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LIGHT PROPAGATION IN MULTILAYER WAVEGUIDES

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Abstract

In this paper the light propagation in magnetooptical sandwiches is discussed. Based on 4x4 Yehs matrix formalism [1], [2] the conditions of waveguiding are investigated in absorbing and anisotropic film systems. The attention is paid on three-layered sandwich structure with ultrathin magnetooptical film between dielectric layers. The waveguiding in such systems in the polar, longitudinal and transversal magnetooptical configuration is analysed in [3]. For transversal configuration we can separate waveguide dispersion relations for two independent polarizations. In the cases of longitudinal and polar geometry the separation is not possible and relations are more complicated [4]. In the second part of the article the recurrent approach of building of waveguide condition in multilayer is presented.

1. Absorption and anisotropy influence on guided waves

The guided modes are investigated by analysing condition

$$M_{11}M_{33} - M_{13}M_{31} = 0 \tag{1}$$

where M_{ij} are elements of Yeh's total matrix [1]. The equation (1) is dispersion relation of guided modes. In case of systems, containing also absorbing films, guided waves are attenuated. The amplitude of left side of Eq. (1) (which is called the waveguide term) reaches minimum value in the points, when phase matching condition is fulfilled. The values of these minima correspond to the degree of guided modes attenuation during their propagation.

The modes are well guided when the minima values turn to zero. For nonabsorbing structures these minima reach zero. To illustrate absorption effect for guided waves the amplitude of waveguide term is calculated for three layer sandwich structure AlN / Fe / AlN / glass (see Fig. 1). The minima (in order from left side) represent modes: TM_3, TE_3, TM_2, TM_1, TE_1, TM_0. The modes TM_3, TE_2, TM_1, TE_0 are more attenuated, the quality of wawveguiding is closely connected with field distribution for each mode.

In the second figure (Fig. 2) the influence of magnetooptical anisotropy on amplitude difference is presented. It is very small, even for well guided modes this difference goes to zero. In general, we can say, that for such modes magnetooptical effect vanish. In Tab. 1 are shown values of amplitudes minima (guided modes) and their positions (effective indices of refraction) for our basic magnetooptic configurations. We can see that the effective refraction index shift of guided modes is very small. These facts entire us to use more simple isotropic dispersion relations also during analysis of waveguiding in corresponding anisotropic media. Formalism, presented in second part of the paper, can be useful for deriving of dispersion relations for complex multilayered structures.

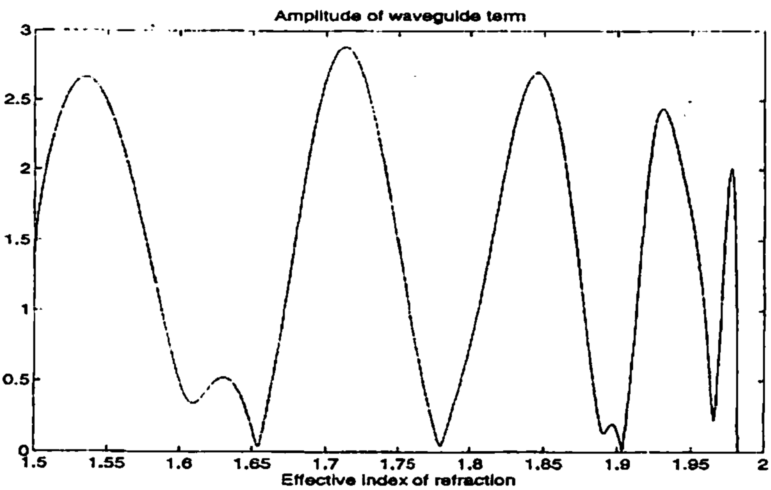

Fig. 1. Amplitude of waveguide term for λ=632.8 nm for following structure:
AlN (t = 500 nm, N = 1.98) / Fe (t = 3 nm, N=2.86 -i 3.68) / AlN(t=500 nm, N=1.98) / glass (n=1.5)

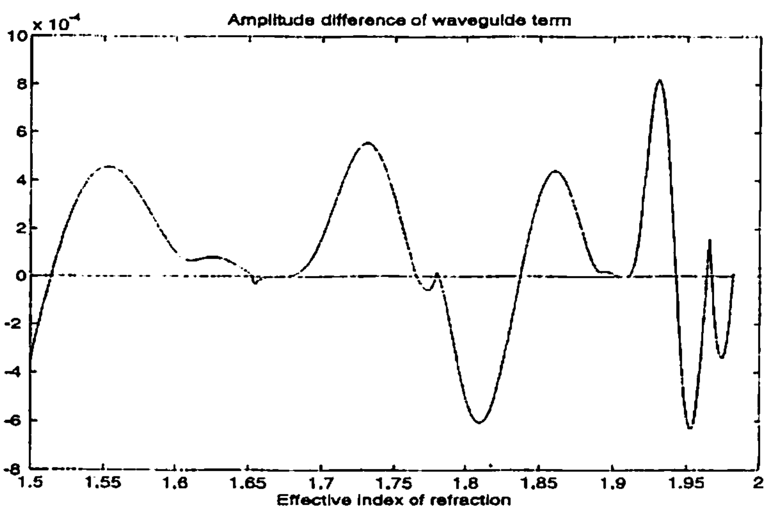

Fig. 2 Amplitude difference of waveguide term in longitudinal polarisation (Q = 0.034 - i 0.02) relative to corresponding isotropic system, presented in Fig. 1

Mode order	isotropic	anisotropic transversal	anisotropic longitudinal	anisotropic polar	waveguide term value
TM_3	1.6091	-/+ 5.8 . 10^{-5}	+ 1.7 . 10^{-7}	+ 1.6 . 10^{-4}	0.3395
TE_3	1.6534	0	+ 3.5 . 10^{-7}	- 2.8 . 10^{-4}	0.0327
TM_2	1.7796	+/- 2.7 . 10^{-5}	+ 6.0 . 10^{-7}	+ 2.8 . 10^{-4}	0.0392
TM_1	1.8908	-/+ 3.2 . 10^{-5}	+ 6.6 . 10^{-8}	+ 7.3 . 10^{-5}	0.1314
TE_1	1.9031	0	-3.4 . 10^{-8}	+ 2.8 . 10^{-4}	0.0053
TM_0	1.9655	+/- 2.2 . 10^{-6}	+ 2.7 . 10^{-4}	+ 8.2 . 10^{-5}	0.2204

Tab. 1. Effective refraction indices of guided modes: absolute values of isotropic configuration and anisotropy shift relative to isotropic configuration. The signes +/- represent the different directions of these shifts for opposite direction of magnetisation.

2. Dispersion relations in isotropic multilayers

Through the extension of the Yeh's and Višňovský's formalism [1,2] the dispersion properties of multilayered waveguides for TE and TM modes are expressed in simple and general analytic forms. These formulas describe the variation of the effective refractive indices with respect to the variation of the refractive index of any layer or the thickness of the guiding system. We will consider the structure of m homogenous layers separated by the interfaces with z_n ($n = 0, 1, \dots, m$) coordinates in a Cartesian system, $z_{n-1} < z_n$. The complex refractive index is

$$N^{(n)} = n^{(n)} - i\, k^{(n)} \tag{2}$$

and

$$N_y = N^{(n)}\, a_y^{(n)}, \tag{3}$$

$$N_z^{(n)} = N^{(n)}\, a_z^{(n)}, \tag{4}$$

for $n = 0, 1, \dots, m+1$; $a_y^{(n)}$ and $a_z^{(n)}$ describe the complex direction cosines. Let factor $P^{(n)}$ describes the phase change of wave during its propagation through n - th layer,

$$P^{(n)} = \exp\left(i \frac{\omega}{c} N_z^{(n)} t_n\right). \tag{5}$$

Here t_n is the thickness of n - th layer. The reflectance and transmitance at j - k interface is denoted by Jones matrix, which fulfils the relation

$$\begin{pmatrix} A_1^k \\ A_2^j \end{pmatrix} = \begin{pmatrix} t^{jk} & r^{kj} \\ r^{jk} & t^{kj} \end{pmatrix} \cdot \begin{pmatrix} A_1^j \\ A_2^j \end{pmatrix}. \tag{6}$$

Here A_1^j, A_2^k are amplitudes of incident waves in the k - th and j - th medium, A_1^k, A_2^j are waves, propagating from the j/k boundary. Equation (6) can be transformed to the different form (with using identity $t^{jk}t^{kj} - r^{jk}r^{kj} = 1$)

$$\begin{pmatrix} A_1^j \\ A_2^j \end{pmatrix} = \frac{1}{t^{jk}} \begin{pmatrix} 1 & -r^{kj} \\ r^{jk} & 1 \end{pmatrix} \cdot \begin{pmatrix} A_1^j \\ A_2^j \end{pmatrix} = \mathbf{S}^{jk} \cdot \begin{pmatrix} A_1^j \\ A_2^j \end{pmatrix}. \tag{7}$$

The total transfer matrix of multilayer system is specified by equation

$$\begin{pmatrix} A_1^0 \\ A_2^0 \end{pmatrix} = \mathbf{S}^{01}\mathbf{P}^1\mathbf{S}^{12}\mathbf{P}^2 \dots \mathbf{P}^{m-1}\mathbf{S}^{m-1,m} \cdot \begin{pmatrix} A_1^m \\ A_2^m \end{pmatrix} = \mathbf{S} \cdot \begin{pmatrix} A_1^m \\ A_2^m \end{pmatrix}, \tag{8}$$

where

$$\mathbf{P}^n = \begin{pmatrix} P^n & 0 \\ 0 & \dfrac{1}{P^n} \end{pmatrix} \tag{9}$$

is propagation matrix of n-th layer. The condition

$$S_{11} = 0 \tag{10}$$

present the dispersion relation of the waveguide formed by the structure. To derive analytic form of S_{11} we can use the following method. It is convenient to define

$$X_j = r^{j,j-1} r^{j,j+1} \left[P^{(j)}\right]^2, \tag{11}$$

$j = 1, 2, \dots, m$.

Equation (11) provides the required information about the interaction of the plane electromagnetic wave in j - th area. Turning to the waveguide condition in multilayered system, we can write on the base of Eq. (11)

$$\prod_{j=1}^{m} \left(1 - X_j\right) \tag{12}$$

Finally, the multiplication of each partial products $X_i X_{i+1}$ $(i \in \langle 1, m-1 \rangle)$ with succeed indices (for immediately adjoining layers) by elements

$$-\frac{1}{r^{i,i+1} r^{i+1,i}} \tag{13}$$

in expanded term (12) allow us to obtain guided mode condition. According to Eq. (10) the final product of this evaluation equal to zero represents condition of the waveguiding in relevant planar structure.

Example of four-layered planar system illustrates the use of discussed process:

$$\begin{aligned}\prod_{j=1}^{4} \left(1 - X_j\right) = 1 - X_1 - X_2 - X_3 - X_4 + X_1 X_2 + X_2 X_3 + X_3 X_4 + X_1 X_3 + X_1 X_4 + \\ + X_2 X_4 - X_1 X_2 X_3 - X_2 X_3 X_4 - X_1 X_3 X_4 - X_1 X_2 X_4 - X_1 X_2 X_3 X_4\end{aligned} \tag{14}$$

Multiplying of partial multiples $X_i X_{i+1}$ according to (13) gives

$$\begin{aligned}1 - X_1 - X_2 - X_3 - X_4 - \frac{1}{r^{12} r^{21}} X_1 X_2 - \frac{1}{r^{23} r^{32}} X_2 X_3 - \frac{1}{r^{34} r^{43}} X_3 X_4 + \\ + X_1 X_3 + X_1 X_4 + X_2 X_4 - \frac{1}{r^{12} r^{21} r^{23} r^{32}} X_1 X_2 X_3 - \frac{1}{r^{23} r^{32} r^{34} r^{43}} X_2 X_3 X_4 - \\ + \frac{1}{r^{34} r^{43}} X_1 X_3 X_4 + \frac{1}{r^{12} r^{21}} X_1 X_2 X_4 - \frac{1}{r^{12} r^{21} r^{23} r^{32} r^{34} r^{43}} X_1 X_2 X_3 X_4 = 0,\end{aligned} \tag{15}$$

which is the guided mode condition in planar structure consisting of four thin films. Analogicaly we can realise the waveguide conditions for planar systems, whose are prepared by multiple repetition of two-layered or three-layered structures.

The first results show, that presented method can be used also for transversal geometry of magnetisation. A more detail discussion is planned in a future paper.

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