A Frequency-Dependent Transmission-Line Simulator Using S-Parameters

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Abstract: In this study an efficient approach is presented for the transient simulation of lossy, frequency-dependent multiconductor transmission lines. The method is based on the approximation of the modal scattering parameters of the system. Rational function approximation and recursive convolution are used to generate the time-domain stamps of the multiconductor system

Keywords: scattering parameters, transient, skin effect, delay

Introduction

The simulation of transients on coupled interconnects has received extensive attention during the past decade. Recently, the focus has shifted towards the ability to simulate lines with frequency dependent parameters. As a result of losses and higher frequency operation, this frequency dependence can no longer be ignored. The treatment of frequency-dependent coupled lossy transmission lines was covered in [1] in which approximations were performed on the propagation function and characteristic impedance matrices using an interpolation scheme. More recently, an enhancement of the method was proposed by [2] that improves the approximation of the propagation function matrix when the skin effect in the lines is significant.

One key element in the success of the methods in [1] and [2] is the matrix delay separation scheme that allows the approximation of a delayless modal propagation function matrix. When the frequency dependence of the line parameters are significant the accuracy of the scheme cannot always be guaranteed. In this study, we apply the approximation and numerical integration schemes of [1] and [2] to a scattering parameter formulation to generate the time-domain stamp for multiconductor frequency-dependent lines. Scattering parameters offer the unique advantage of the flexibility in the choice of a reference system. This flexibility can be exploited to minimize the approximation effort thus optimizing the accuracy of the results. In this work, such an approach is used to formulate the equations and generate the time-domain stamp for a lossy, frequency-dependent multiconductor transmission line system.

We first present a single line analysis in which we bring out the advantages of a scattering parameter formulation. Next, the analysis is generalized to lossy, frequency-dependent multiconductor lines. Examples and comparisons are given to evaluate the performance of the method

Single-Line Scattering Parameter Formulation

The scattering parameters of a lossy transmission line of length l, complex characteristic impedance Z_O and propagation constant γ satisfy the relation (see Figure 1)

$$B_1 = S_{11}A_1 + S_{12}A_2 \tag{1a}$$

$$B_2 = S_{21}A_1 + S_{22}A_2 \tag{1b}$$

where A_1 and A_2 are the incident voltage waves at the reference planes measured in the reference lines of characteristic impedance Z_{ref}. B_1 and B_2 are the reflected waves due to A_1 and A_2 respectively. These scattering parameters are given by [3]

$$S_{11} = S_{22} = \frac{(1 - X^2)\Gamma}{1 - X^2\Gamma^2} \tag{2a}$$

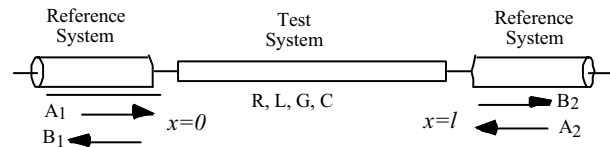$$S_{12} = S_{21} = \frac{(1 - \Gamma^2)X}{1 - X^2\Gamma^2} \tag{2b}$$

Figure 1. Transmission-line network for scattering parameter determination.

where

$$\Gamma = \frac{Z_c - Z_{ref}}{Z_c + Z_{ref}} \tag{3}$$

and $X = e^{-\gamma l}$ is the complex propagation function of the line. This expression contains propagation and attenuation components. The complex propagation constant γ is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{4}$$

and the complex characteristic impedance of the line is:

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \tag{5}$$

where R, L, G and C are the resistance, inductance, conductance and capacitance per unit length respectively. In general they are functions of frequency. The choice of the reference impedance is arbitrary; however, if we choose

$$Z_{ref} = \sqrt{\frac{L}{C}}, \tag{6}$$

we observe that the scattering parameters of the line are "optimized"; in particular, for high frequency, since

$$Z_c \rightarrow \sqrt{\frac{L}{C}}, \tag{7}$$

we observe that

$$S_{11} \rightarrow 0 \quad and \quad S_{12} \rightarrow e^{-j\omega\sqrt{LC}l} = X_o \tag{9}$$

where X_o is the asymptotic propagation function of the line. This factor corresponds to a simple delay in the time domain. As a first step in our formulation, S_{11} is split into two terms to give

$$B_1 = S_{11}^{(a)}A_1 + S_{11}^{(b)}A_1 + S_{12}A_2 \tag{10a}$$

$$B_2 = S_{12}A_1 + S_{11}^{(a)}A_2 + S_{11}^{(b)}A_2 \tag{10b}$$

such that the split terms are given by:

$$S_{11}^{(a)} = \frac{\Gamma}{1 - \Gamma^2 X^2} \tag{11}$$

$$S_{11}^{(b)} = \frac{-\Gamma X^2}{1 - \Gamma^2 X^2} \tag{12}$$

and S_{12} is still given by:

$$S_{12} = \frac{\left(1 - \Gamma^2\right)X}{1 - \Gamma^2 X^2} \tag{13}$$

We next multiply $S_{11}^{(b)}$ by $X_o^2 X_o^{-2}$ so as to define $\hat{S}_{11}^{(b)}$

$$\hat{S}_{11}^{(b)} = \frac{-\Gamma X^2 X_o^{-2}}{1 - \Gamma^2 X^2} \tag{14}$$

Analogously, we multiply S_{12} by $X_o X_o^{-1}$ and define

$$\hat{S}_{12} = \frac{\left(1 - \Gamma^2\right)X X_o^{-1}}{1 - \Gamma^2 X^2} \tag{15}$$

The two-port network formulation now becomes

$$B_1 = \hat{S}_{11}^{(a)}A_1 + \hat{S}_{11}^{(b)}X_o^2 A_1 + \hat{S}_{12}X_o A_2 \tag{16a}$$

$$B_2 = \hat{S}_{12}X_o A_1 + \hat{S}_{11}^{(a)}A_2 + \hat{S}_{11}^{(b)}X_o^2 A_2 \tag{16b}$$

Since the factors X_o and X_o^{-2} translate to simple shift in the time domain, the inversion process must thus focus on the approximation of the terms $\hat{S}_{11}^{(a)}$, $\hat{S}_{11}^{(b)}$ and $\hat{S}_{12}$. Little analytical effort shows that if the reference impedance is chosen as per (6), then, in the asymptotic high-frequency limit, we get,

$$\hat{S}_{11}^{(a)} \rightarrow 0, \ \hat{S}_{11}^{(b)} \rightarrow 0, \ and \ \hat{S}_{12} \rightarrow 1$$

These asymptotic quantities represent the nominal values about which the augmented scattering parameters will fluctuate as a function of frequency. In most practical applications, these functions are weak and monotonous; this thus provides the basis for an accurate approximation over a wide frequency range. The functions are approximated as rational functions as per [1]. The approximation is performed by using an interpolation scheme to approximate the real parts of the scattering parameters. Results for these approximations are shown in Figures 2 for the augmented scattering parameters for a lossy line with skin effect.

N-Line Scattering Parameter Formulation

Given the R, L, G and C matrices of a lossy n-line system, the frequency-domain solution leads to the introduction of the complex propagation function matrix $X(u)$ given by

$$X(u) = \begin{bmatrix} e^{-(\alpha_1 + j\beta_1)u} & & & \\ & e^{-(\alpha_2 + j\beta_2)u} & & \\ & & \bullet & \\ & & & e^{-(\alpha_n + j\beta_n)u} \end{bmatrix} \tag{17}$$

for any real scalar u. The complex propagation constant, $\alpha_i + j\beta_i$, is associated with the ith mode and is the ith entry of the diagonal eigenvalues matrix Λ_m. Modal and line impedance matrices can also be defined as Z_m and Z_c [4]. They are given by:
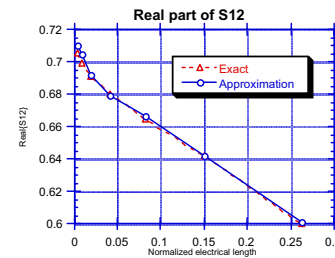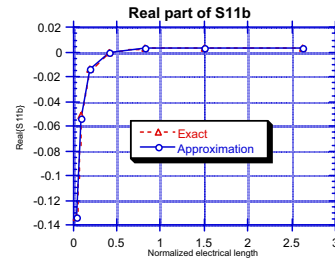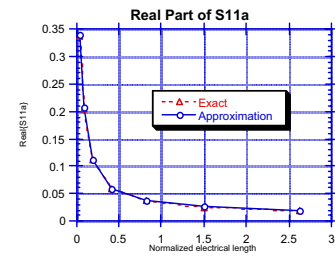

$$Z_m = \Lambda_m^{-1}E Z H^{-1} \tag{18}$$

$$Z_c = E^{-1}Z_m H = E^{-1}\Lambda_m^{-1}E Z . \tag{19}$$

E and H are the complex voltage and current eigenvector matrices respectively. Once the important propagation parameters of the coupled lossy line system are determined, we can derive the scattering-parameter representation of such a system. The derivation of the scattering parameter matrices is detailed in [4]. The procedure consists of defining a lossless, ideal array of coupled transmission lines with inductance and capacitance matrices give by L_o and C_o respectively. Such a system has voltage and current transformation matrices given by E_o and H_o respectively. The eigenvalue matrix of the system is Λ_o and its modal impedance matrix is Z_o. From its

Figure 2. Plots of $S_{11}(a)$, $S_{11}(b)$ and S_{12} for exact and approximated values versus normalized electrical length (length/λ). The line characteristics are L_s=418 nH/m, C_s = 0..092, R_{dc}=200 Ω, R_s=200 ohms(m- GHz$^{1/2}$)

eigenvalue matrix, a diagonal propagation function matrix $X_o(u)$ can be derived and is given by

$$X_o(u) = \begin{bmatrix} e^{-\frac{\omega u}{v_{o1}}} & & & \\ & e^{-\frac{\omega u}{v_{o2}}} & & \\ & & \bullet & \\ & & & e^{-\frac{\omega u}{v_{on}}} \end{bmatrix} \qquad (20)$$

for any real scalar u; v_{oi} is the propagation velocity mode associated with the ith mode. It can be shown that $\lambda_{oi} = 1/v_{oi}$ where λ_{oi} is the eigenvalue associated with the ith mode. In order to derive the scattering parameters of the coupled lossy line system, two identical arrays of the ideal multiconductor system are connected to either side of the lossy array to be analyzed as shown in Figure 3, a network can be constructed such that all modes are matched upon incidence, which insures no reflections from the source side of the arrays. These matching networks have a conductance matrix, Y_T, given by
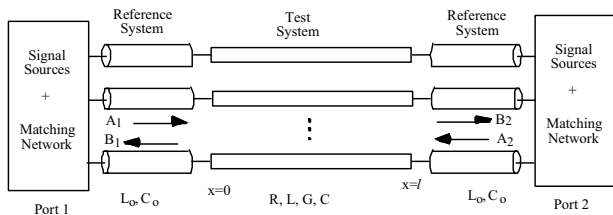
$$Y_T = L_o^{-1} E_o^{-1} \Lambda_o E_o \qquad (21)$$

Figure 3. Array of coupled lossy transmission for scattering parameter analysis.

in which $[Y_T]_{ij}$ is the conductance between the ith and jth lines and $[Y_T]_{ii}$ is the conductance between ith line and ground at the port being considered. Modal coefficient vectors A_1, B_1, A_2, and B_2 are defined at the transition planes; A_1 and B_1 are the incident and reflected modal wave vectors to the lossy array from Port 1, and A_2 and B_2 are the incident and reflected modal wave vectors from Port 2. Next, a set of relations is sought between the incident modal wave vectors from one port and the reflected modal wave vectors at either port. From this, a frequency domain modal scattering parameter array can be defined to describe a $2n$ by $2n$ matrix. Such an array satisfies the relation

$$B_1 = S_{11} A_1 + S_{12} A_2 \qquad (22a)$$

$$B_2 = S_{21} A_1 + S_{22} A_2 \,. \qquad (22b)$$

In the case of a lossy multiconductor system, the scattering parameter formulation is performed in the modal space [4]. The modal scattering parameter matrices are given by

$$S_{11} = S_{22} = T^{-1}[\, \Gamma - \Psi \Gamma \Psi \,][1_n - \Gamma \Psi \Gamma \Psi \,]^{-1} T \qquad (23a)$$

$$S_{12} = S_{21} = 2 E_o E^{-1}[1_n - \Gamma]\Psi[1_n - \Gamma \Psi \Gamma \Psi \,]^{-1} T \qquad (23b)$$

in which

$$\Gamma = [1_n + EE_o^{-1} Z_o H_o H^{-1} Z_m^{-1} \,]^{-1} \, [1_n - EE_o^{-1} Z_o H_o H^{-1} Z_m^{-1} \,] \qquad (24a)$$

$$T = [1_n + EE_o^{-1} Z_o H_o H^{-1} Z_m^{-1} \,]^{-1} EE_o^{-1} \qquad (24b)$$

$$\Psi = \begin{bmatrix} e^{-(\alpha_1 + j\beta_1)l} & & & \\ & e^{-(\alpha_2 + j\beta_2)l} & & \\ & & \bullet & \\ & & & e^{-(\alpha_n + j\beta_n)l} \end{bmatrix} \qquad (25)$$

where 1_n is the unit matrix of dimension n and l is the length of each line in the test array.

The associated augmented modal scattering parameters matrices are given by

$$\hat{S}_{11}^{(a)} = T^{-1} \Gamma \left(1 - \Gamma \Psi \Gamma \Psi \right)^{-1} T \qquad (26a)$$

$$\hat{S}_{11}^{(b)} = T^{-1} \Psi \Gamma \Psi \left(1 - \Gamma \Psi \Gamma \Psi \right)^{-1} T \Psi_o^{-1} \Psi_o^{-1} \qquad (26b)$$

$$\hat{S}_{12} = E_o E^{-1} \left(1 - \Gamma \right) \Psi \left(1 - \Gamma \Psi \Gamma \Psi \right)^{-1} T \Psi_o^{-1} \qquad (26c)$$

where $\Psi_o = X_o(l)$ is associated with the lossless reference system. The augmented scattering parameter matrices can be approximated using the same method as in the single-line case. The inversion into the time domain is then performed from the matrix equivalent of equations (16) using recursive convolution.

Applications

The accuracy and efficiency of the method were tested through testing an array of lossy lines. The configuration is shown in Figure 4 and consists of 5 coupled lines with 4 drive lines and one sense line. The line characteristics are shown in Table I. In particular, the resistance per unit length was chosen to take the form:
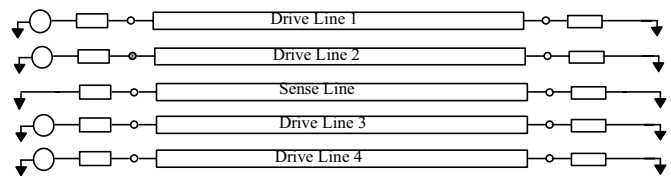

Figure 4. Coupled line structure with resistive terminations. The pulse characteristics are: width=20 ns, rise and fall times=1 ns, magnitude=1V. Near end terminations: $Z_s=50\Omega$; far end terminations: $Z_L=1k\Omega$ The length of the lines is l=14 in.

Table I. Coupled-line characteristics

	L (nH/m)	C (pF/m)	G (mhos/m)	R (Ω/m, Ω/m-GHz^p)
Self	418	96	0.0	R_{dc}=150, R_{skin}=100,p=0.5
Mutual	150	22	0.0	0.0
Non-adjacent	25	0.6	0.0	0.0

$$R_s(f) = R_{skin} f^p \qquad (27)$$

where R_s is the self resistance per unit length in ohms per meter, f is the frequency in GHz, R_{skin} is the skin effect resistance factor and p is an exponent. Results are shown in Figure 5. For the chosen parameters, sufficient agreement was observed [between] the two methods. Future work will involve the study of strong variations of the skin effect resistance as well as the inclusion of conductance terms.

Conclusions

An efficient and accurate method for simulating transmission lines is presented. The method uses a scattering parameter formulation. The s-parameters are augmented and approximated in the frequency domain before a recursive convolution is used to yield the time-domain transients. Comparison with previous methods confirm the accuracy and robustness of the method.
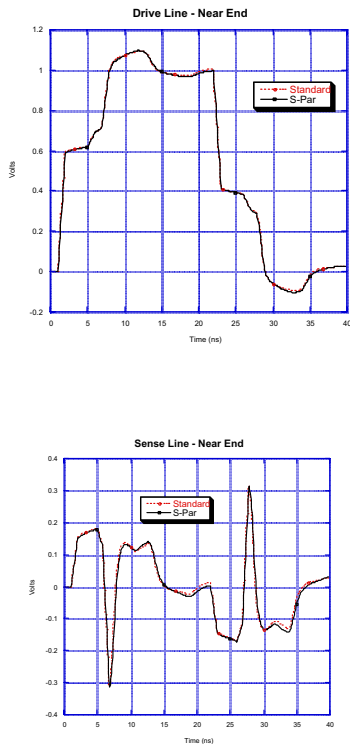
Drive Line - Near End

Sense Line - Near End

Figure 5. Simulation results for circuit in Figure 4 using both standard and scattering parameter approach. Top: drive lines waveforms at near end, Bottom: sense line waveforms at near end.

VI. References

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