MUTUAL IMPEDANCES BETWEEN NON-NARROW DIPOLES

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1. Introduction

Simple expressions for the mutual impedances between infinitely thin, free half-wavelength dipoles have been recently introduced by the author [1]. They were derived from an asymptotic expansion of a complicated bidimensional Fourier integral. Very simple algebraic terms are involved, which enable to perform calculations with a hand calculator. It is the aim of the present work to incorporate the width of the dipoles within the analysis. Like in the previous article [1], this will be performed only in a crude way to obtain a simple formula that is a generalization of the former one.

The Mathematical Analysis

The mutual impedance between two identical dipoles can be expressed as the bidimensional Fourier transform of the inner product of the plane wave spectrum by another plane wave spectrum obtained from it by reversing the sense of propagation of each component wave . For a cosinusoidal current distribution along each half-wavelength dipole, the mutual impedance is given by :

$$z_{12}(d_x,d_y)=k_o\eta/(2\pi^2)\bullet\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(k\ )/(k_o^2-k_x^2-k_y^2)^{0.5}\bullet\exp(-j(k_xd_x+k_yd_y))dk_xdk_y \quad (1)$$

where $f(k\ )=\cos^2(\pi k_x/(2k_o))/(k_o^2-k_x^2)\cdot\mathrm{sinc}^2(k_y w)$

It has been shown for parallely located dipoles (d =0) that this double integral is estimated by the simple expression [1] :

$$z_{12}(S_1) = 30\bullet \exp(-jS_1)\bullet(4j/S_1-(\pi^2-6)(j/S_1^3-1/S_1^2)) \tag{2}$$

where $S_1=(1+S^2)^{0.5} = (1+(k_0 d_y)^2)^{0.5}$.

This compares reasonably well to the exactly computed integral (1) as shown in [1].

To take care of the width term within (1), it is required also to use the familiar transform of the 'sinc' function. Employing the convolution theorem,

$$\int_{-\infty}^{\infty} J_0[(1-(k_y/k_0)^2)^{0.5}]\cdot \mathrm{sinc}\,(k_y w)\cdot \exp\,(-jd_y k_y)\cdot dk_y = -j\cdot \Psi_1(d_y,w) \tag{3}$$

for $\Psi_1(d_y,w) = 1/w\cdot\int_{-w}^{w} \exp\,(-jkR)/R\cdot dy$

and $R = [a^2+(d_y-y)^2]^{0.5}$, $a = 1/k_0$.

As suggested by Harrington [2], it seems best to use a Maclaurin series expansion of the integrand and then perform integration. Thus,

$$\Psi_1(s,w)\sim 2\,\exp\,(-jkr)/r\cdot[\,A_0+jk_0 w\,A_1+(k_0 w)^2 A_2+o((k_0 w)^4)]$$

$$\text{where}\quad r = (1/k_0^2+d_y^2)^{0.5},\quad s = k_0 d_y\,, \tag{4}$$

$$A_0 = 1 + 1/6\cdot(k_0 w)^2/(1+s^2)^{0.5}\cdot[-1+3s^2/(1+s^2)]$$

$$A_1 = 1/6\cdot(k_0 w)/(1+s^2)^{0.5}[-1+3s^2/(1+s^2)]$$

$$A = -1/6\cdot s^2/(1+s^2)$$

A second convolution with the 'sinc $(k_y w)$' function yields :

$$\int_{-\infty}^{\infty} J_0[(1-(k_y/k_0)^2)^{0.5}]\cdot \mathrm{sinc}^2(k_y w)\cdot \exp\,(-jd_y k_y)\cdot dk_y =$$

$$-\{[1+(-1/6+j/3)(k_0 w)^2]\cdot\Psi_1(d_y,w) + (1-j)\cdot(k_0 w)^2/2\cdot\Psi_3(d_y,w) -(k_0 w)^2/2\cdot\Psi_5(d_y,w)\} \tag{5}$$

where $\Psi_n(d_y, w) = 1/w \int_{-w}^{w} \exp(-jkR)/R^n \cdot dy$

To evaluate $\Psi_n(d_y, w)$, let $\Psi_n(k) = \Psi_n(d_y, w)$

so that the following recursive relation is valid :

$$\Psi_n(k) = \Psi_n(0) - j\int_0^k \Psi_{n-1}(x)\ dx \tag{6}$$

The set of $\{\Psi_n(0)\}$ are computed seperately.

Retaining terms up to second order in 'w',

$$Z_{12}(S_1) \simeq 30 \cdot \exp(-j \cdot S_1) \cdot \{ 1 + [-1/6 - j/(6 \cdot S_1) + j \cdot s^2/(2 \cdot S_1^3)] \cdot (k_o w)^2 \} \cdot [4j/S_1 - (\pi^2 - 6) \cdot (j/S_1^3 - 1/S_1^2)] \tag{7}$$

where $S_1 = (1+s^2)^{0.5}$, $s = k_o d_y$.

Numerical Results

Some numerical values of the mutual impedances for different widths of the dipoles will now be given using the previously derived asymptotic relation (7). They are shown in figures 1,2 as a function of the transverse distance between their centres. The accuracy error is estimated to be of the order of 15%. It comes mainly due to using truncated series expansions for 'w'. Introducing higher order terms within the analysis may improve the accuracy on account of the simplicity of the final expression.

Conclusion

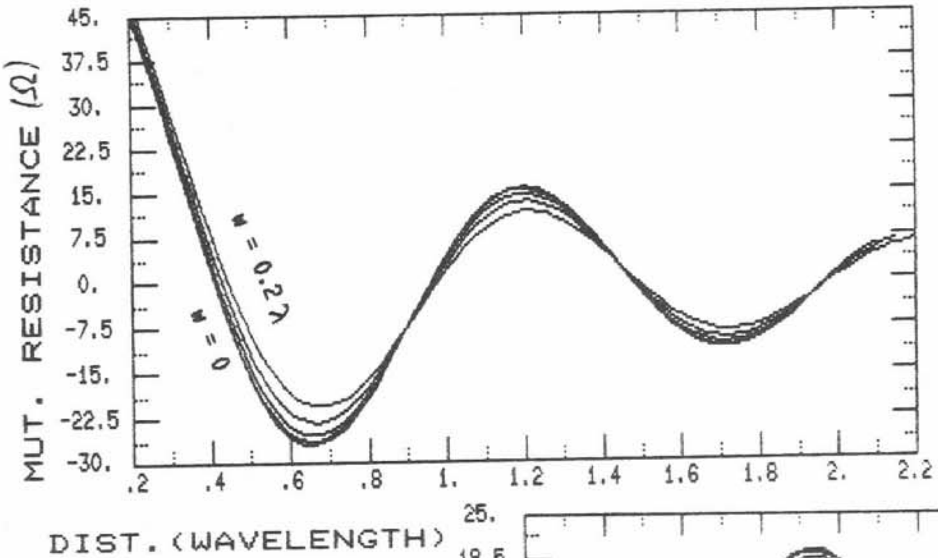
A method of evaluating the mutual impedances between non-narrow dipoles has been presented. It is a generalization of a previous one that was derived for the narrow ones. A complicated bidimensional Fourier integral is calculated in terms of a very simple algebraic expression. A hand calculator is sufficient for this purpose. Further effort should be carried out in applying this method for other configurations (non-parallel dipoles).

References

(1) A.E. Gera : "Mutual Impedances Easily Evaluated", IEE Conf. Publ. 274, 1987, pp.385-389.

(2) R.F. Harrington : "Matrix Methods for Field Problems", in Lectures on Comp. Methods in Electrom., The SCEE Press, 1981.

(3) P. Perlmutter,S. Shtrikman,and D. Treves : "Calculation of Mutual Coupling between Microstrip Antennas using an Electric Surface Current Model", Int. Rept., Weizmann Inst., Rehovot, Israel, 1984.



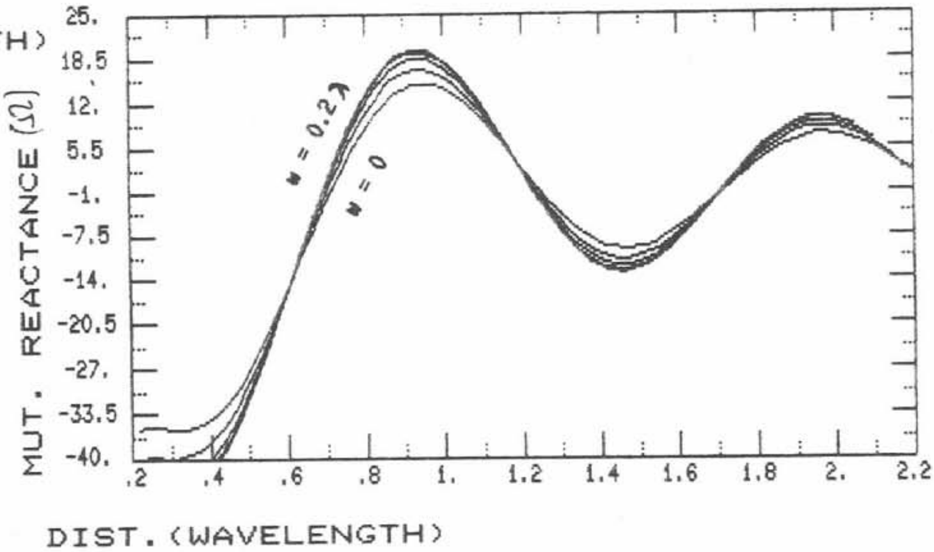


Figure 1,2 : The Mutual Impedance between parallel half-wavelength dipoles for different widths : w = 0, 0.05, 0.1, 0.15, 0.2 λ