A COMPARISON BETWEEN RADIATION AND OHMIC LOSSES IN HIGHLY-CONDUCTING INTERCONNECTS FOR HIGH-SPEED APPLICATIONS

A. Maffucci*, G. Miano°, F. Villone*

* DAEIMI, Università di Cassino, via Di Biasio 43, 03043 Cassino, FR, Italy

° DIEL, Università di Napoli Federico II, Via Claudio 21, 80125 Napoli, Italy

E-mail: maffucci@unicas.it, miano@unina.it, villone@unicas.it

Abstract: This paper deals with the problem of estimating the losses in highly-conducting wires, in a frequency range where the transverse dimension is comparable to the signal wavelength. In this case the *standard* transmission line model fails since it cannot predict high-frequency effects like radiation in the transverse plane. Here we use a recently proposed *enhanced* transmission line model which, although it is simple as the standard one, can take into account properly these effects. This model is obtained starting from a full-wave analysis of the propagation along perfectly conducting wires, and using an integral formulation based on the electromagnetic potentials. This model is here upgraded in order to take into account the effect of an high but finite wire conductivity. By means of the modified model, we compare the ohmic losses in the wires to the radiation losses in cases of practical interest.

Key words: Interconnects, high-frequency effects, full-wave models, integral formulation.

1. Introduction

In high-speed applications the losses associated to the propagation along electric interconnects cause serious degradation of signals (crosstalk levels, pulse distortion, switching noise, …), hence a correct estimation of such losses is essential to study the overall system performance. If the propagation is of the quasi-TEM type, the correct estimation of losses is a consequence of an accurate extraction of the transmission line parameters (*e.g.*, [1]-[2]). In many applications of practical interest, however, the quasi-TEM hypothesis no longer holds, due, for instance, to an high operating frequency or to strong non-uniformities of the interconnect. In such cases, besides the losses in the conductors and the dielectrics, there could be the need to take into account losses associated to unwanted radiation. The accurate prediction of the losses requires, in principle, a full-wave analysis, with a tremendous increase of the computational cost (*e.g.*, [3]). Therefore, many authors have proposed generalized transmission line model able to describe the high-frequency behavior of interconnects with low computational cost (e.g., [4]).

The authors have recently proposed an *enhanced* transmission line (*ETL*) model to study the propagation along two parallel perfectly conducting wires in a homogeneous dielectric, when the distance between the wires is comparable to the wavelength ([5]-[6]). The model has been obtained, with suitable approximations, starting from a full-wave analysis and using an integral formulation based on the electromagnetic potentials, hence it can predict high-frequency effects like radiation or dispersion that cannot be foreseen by the standard transmission line (*STL*) model. The *ETL* model has the same mathematical structure as the *STL* one, hence the computational cost of its simulation is relatively low.

In this paper we first upgrade the *ETL* model in order to take into account the effect of an high but finite conductivity of the conductors (Section 2). The modified model is then used to compare the ohmic and the radiation losses in a case study (Section 3). In this way we show in what frequency ranges the effects of a finite conductivity have to be considered.

2. The "enhanced" transmission line model

2.1 Derivation of the ETL model

In the following we only report the main steps followed to derive the ETL model. More details may be found in [5] and [6]. The considered structure is depicted in Figure 1: two parallel lossless conductors of length $2l$, geometrically long, with circular cross sections; h and a are, respectively, the distance between the conductors and their radii. The wires connect two terminal devices which are not depicted.

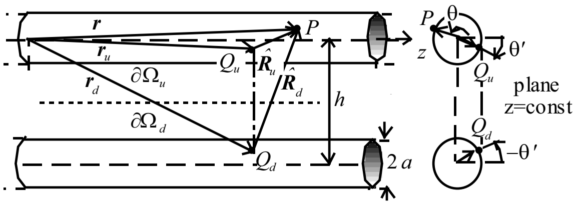

Fig. 1. Interconnect geometry.

We assume that the interconnect may be described as a two-port. We use a cylindrical coordinate system (r, θ, z) with the *z*-axis parallel to the conductor axes.

The electromagnetic field can be represented, in the frequency domain, through the potentials $\mathbf{A}$ and φ :

$$\mathbf{E} = -j\omega\mathbf{A} - \nabla\varphi, \quad \mathbf{B} = \nabla \times \mathbf{A}. \tag{1}$$

The potentials are given by the two integral relations (Lorenz gauge condition):

$$\mathbf{A}(P) = \mu \iint_S G(r)\mathbf{J}_s(r)dS, \ \varphi(P) = \frac{1}{\varepsilon}\iint_S G(r)\sigma(r)dS, \tag{2}$$

where G is the Green function for the homogenous space with propagation constant $k = \omega\sqrt{\varepsilon\mu} = \omega / c$:

$$G(r) = \frac{\exp(-jkr)}{4\pi r}. \tag{3}$$

The distributions of surface charge and current densities are determined by imposing the boundary conditions and the charge conservation law

$$(-j\omega\mathbf{A} - \nabla\varphi)\big|_S \times \hat{\mathbf{n}} = \mathbf{0}, \nabla^{(s)} \cdot \mathbf{J}_s = -j\omega\sigma \quad , \tag{4}$$

where $\nabla^{(s)} \cdot$ is the surface divergence operator. The fundamental hypothesis for both the *STL* and the *ETL* models is that the current density field has only the *z*-component, as well as the vector potential:

$$\mathbf{J}_s = \hat{\mathbf{z}}J_s, \quad \mathbf{A} = \hat{\mathbf{z}}A. \tag{5}$$

The field is of *transverse magnetic* type, hence in each plane *z*=constant the voltage between the two ideal conductors is uniquely defined. Having identified with (θ, z) a generic point lying on one of the two lateral surfaces, from (5) we get:

$$\frac{\partial\varphi}{\partial z} + j\omega A = 0, \ \frac{\partial\varphi}{\partial\theta} = 0, \ \frac{\partial J_s}{\partial z} = -j\omega\sigma. \tag{6}$$

Therefore, potentials depend solely on the variable *z*. Let us indicate with subscripts 1 and 2 the two conductors: at a given *z* the variables

$$V(z) \equiv \varphi_1(z) - \varphi_2(z), \quad \Phi(z) \equiv A_1(z) - A_2(z) \tag{7}$$

are, respectively, the voltage between the two conductors and the *p.u.l.* flux of the magnetic field linked with the two conductors. In our case the common mode current intensity values must be equal to zero because the interconnect is ended with two one-ports and there is no external excitation. The currents and the charges on the two conductors are

$$I_1 = -I_2 \equiv I \ , \ Q_1 = -Q_2 \equiv Q. \tag{8}$$

From equations (6) we immediately get

$$-\frac{dV}{dz} = j\omega\Phi, \quad -\frac{dI}{dz} = j\omega Q \tag{9}$$

while from relations (2) we obtain, respectively,

$$\varepsilon V(z) = 2a\int_{-l}^{+l} dz' \int_0^{2\pi} d\theta' K(\theta, \theta'; z - z')\sigma(z', \theta') \tag{10}$$

$$\mu^{-1}\Phi(z) = 2a\int_{-l}^{+l} dz' \int_0^{2\pi} d\theta' K(\theta, \theta'; z - z')J(z', \theta') \tag{11}$$

The kernel K is given by

$$K(\theta, \theta'; \zeta) = G_s(\theta - \theta'; \zeta) - G_m(\theta, \theta'; \zeta), \tag{12}$$

where G_s and G_m are respectively the Green functions when the source and field points are located on the same or on different conductor surfaces.

The transmission line model that we have obtained is described by equations (9), which can be regarded as *governing* equations, and (10)-(11), which are *constitutive relations*. The model contains the STL model as a particular case. The *STL* model, in fact, is described by the same governing equations (9) and by the following constitutive relations:

$$\Phi(z) = LI(z), \quad V(z) = Q(z)/C, \tag{13}$$

where the C and L are, respectively, the *p.u.l.* capacitance and the inductance of the interconnect. These relations may be derived from (10)-(11) when the distance h between the conductors is electrically short, that is, $kh << 1$. In this limit the *p.u.l.* flux (the voltage) at a given abscissa *z* is approximately equal to the *p.u.l.* flux (the voltage) produced by the same couple of conductors, but of infinite length, assuming a uniform surface current (charge) density.

The *ETL* model is described by the governing equations (9) and by constitutive relations which are derived from (10)-(11) by considering finite length conductors and $kh \approx 1$ (the distance h is comparable to the wavelength). To derive these relations we first consider the dependence of J and σ on the variables θ and z are of a separable type. Then, assuming $h/a > 2.5$ and $hk < 5$ we approximate the integrals (10)-(11) to their average values on the variable θ. Then the constitutive equations of the *ETL* model are

$$\Phi(z) = \mu\int_{-l}^{+l} H(z - z')I(z')dz', \tag{14}$$

$$V(z) = \frac{1}{\varepsilon}\int_{-l}^{+l} H(z - z')Q(z')dz'. \tag{15}$$

In [6] we give the general expression of kernel *H,* which may be split into the sum of a "static" and "dynamic" part $H = H_{stat} + H_{dyn}$. Here we report an approximate expression obtained by disregarding the proximity effect ($a/h << 1$) and assuming $ka << 1$:

$$H_{stat}(\zeta) \cong \frac{1}{\pi^2}\frac{\kappa[m(\zeta)]}{R_s(\zeta)} - \frac{1}{2\pi}\frac{1}{R_m(\zeta)}, \tag{16}$$

$$H_{dyn}(\zeta) \cong -\frac{jk}{\pi}\exp\left[-j\frac{kR_m(\zeta)}{2}\right]\sin c\left[\frac{kR_m(\zeta)}{2}\right] \tag{17}$$

where $\kappa(m)$ is the complete elliptic integral of the first type, $m(\zeta) = \zeta^2/(4a^2 + \zeta^2)$,

$$R_m(\zeta) = \sqrt{h^2 + \zeta^2}, \ R_s(\zeta) = \sqrt{4a^2 + \zeta^2}. \tag{18}$$

The obtained *ETL* model, described by two coupled differential-integral equations of the first kind (9), (14)-(15), can describe properly effects like radiation and dispersion which are not predicted by the *STL* one (*see* [5],[6]). The difference between this model and the other proposed in literature (*e.g.,* [4],[5]) is in the correct evaluation of the singularities of the kernel: since $\kappa(m) \approx -0.5 \ln m$ for $m \to 0$, H_{stat} has a singularity of logarithmic type at $\varsigma = 0$, while H_{dyn} is regular. This is the characteristic singularity associated with surface distributions and plays a very important role in the radiation problems.

2.2 The modified ETL model

Let us now investigate how the *ETL* may be modified when considering conductors with non-zero conductor resistivity η (we assume homogeneous conductors).The electric field **E** is related to the current volumetric density field **J** through:

$$\mathbf{E} = \eta \mathbf{J} \, . \tag{20}$$

In such a case the current is diffused throughout the conductor section, and the tangential component of the electric field at the conductor surfaces is no longer zero. Moreover, by combining (20) with the charge conservation law and Gauss's law is easy to show that **J** must be solenoidal inside the conductors. Therefore, the electric charge can be accumulated only on the conductor surfaces and so the normal component of **J** on the conductor lateral surfaces is not generally zero. It clearly follows that we cannot contemporary satisfy the hypotheses: *i*) the current density has only the *z*-component; *ii*) this component also depends on the variable *z*. Other current density components, both radial and azimuthal appear, hence no transmission line type model may be derived, unless some reasonable approximations are made.

Let us assume for a moment that **J** has only the *z*-component and that it does not depend on *z*:

$$\tilde{\mathbf{J}} = \tilde{J}(\rho, \theta)\hat{\mathbf{z}} \, , \tag{21}$$

where ρ is the radial coordinate. This assumption will be removed later. The equation for the current is

$$\frac{\partial^2 \tilde{J}}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 \tilde{J}}{\partial \theta^2} + \frac{1}{\rho}\frac{\partial \tilde{J}}{\partial \rho} - \gamma^2 \tilde{J} = 0 \quad \text{for } \rho \le a \, , \tag{22}$$

$$\gamma^2 = j\frac{2}{\delta^2} - k^2 \approx j\frac{2}{\delta^2} \, . \tag{23}$$

In (23) we have introduced the *penetration depth* $\delta = \sqrt{2\eta / \omega\mu}$ and taken into account that, in our cases of interest, $k^2 << 2/\delta^2$ for $\omega << 1/\varepsilon\eta$.

The next step is to express the current density as function of its distribution at the conductor surface $\tilde{J}(a, \theta)$, and then to derive a *surface impedance* relating the electric field on the conductor surface to the total current crossing the wire.

The general solution of (22) can be found in the form

$$\tilde{J}(\rho, \theta) = \sum_{n=-\infty}^{+\infty} C_n g_n(\rho) \exp(jn\theta), \tag{24}$$

where $g_n(\rho)$ is solution of a Bessel's equation and C_n are constants. For our frequency range of interest we can assume $\delta << a$ (for the copper, at 1 GHz, it is $\delta \approx 1 \mu\text{m}$), hence the current density differs from zero in a layer that is few δ thick. In such a case:

$$g_n(\rho) \cong \left(\frac{1}{\sqrt{\rho / a}} \exp\left(-\frac{a-\rho}{\delta} \right) \right) \exp\left(-j\frac{a-\rho}{\delta} \right), \tag{25}$$

which is valid for those values of n such that $n \le N_c \equiv \sqrt{0.2a/\delta}$ (for example, for $a = 1\,\text{mm}$ and $\delta = 1\,\mu\text{m}$ it must be $n \le 15$). Furthermore, we can retain that the current density distribution along coordinate θ is, with good approximation, described by the same shape function $f(\theta)$ obtained in the perfect conductor case, hence:

$$\tilde{J}(a, \theta) = \tilde{J}_m(a) f(\theta), \tag{26}$$

where $\tilde{J}_m(a)$ is the average value on θ. The terms C_n are nothing but the coefficients of Fourier expansion of $f(\theta)$, unless for the constant factor $\tilde{J}_m(a)$. Consequently, when the amplitude of the harmonics of $f(\theta)$ of the order greater than N_c are negligible, by combining (24) and (25) and by integrating on the conductor section, we relate $\tilde{J}_m(a)$ to the current:

$$\tilde{J}_m(a) = T\tilde{I}, \qquad T = \frac{1+j}{2\pi a \delta} \, . \tag{27}$$

We may now easily take into account the general case in which the current density may depend on *z* simply by using the same coefficient *T*:

$$J_m(a, z) = TI(z), \tag{28}$$

This is possible in our case, since the smallest characteristic wavelength is greater than δ.

Now, by assuming again that **J** (and hence **A**) is directed along *z*, at the edge of the conductor also **E** is prevalently directed along *z*. Therefore, the scalar potential is almost independent of θ, and hence the first of (6) for the generic conductor becomes:

$$\frac{d\varphi}{dz} + j\omega A(a, \theta, z) = E(a, \theta, z) = \eta J(a, \theta, z) \tag{29}$$

where *A*, *E* and *J* are the components along *z*. At this point, averaging on θ and using (28) we obtain:

$$\frac{d\varphi}{dz} + j\omega A_m(a, z) = \eta TI(z) \, , \tag{30}$$

from which, introducing differential mode variables:

$$-\frac{dV}{dz} = j\omega\Phi(z) + Z_s(\omega) I(z), \tag{31}$$

where Z_s is the *p.u.l. surface impedance* defined as

$$Z_s(\omega) = 2\eta T = \frac{1}{\pi a}(1+j)\sqrt{\frac{\omega\mu\eta}{2}} \, . \tag{32}$$

Relation (31) substitutes the first of (9), while the other equations remain unaltered, including (14), which links the *p.u.l.* flux to the current intensity.

3. Case study

We study a pair of copper wires with $a = 10^{-3}\, m$, $h = 10^{-2}\, m, l = 10^{-3}\, m$ and $\eta = 1.7 \cdot 10^{-8}\, \Omega m$. The line is fed by a unitary current source [arb] and is opened at the other end. We consider the frequency range $0.1 f_0 \div 2.5 f_0$, ($f_0 = 1.5\, GHz$), where the *STL* model fails while the *ETL* one may be used [6].

We have evaluated the ohmic power absorbed by the line. This has been done both by integrating the ohmic power density along the conductors, and by making the difference of the values of the power $P_{in} = \mathrm{Re}(VI^*)/2$ absorbed at $z = 0$, evaluated with and without the resistivity effects (to remove the contribution of the radiated power).
The model has been solved numerically by means of the collocation method [6]. First we have evaluated the current distribution, then the voltage distribution is obtained according to Lorenz gauge as

$$V(z) = j \frac{c}{k} \frac{d\Phi}{dz}. \tag{33}$$

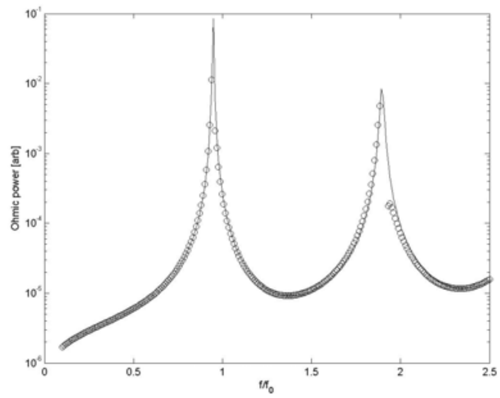

Fig.2. Ohmic power, calculated as (solid) integrating the ohmic power density, and (circles) from P_{in}.

Figure 2 shows that the results agree (the difference at some frequencies is due to cancellation problems). Figure 3 shows the behavior of the ohmic power, the radiated mean power and the total mean power absorbed at $z = 0$. It is evident that the absorbed power is due to the ohmic losses for low frequencies and to the radiation loss for high frequencies. Note that there is a *transition* frequency above which the radiated power becomes greater than the ohmic one. The ratio between ohmic and radiated power is plotted in Fig.4. The effect of a finite resistivity is relevant for frequency ranges where the interconnect may still be described by the *STL* model. When the transverse dimension is comparable to the wavelength, the loss is mainly due to radiation.

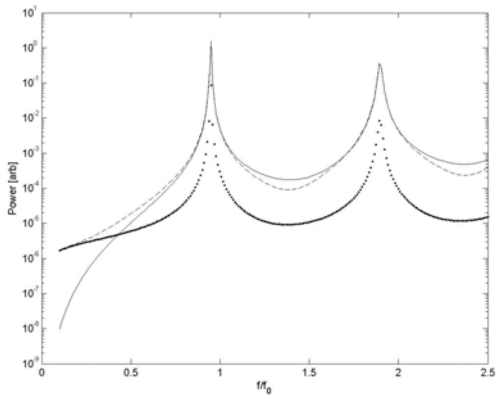

Fig.3. Ohmic power (dotted), radiated mean power (solid), absorbed mean power at z=0 (dashed).

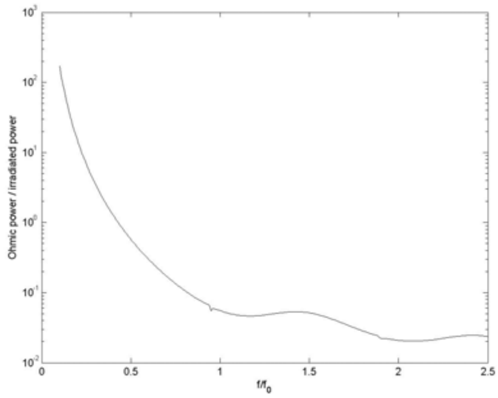

Fig.4. Ratio between ohmic and radiated mean power

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