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RECTANGULAR WAVEGUIDE LOADED WITH A STRATIFIED INHOMOGENEOUS MEDIUM

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1 Introduction

Propagation properties of dielectric waveguides have been widely studied by using mode matching method [1], integral equation method [2], finite element method, and finite difference method. The mode matching methods are effective for stratified media with a piecewise-constant permittivity profile. However, they can not be applied to analyze guiding structures with the permittivity profile a continuous function of coordinates.

Similar techniques have also been applied to study the propagation properties of dielectric guides enclosed in a metallic waveguide. Take for example, variation-iteration method [3], Rayleigh-Ritz technique [4], finite element method [5], and finite difference method [6]. Similar to the dielectric waveguide problems, all these methods require fine cells to properly represent the field distributions in the waveguide, hence a large matrix equation needs to be solved for the unknowns. Conventional mode matching methods require less unknowns (mode amplitudes), but do not have the flexibility to handle inhomogeneities like the other methods. When the permittivity profile is a continuous function of coordinates, conventional mode matching methods can not be applied.

In this paper, we use an efficient mode matching method to analyze the propagation properties of a rectangular waveguide loaded with inhomogeneous dielectrics as shown in Figure 1. The permittivity profile in each layer can be a continuous function of x. The eigen modes in each layer are first solved numerically, and are then used to represent the fields in that layer. By matching the tangential fields at interfaces between two contiguous layers, the dispersion relation of the whole guiding structure is obtained.

2 Formulation

In Figure 1, we show the configuration of a rectangular waveguide loaded with three layers of inhomogeneous dielectrics. The whole structure is uniform in the y direction. The dielectric constant in each layer is a piecewise continuous function of x, and is independent of y and z.

First, we obtain the eigenmodes of D_x and B_x in an inhomogeneous layer which extends to infinity in the $\pm z$ direction. These eigenmodes are expanded as a linear combination of a set of basis functions. In each inhomogeneous layer, the field components D_x, B_x, E_y, and H_y can be expressed in terms of these eigenmodes. Impose the boundary conditions that tangential field components are continuous across the interface between two contiguous layers, and take the inner product of each basis function with these boundary conditions to obtain a determinantal equation. The dispersion relation is then obtained by solving the determinantal equation.

3 Numerical Results

We first calculate the propagation constant of an inhomogeneous dielectric slab attaching to the bottom of a rectangular waveguide. When the dielectric slab is homogeneous, both TM_z and TE_z modes exist. Their propagation constants can be calculated by solving a transcendental equation derived from matching boundary conditions at the dielectric-air interface. When the dielectric profile of the slab is gradually changed to an inhomogeneous one, each TM_{znm} (TE_{znm}) mode will evolve to a hybrid mode, denoted as E^z_{nm} (H^z_{nm}) mode in this paper [2].

In Figure 2, we show the propagation constants of the E^z_{01} mode for three homogeneous slab structures and two inhomogeneous slab strctures. The propagation constants obtained by solving the transcendental equation for a homogeneous slab are also shown for comparison . The propagation constants of case 4 ($\epsilon_a = 2.45\epsilon_o$, $\epsilon_b = 2.7\epsilon_o$) is higher than that of case 1 ($\epsilon_a = \epsilon_b = 2.45\epsilon_o$) and is lower than that of case 2 ($\epsilon_a = \epsilon_b = 2.7\epsilon_o$). Similarly, the propagation constant of case 5 is higher than that of case 2 and lower than that of case 3.

In Figure 3, similar results are observed with the H^z_{10} mode for the same structures as in Figure 2. The cutoff frequency of the H^z_{10} mode is higher than that of the E^z_{01} mode. The dielectric profile has a more significant effect on the propagation constant of the H^z_{10} mode than on the E^z_{01} mode near the cutoff frequency.

Next, we calculate the propagation constants of an inhomogeneous slab hanging in the middle of a rectangular waveguide. In Figure 4, we show the propagation constants of the E^z_{01} mode for five different slab profiles. In cases 2 and 3, a strip with higher dielectric constant replaces the middle part of the dielectric slab. In cases 4 and 5, a parabolic permittivity profile is assumed. The dispersion curves for cases 1 to 3 are very close to one another near the cutoff frequency. The dispersion curves for cases 4 and 5 are close to each other near the cutoff frequency which is significantly different from that of cases 1 to 3.

In Figure 5, we show the propagation constants of the H^z_{10} mode for the same structures as in Figure 4. The cutoff frequencies are higher than those of the corresponding E^z_{01} mode. Near the cutoff frequencies, the separation among the dispersion curves for the five cases are clearly observed.

4 Conclusions

The propagation properties of a rectangular waveguide loaded with a multilayered inhomogeneous dielectric have been analyzed by using an efficient mode matching method. This method can be applied to analyze the structures with the dielectric constant a continuous function of the lateral coordinate, which can not be solved by using conventional mode matching methods. The dispersion curves of the E^z_{01} and H^z_{10} modes for several inhomogeneous dielectric waveguides have also been compared.

References

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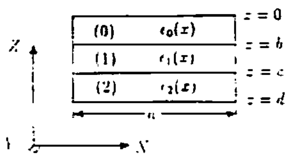

Figure 1: Geometrical configuration of a rectangular waveguide loaded with three layers of inhomogeneous dielectrics.

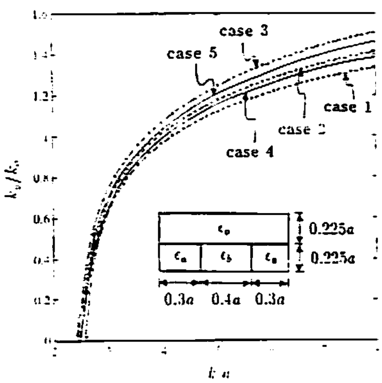

Figure 2: Normalized propagation constant of the E^{z}_{01} mode of an inhomogeneous dielectric slab guide in a rectangular waveguide, (1) $\epsilon_a = \epsilon_b = 2.45\epsilon_o$, (2) $\epsilon_a = \epsilon_b = 2.7\epsilon_o$, (3) $\epsilon_a = \epsilon_b = 3\epsilon_o$, (4) $\epsilon_a = 2.45\epsilon_o$, $\epsilon_b = 2.7\epsilon_o$, (5) $\epsilon_a = 2.45\epsilon_o$, $\epsilon_b = 3\epsilon_o$, * : closed form solution.

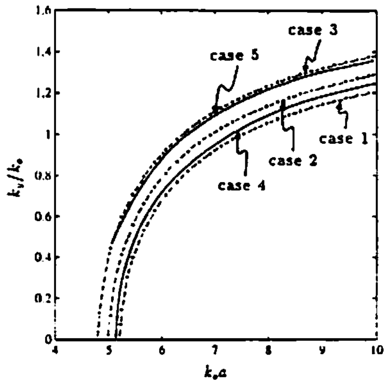

Figure 3: Normalized propagation constant of the $H^{\bar{z}}_{10}$ mode of an inhomogeneous dielectric slab guide in a rectangular waveguide, all the parameters are the same as in Figure 2.

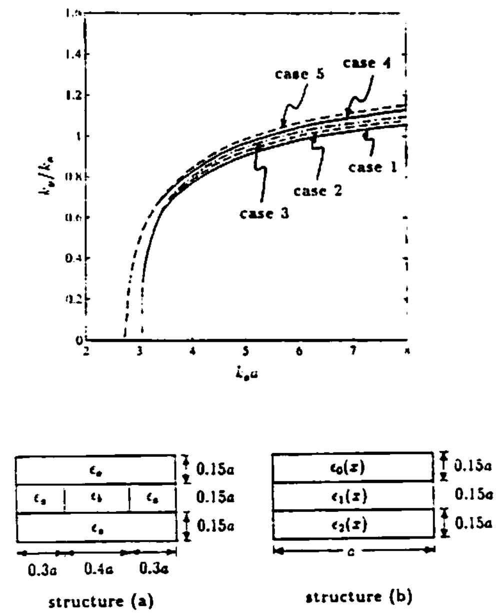

Figure 4: Normalized propagation constant of the $E^{\bar{z}}_{01}$ mode of an inhomogeneous dielectric slab guide in a rectangular waveguide, (1) structure a, $\epsilon_a = \epsilon_b = 2.45\epsilon_o$, (2) structure a, $\epsilon_a = 2.45\epsilon_o$, $\epsilon_b = 2.7\epsilon_o$, (3) structure a, $\epsilon_a = 2.45\epsilon_o$, $\epsilon_b = 3\epsilon_o$, (4) structure b, $\epsilon(x)/\epsilon_o = 2.45 + 4(\epsilon_m - 2.45)x(a-x)/a^2$, $\epsilon_m = 3.45$, (5) same as (4) except $\epsilon_m = 3.95$.

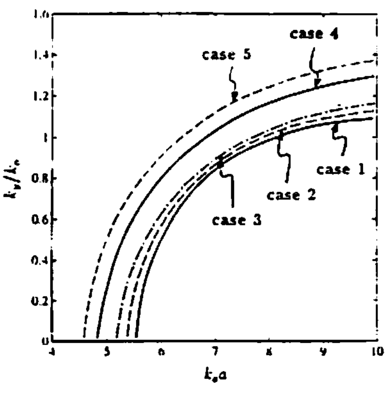

Figure 5: Normalized propagation constant of the $H^{\bar{z}}_{10}$ mode of an inhomogeneous dielectric slab guide in a rectangular waveguide, all the parameters are the same as in Figure 4.