

AN EFFECTIVE DESIGN METHOD FOR SUM AND DIFFERENCE PATTERN OF ARRAY OF DBF IN SUBARRAY

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Abstract—The realization of the DBF (digital beamforming) technique can complicate the equipments, therefore, in many cases, its application is primarily limited at the level of subarray. In this contribution we analyze the sum and difference pattern function of the DBF in subarray and deduce the condition for the formation of grating lobes. And we modify the conventional weighting methods for suppressing sidelobe, which are employed in DBF, namely, according to the relative positions of the subarrays we modify the weighting coefficients in the array to be suitable for the DBF in subarray. This kind of modified methods is adaptable for non-uniform array. Both of the analyses and the simulation results demonstrate that presented methods are effective.

Key words: Digital beam forming; DBF in subarray; antenna pattern; modified weighting method

I. INTRODUCTION

DBF is a key technique in signal processor, and one of the most important mark for the modern radar. The structure of DBF is limited by many factors, such as, the cost, the amount of calculation, etc.. Because the realization of the DBF technique can complicate the equipments, in many cases the application of this technique is primarily limited at the subarray level, and is not widely employed at array element level. The DBF in subarray differs obviously from the DBF composed of array elements in the pattern function design^[1,2,5] and in the channel mismatch correction^[3,4,5].

In this paper we study some problems of the analyses and the design of pattern function for the DBF in subarray, including the sum and difference pattern function, the condition for grating lobe formation, and a kind of modified weighting methods for suppressing sidelobe.

II. A KIND OF MODIFIED WEIGHTING METHODS ADAPTING TO DBF IN SUBARRAY

Some commonly used weighting methods, f.i., the Hamming weighting etc. are usually used to suppress sidelobe of directivity pattern of array. However these methods are only suitable for the DBF composed of array elements. For the DBF in subarray, the non-uniform array is usually formed because the numbers of the array elements in different subarrays are different. The above mentioned methods are not valid because the non-uniformity of the array structure changes the corresponding relation between the weighting coefficients and the subarrays.

Here we modify the conventional methods, namely adjust the weighting coefficients according to the array structure in order that the weighting coefficients match the subarrays.

We use $w(n)$ to represent the conventional weighting function, and then it is the function of the serial number of the array element n , and the total element number of the array N :

$$w(n) = g(n, N). \quad (1)$$

For the DBF in subarray, we assume that all the array elements are arranged with the equal distance d , the number of the total subarrays in the DBF is N , and M_n represents the number of the array elements in the n th subarray. In order to suit weighting function to DBF in subarray, we modify $w(n)$ to be $w'(n)$

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with

$$w'(n) = g(l_d(n), L_d), \quad (2)$$

where

$$l_d(n) = \begin{cases} 0, & n=0 \\ M_0/2 + M_1/2, & n=1 \\ M_0/2 + \sum_{k=1}^{n-1} M_k + M_n/2, & 2 \leq n \leq N-1 \end{cases} \quad \text{and} \quad L_d = M_0/2 + \sum_{k=1}^{N-2} M_k + M_{N-1}/2. \quad (3)$$

The process of modification is as the follows. At first we assume that a uniform array is formed between the leftmost and the rightmost subarrays and the distance between the array elements are equal to those of the original ones. Then the uniform array is weighted with the conventional weighting function and thus a window is formed. After that the window is sampled with the sampling points corresponding to the relative positions of each subarray in the array. The sampling values are the modified weighting coefficients. The above process is shown in the Fig.1.

In the window formed with the conventional weighting coefficients, the abscissa shows the serial number of the array elements. It is seen that the weighting coefficients are arranged with the same distance. For the non-uniform array composed of subarrays, the modified weighting coefficients form an another window. The values of the abscissa are determined by the relative positions of the subarrays in the array. It is obvious that here the weighting coefficients are not arranged with the same distance any more.

No matter how the structure of the array composed of subarrays is, the two windows overlap, which indicates that the modified weighting and the conventional weighting methods are equivalent for the choice of the weighting coefficients, namely the modified weighting can adjust the coefficients according to the array structure, therefore, it is self-adaptable for non-uniform arrays.

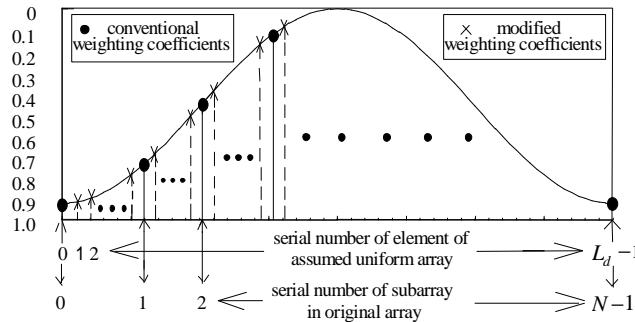


Fig.1 Diagram of the modification process for the conventional weighting.

III. ANALYSES OF PATTERN FUNCTIONS

A. Sum Beam Pattern Function

We assume that the angle between the sum beam direction and the normal direction of the array is q_k . The pattern function of sum beam of the DBF in subarray is

$$F_{\Sigma}(q, q_k) = \sum_{n=0}^{N-1} w'(n) f(n, q) e^{j \frac{2p}{l} l(n) (\sin q - \sin q_k)}, \quad (4)$$

where $w'(n)$ is used to suppress sidelobe, and the grating lobes which are probably to appear, $f(n, q)$ is the pattern function of the n th subarray.

We assume that the array factor of the n th subarray is $f_1(n, q)$, then have

$$f_1(n, q) = \sum_{m=0}^{M_n-1} e^{j \frac{2p}{l} m d (\sin q - \sin q_k)}. \quad (5)$$

We assume: 1. all array elements are same, and pattern functions are assumed to be $f_2(q)$. 2. the

coupling effect between the array elements is ignored, then

$$f(n, \mathbf{q}) = f_1(n, \mathbf{q}) f_2(\mathbf{q}) . \quad (6)$$

B. The Condition for the Formation of Grating Lobes

If the element numbers of all subarrays are same, then $f_1(n, \mathbf{q})$ is independent of n . We substitute here $f_1(\mathbf{q})$ for $f_1(n, \mathbf{q})$ and let $M_n = M$, then from (5) we obtain

$$f_1(\mathbf{q}) = \sum_{m=0}^{M-1} e^{j \frac{2p}{l} m d (\sin \mathbf{q} - \sin \mathbf{q}_k)} = \frac{\sin[M(p/l)d(\sin \mathbf{q} - \sin \mathbf{q}_k)]}{\sin[(p/l)d(\sin \mathbf{q} - \sin \mathbf{q}_k)]} . \quad (7)$$

In this case (4) is simplified as:

$$F_{\Sigma}(\mathbf{q}, \mathbf{q}_k) = f_1(\mathbf{q}) f_2(\mathbf{q}) \sum_{n=0}^{N-1} w'(n) e^{j \frac{2p}{l} M n d (\sin \mathbf{q} - \sin \mathbf{q}_k)} \quad (8)$$

If $w'(n) = 1 (0 \leq n \leq N-1)$ and all array elements have the same directivity function in all directions, then we have

$$F_{\Sigma}(\mathbf{q}, \mathbf{q}_k) = f_1(\mathbf{q}) \sum_{n=0}^{N-1} e^{j \frac{2p}{l} M n d (\sin \mathbf{q} - \sin \mathbf{q}_k)} = \frac{\sin[MN(p/l)d(\sin \mathbf{q} - \sin \mathbf{q}_k)]}{\sin[(p/l)d(\sin \mathbf{q} - \sin \mathbf{q}_k)]} . \quad (9)$$

If the following relation is satisfied:

$$(p/l)d(\sin \mathbf{q} - \sin \mathbf{q}_k) = \pm k p \quad (k \text{ is positive integer}), \quad (10)$$

we obtain

$$F_{\Sigma}(\mathbf{q}, \mathbf{q}_k) = MN . \quad (11)$$

In this case the beam has multilobe. If $(p/l)d(\sin \mathbf{q} - \sin \mathbf{q}_k) = 0$ is satisfied, then at $\mathbf{q} = \mathbf{q}_k$ we find mainlobe, and the rest are the grating lobes.

Therefore, if the following conditions are at the same time satisfied, the grating lobes will appear: 1. no amplitude weighting is used; 2. the numbers of the array elements in the all subarrays are same; 3. all array elements have the pattern function which is same in all directions.

C. Difference Beam Pattern Function

Difference beam can be formed from two sum beams. We assume that the null direction of the difference beam is \mathbf{q}_{Δ} , and angles between the sum beam and the null direction are $\pm \Delta \mathbf{q}$, respectively, then the difference beam pattern function is

$$\begin{aligned} F_{\Delta}(\mathbf{q}, \mathbf{q}_{\Delta}, \Delta \mathbf{q}) &= f_2(\mathbf{q}) \sum_{n=0}^{N-1} w'(n) f_1(n, \mathbf{q}) e^{j \frac{2p}{l} l(n) [\sin \mathbf{q} - \sin(\mathbf{q}_{\Delta} + \Delta \mathbf{q})]} - f_2(\mathbf{q}) \sum_{n=0}^{N-1} w'(n) f_1(n, \mathbf{q}) e^{j \frac{2p}{l} l(n) [\sin \mathbf{q} - \sin(\mathbf{q}_{\Delta} - \Delta \mathbf{q})]} \\ &= -j 2 f_2(\mathbf{q}) \sum_{n=0}^{N-1} w'(n) f_1(n, \mathbf{q}) \sin[\mathbf{a} \cdot (2p/l) l(n)] e^{j \frac{2p}{l} l(n) [\sin \mathbf{q} - \sin(\mathbf{q}_{\Delta}) \cos(\Delta \mathbf{q})]} , \end{aligned} \quad (12)$$

where

$$\mathbf{a} = \cos(\mathbf{q}_{\Delta}) \sin(\Delta \mathbf{q}) .$$

IV. COMPUTER SIMULATIONS

Table I shows the array structure of a DBF, which includes 72 array elements. These array elements are grouped to be 26 subarrays. Each array element is the half-wave vibrator, and $d = l/2$.

In Table II the Hamming weighting and the modified Hamming weighting coefficients are listed (The coefficients are symmetric, and the half of them are given). For this array, the modified coefficients are larger than the corresponding original weighting coefficients, because the element number of the subarrays on the both sides is the maximum, and the nearer a subarray to the array center, the less is the number of the elements in the subarray.

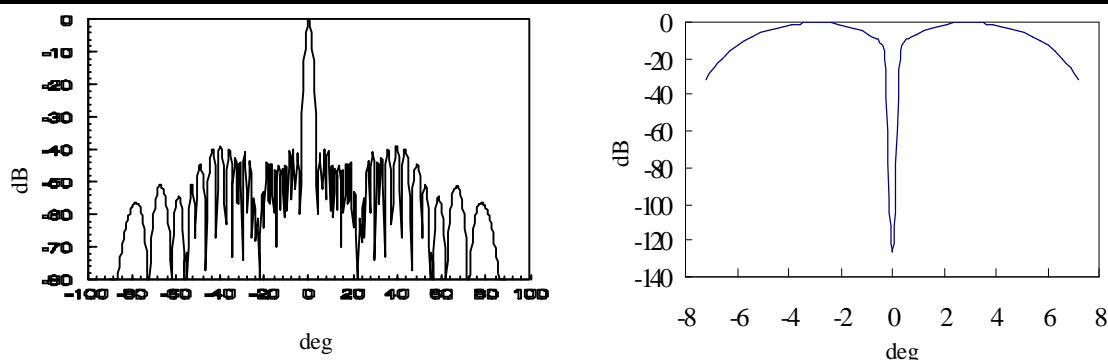
Fig.2 shows the sum and the difference beam pattern of the modified Hamming weighting for the non-uniform subarray shown in Table I . Table III gives the properties of the sum and the difference beam pattern. For the sum beam, the sidelobe level is -38.90 dB; for the difference beam, the null depth level is -125.9 dB. We see that due to the difference of the directivity patterns of the subarrays, the modified weighting for the non-uniform subarrays makes sidelobe level higher than that caused by the conventional weighting for the uniform array composed of the array elements.

Table I THE DIVISION OF THE SUBARRAYS (THE ARRAY STRUCTURE IS SYMMETRIC)

Serial number of subarray	0	1	2	3	4	5	6	7	8	9	10	11	12
Serial number of element	0-3	4-7	8-11	12-14	15-17	18-20	21-23	24-25	26-27	28-29	30-31	32-33	34-35

Table II THE COMPARISON OF THE HAMMING WEIGHTING AND THE MODIFIED HAMMING WEIGHTING COEFFICIENTS

Serial number of subarray	0	1	2	3	4	5	6	7	8	9	10	11	12
Hamming weighting	0.080	0.094	0.137	0.205	0.294	0.398	0.511	0.626	0.736	0.833	0.912	0.968	0.996
Modified Hamming weighting	0.080	0.111	0.200	0.316	0.435	0.561	0.686	0.782	0.850	0.907	0.952	0.982	0.998



(a) The directivity pattern of the sum beam($q_{\Sigma} = 0^{\circ}$) (b) The directivity pattern of the difference beam($q_{\Delta} = 0^{\circ}, \Delta q = 3^{\circ}$)

Fig.2 The directivity patterns of the sum and the difference beam obtained by the modified Hamming weighting

Table III THE PROPERTIES OF THE SUM AND THE DIFFERENCE BEAM PATTERNS USING THE MODIFIED HAMMING WEIGHTING

Sum Beam			Difference Beam
Sidelobe level(dB)	3dB Spreading of mainlobe	SNR loss(dB)	Null depth level (dB)
-38.90	1.530	1.402	-125.9

V. CONCLUSIONS

We present a kind of weighting methods to improve the conventional methods for suppressing sidelobes of the DBF in subarray. The modified methods adjust the weighting coefficients according to the array structure and are adaptable for non-uniform array. The sum and difference directivity patterns of high quality can be achieved by using these methods. The computer simulations show that our methods are rather effective for DBF in subarray.

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