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SPATIO-POLARIMETRIC CORRELATIONS IN SINGLE-FEED DIVERSELY POLARIZED ARRAYS

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1. Introduction

It is known that the use of dual polarizations can enhance the performance of direction-of-arrival (DOA) estimation in array processing when the sources have different polarizations [1, 2]. Recently, polarization diversity has also found applications in nonstationary signal array processing problems [3]. In [4], the spatial and polarimetric correlation characteristics have been investigated for double-feed dual-polarized arrays (referred to as *dual-polarized array* hereafter), and their performance was compared to that of single-polarized arrays with twice the number of array sensors.

In this paper, we use the spatial and polarimetric correlations to consider the performance of single-feed linear arrays consisting of alternating circularly polarized antennas (referred to as *diversely polarized array* hereafter) for DOA estimations of linearly polarized sources. The spatio-polarimetric correlation coefficients and the DOA estimation performances are compared to those of the double-feed dual-polarized arrays. It is shown that, while an diversely polarized array shows comparable spatial and polarimetric resolution when two sources have close spatial signatures, it may produce grating lobes in the spatio-polarimetric correlation coefficients and, subsequently, false spectrum estimations in DOA estimations. Such problem can be avoided by using smaller interelement spacing at the expense of array aperture reduction.

2. Signal Model

Consider L narrowband linearly polarized signals that arrive at a linear array of N sensors. We consider three types of array sensors: dual-polarized antennas (denoted as D), diversely polarized circular polarization antennas (denoted as C), and single-polarization (vertical) antennas (denoted as S). A diversely polarized array uses alternating left-hand and right-hand circularly polarized antennas. For simplicity of analysis, N is assumed to be even so that the same number of left-hand and right-hand circular antennas are used.

For a dual-polarized array, the received signal vector at is expressed as

$$\mathbf{x}_D(t) = \begin{bmatrix} \mathbf{x}_D^{[v]}(t) \\ \mathbf{x}_D^{[h]}(t) \end{bmatrix}, \qquad \mathbf{x}_D^{[i]}(t) = \mathbf{A}\mathbf{s}^{[i]}(t) + \mathbf{n}^{[i]}(t), \tag{1}$$

where $^{[i]}$ denotes the polarization with $i = v$ (vertical) or h (horizontal), $\mathbf{s}^{[i]}(t)$ and $\mathbf{n}^{[i]}(t)$ are, respectively, the source signal vector and the noise vector of polarization i, and $\mathbf{A}$ is the structured mixing matrix which is independent of polarization. The lth column of $\mathbf{A}$ is the steering vector of lth signal, expressed as $\mathbf{a}_l = [1 \ e^{-j\theta_l} \cdots e^{-j(N-1)\theta_l}]^T$ for $l = 1, \cdots, L$ where ^{T} is the transpose operator and $\theta = (2\pi d/\lambda)\sin(\phi_l)$ with d denoting the interelement spacing, ϕ_l the DOA of the lth signal, and λ the wavelength.

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Denote $\mathbf{P}^{[i]} = \text{diag}[p_1^{[i]}, \cdots, p_L^{[i]}]$ with

$$p_l^{[v]} = \cos\gamma_l \quad \text{and} \quad p_l^{[h]} = \sin\gamma_l \; e^{j\eta_l} \tag{2}$$

being the polarization coefficients of the lth source, where γ_l is the polarization angle that determines the magnitude ratio of the two polarizations, and η_l is the phase difference between them. In this paper, we only consider linearly polarized signals, i.e., $\eta_l = 0$ for all sources. It is clear that $\mathbf{p}_l = [p_l^{[v]} \;\; p_l^{[h]}]^T$ has a unit norm. Using the definition of $\mathbf{P}^{[i]}$, we have $\mathbf{s}^{[i]}(t) = \mathbf{P}^{[i]}\mathbf{s}(t)$ where $\mathbf{s}(t)$ is the source signal vector. Then, the second part of (1) can be rewritten as

$$\mathbf{x}_D^{[i]}(t) = \mathbf{y}_D^{[i]}(t) + \mathbf{n}_D^{[i]}(t) = \mathbf{A}\mathbf{P}^{[i]}\mathbf{s}(t) + \mathbf{n}^{[i]}(t). \tag{3}$$

For the diversely polarized and single-polarized arrays, the received signal vectors become

$$\mathbf{x}_C(t) = \frac{1}{\sqrt{2}}\left[\mathbf{A}\mathbf{P}^{[v]} + j\mathbf{D}\mathbf{A}\mathbf{P}^{[h]}\right]\mathbf{s}(t) + \mathbf{n}(t), \tag{4}$$

$$\mathbf{x}_S(t) = \mathbf{A}\mathbf{P}^{[v]}\mathbf{s}(t) + \mathbf{n}(t), \tag{5}$$

respectively, where

$$\mathbf{D} = \text{diag}[1 \;\; -1 \;\; \cdots \;\; 1 \;\; -1] \tag{6}$$

is a matrix reflecting the polarization alternations.

3. Spatial and Polarization Correlations

For a single-polarization array, it is well known that the spatial correlation between two sources l and m is defined as the normalized inner product of their steering vectors

$$\beta_{S:l,m} = \frac{1}{N}\mathbf{a}_l^H\mathbf{a}_m, \tag{7}$$

where the superscript ^{H} denoted complex conjugate transpose. Note that, for a single-polarized array, there may be a polarization loss ($p_l^{[v]}$), depending on the polarization of the source signal.

For a dual-polarized array, the joint spatio-polarimetric correlation coefficient between two sources is obtained as [4]

$$\beta_{D:l,m} = \beta_{S:l,m} \; \rho_{l,m}, \tag{8}$$

where $\rho_{l,m} = \mathbf{p}_m^H\mathbf{p}_l$ is the polarization correlation coefficient between sources l and m. For the underlying situation where both sources are linearly polarized, the polarization correlation coefficient reduces to

$$\rho_{l,m} = \cos(\gamma_l - \gamma_m). \tag{9}$$

Because $|\rho_{l,m}| \le 1$ and the equality holds only when the two sources have the same polarization, it can be concluded that a dual-polarized array can always reduce the spatio-polarimetric correlation coefficient compared to that of the spatial correlation coefficient of the corresponding single-polarized array. In addition, the dual-polarized array does not have a polarization loss because both polarization components are captured.

For the diversely polarized array, the equivalent spatio-polarimetric signature of the lth signal is obtained from (4) as (The factor of $1/\sqrt{2}$ is omitted for convenience so that the norm of $\mathbf{a}_{C:l}$ remains to be N. It does not affect the spatiao-polarimetric correlation.)

$$\begin{aligned} \mathbf{a}_{C:l} &= \mathbf{a}_l\mathbf{P}^{[v]} + j\mathbf{D}\mathbf{a}_l\mathbf{P}^{[h]} \\ &= \dot{\mathbf{a}}_l\left[\mathbf{P}^{[v]} + j\mathbf{P}^{[h]}\right] + \ddot{\mathbf{a}}_l\left[\mathbf{P}^{[v]} - j\mathbf{P}^{[h]}\right] \\ &= \dot{\mathbf{a}}_l e^{j\gamma_l} + \ddot{\mathbf{a}}_l e^{-j\gamma_l}, \end{aligned} \tag{10}$$

where

$$\begin{aligned}\dot{\mathbf{a}}_l &= [1 \;\; 0 \;\; e^{-j2\theta_l} \;\; 0 \;\; \cdots \;\; e^{-j(N-2)\theta_l} \;\; 0]^T \\ \ddot{\mathbf{a}}_l &= [0 \;\; e^{-j\theta_l} \;\; 0 \;\; e^{-j3\theta_l} \;\; \cdots \;\; 0 \;\; e^{-j(N-1)\theta_l}]^T\end{aligned} \tag{11}$$

are the steering vectors of the subsrray consisting of antennas with only the odd and even index numbers, respectively. It is clear that $\dot{\mathbf{a}}_l$ and $\ddot{\mathbf{a}}_m$ have a norm of $2/N$ and are orthogonal for all l and m. Therefore, the spatio-polarimetric correlation coefficient between sources l and m is obtained as

$$\begin{aligned}\beta_{C:l,m} &= \frac{1}{N}\mathbf{a}_{C:l}^H \mathbf{a}_{C:m} = \frac{1}{N}\left[\dot{\mathbf{a}}_l e^{j\gamma_l} + \ddot{\mathbf{a}}_l e^{-j\gamma_l}\right]^H \left[\dot{\mathbf{a}}_m e^{j\gamma_m} + \ddot{\mathbf{a}}_m e^{-j\gamma_m}\right] \\ &= \frac{1}{2}\left[\dot{\beta}_{l,m} e^{j(\gamma_m-\gamma_l)} + \ddot{\beta}_{l,m} e^{-j(\gamma_m-\gamma_l)}\right] \\ &= \frac{1}{2}\dot{\beta}_{l,m}\left[e^{j(\gamma_m-\gamma_l)} + e^{-j[(\theta_m-\theta_l)+(\gamma_m-\gamma_l)]}\right],\end{aligned} \tag{12}$$

where

$$\dot{\beta}_{l,m} = \frac{2}{N}\left[\dot{\mathbf{a}}_l^H \dot{\mathbf{a}}_m\right] \quad \text{and} \quad \ddot{\beta}_{l,m} = \frac{2}{N}\left[\ddot{\mathbf{a}}_l^H \ddot{\mathbf{a}}_m\right] = \dot{\beta}_{l,m} e^{-j(\theta_m-\theta_l)}. \tag{13}$$

Therefore,

$$|\beta_{C:l,m}| = \left|\dot{\beta}_{l,m}\right| \left|\cos\left[\frac{\theta_m-\theta_l}{2} + (\gamma_m-\gamma_l)\right]\right|. \tag{14}$$

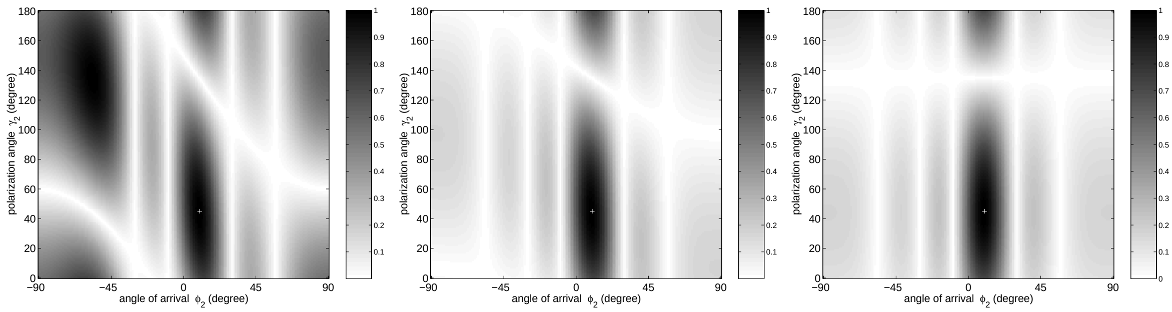
When the two sources are closely spaced (i.e., $\theta_m - \theta_l$ is small), $|\dot{\beta}_{l,m}| \approx |\beta_{S:l,m}|$ and $\cos[(\theta_m-\theta_l)/2 + (\gamma_m-\gamma_l)] \approx \cos[(\gamma_m-\gamma_l)]$ (unless γ_m and γ_l are close to be orthogonal), resulting $|\beta_{C:l,m}| \approx |\beta_{D:l,m}|$. Therefore, the diversely polarized array has comparable performance to that of the dual-polarized array. It is evident, however, that a diversely polarized array with half wavelength interelement spacing cannot avoid the grating problem when the spatial separation between the two sources increases, because the spatial correlation $\dot{\beta}_{l,m}$ is defined at a spatially decimated subarray (i.e., the interelement spacing of the subarray is $2d$). The spatial angles of a grating lobe are determined by $|\dot{\beta}_{l,m}| = 1$ whereas the polarization angles are given by $\gamma_m - \gamma_l = k\pi - (\theta_m - \theta_l)/2$ with k denoting an integer.

To avoid the grating problem, the diversely polarized array must be spatially oversampled such that $d \le \lambda/4$. To maintain the same aperture as that of a dual-polarized array that does not require spatial oversampling, roughly, twice the number of array elements are required for the diversely polarized array. In this case, the spatial-polarization correlation coefficient of the $2N$-element diversely polarized array is comparable to that of the N-element dual-polarized array in the statistical sense. The spatio-polarimetric correlation coefficients of these two arrays, however, have different shapes because the spatial and polarimetric differences are not decoupled in (14).

4. Numerical Results

We first consider the spatio-polarimetric correlation coefficients for both the diversely polarized and dual-polarized arrays. Fig. 1(a) and Fig. 1(b) depict the former whereas Fig. 1(b) illustrates the latter. Parameters are shown in these figures. The spatial correlation coefficient of a single-polarization linear array can be obtained by letting $\gamma_1 = \gamma_2$ in Fig. 1(c). The white '+' mark in each plot indicates the first source. All these plots show similar coefficients around '+' (i.e., the signatures of the two sources are close), but apart from this region the difference is clear because in Fig. 1(a) there is a grating area which the other two do not have.

Next, we compare the MUSIC spectra of the three arrays to confirm the usefulness of the spatio-polarimetric correlation in determining the DOA estimation performance. Fig. 2 shows the results as well as the parameters we have used. The spatial resolutions of all three arrays are comparable, but the diversely polarized array with half-wavelength interelement spacing shows false spectra because of the grating problem (Fig. 2(a)).

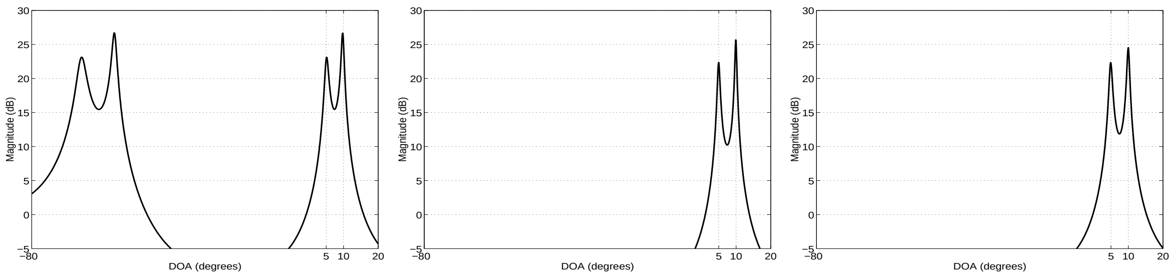


(a) Diversely polarized N=6, $d = \lambda/2$

(b) Diversely polarized N=12, $d = \lambda/4$

(c) Dual-polarized N=6, $d = \lambda/2$

Figure 1 Spatial-polarization correlation coefficients versus the DOA ϕ_2 and polarization angle γ_2 of the second source ($\phi_1 = 10^o$, $\gamma_1 = 45^o$).



(a) Diversely polarized N=6, $d = \lambda/2$

(b) Diversely polarized N=12, $d = \lambda/4$

(c) Dual-polarized N=6, $d = \lambda/2$

Figure 2 Comparison of MUSIC spectra (SNR=10dB, 256 samples, $\phi_1 = 10^o, \phi_2 = 5^o, \gamma_1 = 45^o, \gamma_2 = 90^o$).

5. Conclusion

We have investigated the spatio-polarimetric correlation characteristics of a diversely polarized array that uses single-feed antennas of alternating circular polarizations. Such an array requires spatial oversampling to avoid the grating problem. The performance of a diversely polarized array with $2N$ sensors and $d/2$ interelement spacing is comparable, in the statistical sense, to that of an N-sensor double-feed dual-polarized array with interelement spacing d.

References

[1] E. R. Ferrara and T. M. Parks, "Direction finding with an array of antennas having diverse polarizations," *IEEE Trans. Antennas Propagat.*, vol. 31, pp. 231–236, March 1983.

[2] J. Li and R. J. Compton, "Angle and polarization estimation using ESPRIT with a polarization sensitive array," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1376–1383, Sept. 1991.

[3] Y. Zhang, M. G. Amin, and B. A. Obeidat, "The spatial polarimetric time-frequency distributions and their applicaiton to direction-of-arrival estimation," *Proc. SPIE*, vol. 5205, Aug. 2003.

[4] Y. Zhang and M. G. Amin, "Spatial and Polarization Correlations in Array Processing," submitted to *IEEE Sensor Array and Multichannel Signal Processing Workshop*, Barcelona, Spain, July 2004.