuits is obtained by applying a vector fitting procedure.

### 3.1 Equivalent Circuit Based on SPICE Lossy TL Model

SPICE provides an efficient facility for the simulation of lossy lines by means of a distributed model, allowing freedom from having to determine how many lumps are necessary for accurate results and eliminating possible spurious oscillations. The TLOSSY analog device requires the specification of the p.u.l. capacitance and inductance as constants, but allows the specification of the p.u.l. resistance and conductance as general Laplace expressions. In this way frequency-dependent effects can be modelled, such as skin effect and dielectric losses, whose simulation will be plainly explained, being the main goal of the paper.

The modelling of the distributed losses has been computed implementing the formulas presented in the previous sections; the frequency-dependent Debye and the modified Kobayashi models for the characterization of the dielectric substrate, and the analytical expressions given in [2] for the conductive losses.

The p.u.l. differential capacitance and conductance read respectively

```
.FUNC C(s) { 1/C0*sqrt(EPSeff(s))/Z0}
.FUNC G(s) {2*C(s)*ALFA(s)*pwr(sqrt(Le*C(s)),-1)+s*(C(s)-CI)}
```

where $s$ is the Laplace variable, EPSeff(s) is the effective relative dielectric constant as in (4), Le is frequency-independent external p.u.l. inductance and CI is the constant capacitance corresponding to the infinite frequency permittivity. It should be noted that, in the definition of the function G(s), it has been added the last term which is necessary to model the frequency-dependence of the p.u.l. capacitance, since the capacitance must be set as constant in the TLOSSY model.

The conductive losses have been accounted for by means of a frequency-dependent p.u.l. resistance which reads

```
.PARAM F0={ (1/2*(w*t)/(w+t))**(-2) / (PI *MU0 *sigma)}
.FUNC RL(s) { Rdc*(1+JOTA*F(s)/F0) }
.FUNC RH(s) { Rdc * sqrt(F(s)/F0) * (1+JOTA) }
.FUNC UNIT_STEP_P(s) {0.5*(1+sgn(abs(F(s))-abs(F0))) }
.FUNC UNIT_STEP_N(s) {0.5*(1+sgn(-abs(F(s))+abs(F0)))}
.FUNC R(s) {RL(s)*UNIT_STEP_N(s)+RH(s)* UNIT_STEP_P(s)}
```

where RL and RH are respectively the low and high frequency parts of the p.u.l resistance and F0 is the cut-off frequency. Finally the TLOSSY analog device has been defined as

```
TLOSSY 2 0 3 0 LEN=1e-2 R={R(s)} G={G(s)} L={LI} C={CI}
```

in order to model the propagation on the differential interconnect, in frequency or transient analysis.

### 3.2 Equivalent Circuit Based on the Vector Fitting Procedure

The differential interconnect of length $\ell$ can be represented by the equivalent Π-type circuit shown in Fig. 2 where the frequency-dependent admittances are given by:

$$Y_1(\omega) = \frac{1}{\sqrt{Z'(\omega)/Y'(\omega)}\ \sinh\left(\sqrt{Z'(\omega)Y'(\omega)}\ \ell\right)} \quad (6a)$$

$$Y_2(\omega) = \frac{\cosh\left(\sqrt{Z'(\omega)Y'(\omega)}\ \ell\right) - 1}{\sqrt{Z'(\omega)/Y'(\omega)}\ \sinh\left(\sqrt{Z'(\omega)Y'(\omega)}\ \ell\right)}\ . \quad (6b)$$

By applying the vector fitting procedure [5], the generic admittance $Y_k(\omega)$, with k=1,2, can be approximated by the following rational expressions:

$$\tilde{Y}_k(\omega) = d_k + j\omega e_k + \sum_{i=1}^{N_p} \frac{c_{k,i}}{j\omega - a_{k,i}} \quad (7)$$

where $N_p$ is the order of the approximating function, and $c_{k,i}$ and $a_{k,i}$ are the *i*-th residue and pole, respectively. The rational function (7) can be easily represented by an equivalent RLC network suitable for direct implementation in CAD circuit simulator.

To derive the equivalent circuit, let indicate $N_{pr}$ the number of real poles and $N_{pc}$ the number the complex conjugate pairs. $\tilde{Y}_k(\omega)$ can be written as:

$$\begin{aligned}\tilde{Y}_k(\omega) &= d_k + j\omega e_k + \sum_{i=1}^{N_{pr}} \frac{c_{k,i}}{j\omega - a_{k,i}} \\ &+ \sum_{i=1}^{N_{pc}} \left[\frac{c_{k,i}}{j\omega - a_{k,i}} + \frac{(c_{k,i})^*}{j\omega - (a_{k,i})^*}\right]\end{aligned} \quad (8)$$

where the asterisk denotes complex conjugates. The equivalent circuit associated with (8) is given by the parallel connection of the following branches:

- a conductance $G_{k,0}=d_k$;
- a capacitance $C_{k,0}=e_k$;
- *RL* series circuits associated with the $N_{pr}$ real poles, whose parameters are (see Fig. 3a):

$$La_{k,i} = 1/c_{k,i}$$
$$Ra_{k,i} = -a_{k,i}/c_{k,i}$$

- RLC circuits shown in Fig.3b associated with the $N_{pc}$ complex conjugate pairs, whose parameters are given by:

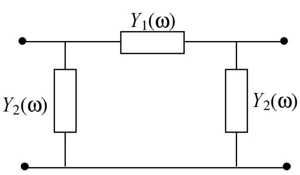
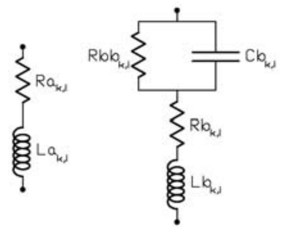



*Fig. 2 – Equivalent Π-type representation.*




*(a) (b)*

*Fig. 3 - Equivalent circuits corresponding to real poles (a) and to complex conjugate pairs (b).*

$$Lb_{k,i} = 0.5/\mathrm{Re}[c_{k,i}] \quad (9)$$

$$Rba_{k,i} = \frac{\mathrm{Im}[c_{k,i}]\mathrm{Im}[a_{k,i}] - \mathrm{Re}[c_{k,i}]\mathrm{Re}[a_{k,i}]}{2\mathrm{Re}[c_{k,i}]^2}$$

$$Cb_{k,i} = \frac{2\mathrm{Re}[c_{k,i}]^3}{\left[\mathrm{Re}[c_{k,i}]^2\left(\mathrm{Re}[a_{k,i}]^2 + \mathrm{Im}[a_{k,i}]^2\right) + \left(\mathrm{Im}[c_{k,i}]\mathrm{Im}[a_{k,i}]\right)^2 - \left(\mathrm{Re}[c_{k,i}]\mathrm{Re}[a_{k,i}]\right)^2\right]}$$

$$Rbb_{k,i} = -\frac{\mathrm{Re}[c_{k,i}]}{Cb_{k,i}\left(\mathrm{Re}[c_{k,i}]\mathrm{Re}[a_{k,i}] + \mathrm{Im}[c_{k,i}]\mathrm{Im}[a_{k,i}]\right)}$$

## 4. Numerical Results

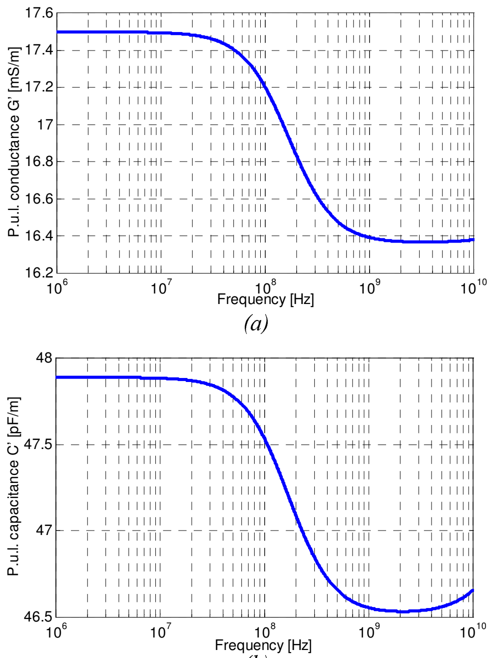
The discussed SPICE-like models are used to analyze the differential interconnect characterized by *w*=1 mm, *s*=0.5 mm, *t*=0.01 mm, and *h*=1 mm (see Fig. 1). The strips are assumed of copper, and the FR4 dielectric substrate is modeled assuming $\varepsilon_s$= 4.5, $\varepsilon_\infty$= 4.19 and $\tau$ = 0.949 ns. The variation with frequency of $G'(\omega)$ and $C'(\omega)$ of the described differential interconnect are shown in Fig. 4.

The vector fitting procedure [5] applied to the admittances of the Π-type equivalent circuit, yields satisfactory approximating functions (see Fig. 5) using two real poles and a complex conjugate pair whose values are reported in Tab. I.

The differential interconnect shown in Fig.1 is considered matched at both ends and driven by a clock source characterized by rise and fall times equal to 50 ps, pulse width 1ns, and period 2 ns. Fig. 6 shows the comparison between the transmitted voltage waveforms computed by means of the lumped circuit and the lossy TL SPICE model. The agreement is reasonably good; the result of the lumped circuit approach is only affected by small spurious oscillations along the falling front due to the fitting procedure which has been carried out up to 10 GHz. Extending the frequency window where to apply the fitting procedure, oscillations will reduce at the extra-cost of additional branches in the equivalent circuits.

## Conclusions

Two SPICE-like models of differential interconnects have been proposed accounting for frequency-dependent conductive and dielectric losses. The comparison between the results of the two SPICE models has shown a good agreement. Nevertheless, the lumped circuit model obtained by the vector fitting is more efficient since the required CPU time is much less than the time required to compute the impulse response of the lossy TL in a transient analysis. The extra-cost of the vector fitting procedure is worthy as far as a few trials are required. This aspect is very important when dealing with complex configurations in which hundreds of interconnects links more complex devices. Moreover, the lumped circuit approach can be easily extended to the analysis of complex interconnect configurations.




(a)

(b)

*Fig. 4 - P.u.l. conductance (a) and capacitance (b) of the differential interconnect.*

*Tab. I – Coefficients of the rational approximations obtained by the vector fitting procedure.*

| | $Y_1$ | $Y_2$ |
|---|---|---|
| $d_k$ | 1.931e-4 | 3.252e-5 |
| $e_k$ | 1.140e-14 | 4.586e-14 |
| $c_{k,1}$ | 1.522e8 | -88.373 |
| $c_{k,2}$ | -2.652e4 | 2.521e3 |
| $c_{k,3}$ | -1.587e8 – $j$ 3.589e4 | 3.173e8 + $j$ 6.08e5 |
| $c_{k,4}$ | -1.587e8 + $j$ 3.589e4 | 3.173e8 - $j$ 6.08e5 |
| $a_{k,1}$ | -5.248e7 | |
| $a_{k,2}$ | -4.146e8 | |
| $a_{k,3}$ | -3.742e8 + $j$ 5.796e10 | |
| $a_{k,4}$ | -3.742e8 - $j$ 5.796e10 | |

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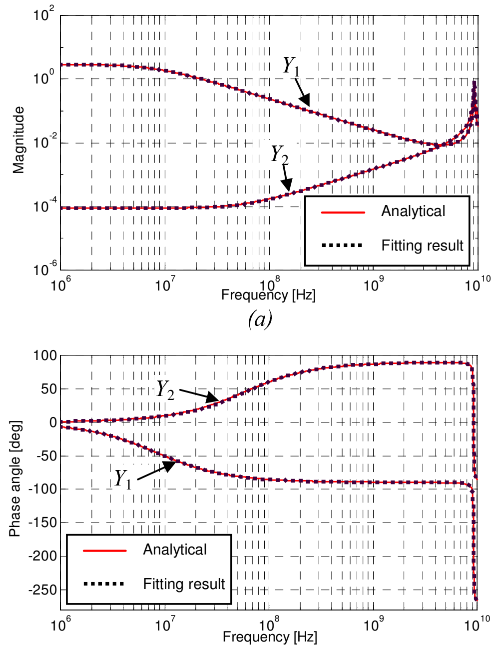


(a)

(b)

*Fig. 5 - Rational approximation of the admittances of the Π type equivalent circuit: magnitude (a) and phase (b).*

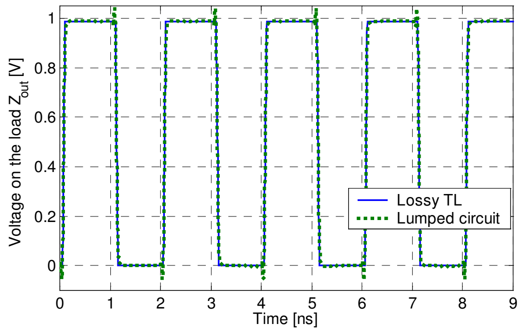


*Fig. 6 – Load voltage on Zout obtained by the two SPICE-like models.*

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