

## MUTUAL COUPLING EFFECT ON PERFORMANCE OF ADAPTIVE ARRAY ANTENNA

Qiaowei Yuan<sup>1</sup>, Qiang Chen<sup>2</sup> and Kunio Sawaya<sup>2</sup><sup>1</sup>Intelligent Cosmos Research Institute Co. Ltd., <sup>2</sup>Tohoku University  
04 Aoba, Aramakizi, Aoba-ku, 980-8579, Sendai, Japan

## I. INTRODUCTION

It has been pointed out that the performance of an adaptive array antenna (AAA) is strongly affected by the mutual coupling between array elements [1]. The results showed that effect of the mutual coupling is particularly serious for small element spacing where the output signal-to-interference-noise ratio (*SINR*) is much lower than that obtained when mutual coupling is ignored and the convergence of least mean square (LMS) becomes slow. Therefore, many researches using the open-circuit voltage method to remove the mutual coupling effect from the received signals have been published. However, since the open-circuit voltage method can not incorporate the scattering of the antenna elements whose driving points are open, it is effective only for the short dipole element. Sakar et al. used the method of moment (MoM) [2] and Hui used redefined impedance matrix to compensate the mutual coupling for AAA [3], [4]. However, Sakar's method requires the DOAs of incident signals which are impossible in practical case and Hui's method needs the current distribution which is difficult to be obtained when the array element has complicated structure.

To avoid the problems mentioned above, a method using the received voltages at the array antenna terminals to estimate the optimum weights is proposed. The input *SINR* and the output *SINR*, the convergence of the adaptive algorithm and the synthesized pattern in the presence of the mutual coupling are investigated by simulations for 6-element dipole array antenna and 2-element monopole array antenna mounted on a mobile handset. Since the mutual coupling is usually removed by multiplying the received voltages with a compensation matrix inserted between the array element terminals and the adaptive processor, the effect of such matrix on the output *SINR* is also studied.

## II. RECEIVED VOLTAGES AND USV CALCULATION

## A. Received voltages

An *M*-element array antenna consisting of elements with arbitrary geometry is used as the receiving antenna as shown in Fig. 1. The received voltage  $v_i$  of the *i*th element terminal can be obtained by the following expression.

$$v_i = a_i(\theta, \phi, p)s(t) + v_i^n \quad (1)$$

where  $s(t)$  denotes the electric field of incident wave from  $(\theta, \phi)$  direction with  $p$  polarization and  $a_i(\theta, \phi, p)$  represents the element pattern of *i*th element.  $v_i^n$  is the voltage caused by the thermal noise in the receiving circuit. For the *M* received voltage vector at the array element terminals, the following matrix can be obtained.

$$[V] = [A]s + [V^n] \quad (2)$$

where  $[V^n]$  represents the noise vector with dimension *M*.  $[A]$  is the vector with dimension *M* composed of the element patterns of array elements and is called the steering vector. Since the steering vector here takes account of the mutual coupling and the individual element radiation property, it is different from the conventional steering vector (CSV) representing the array factor and called the universal steering vector (USV).

## B. USV Calculation

Although the USV is not required when the optimum weights are estimated directly from the received voltages, it is required for obtaining the synthesised pattern. The USV can be obtained by measuring the complex array element pattern when the other array elements are terminated. However, the accurate measurement of the array element pattern is not easy and the numerical calculation for USV was performed by using the method of moments (MoM).

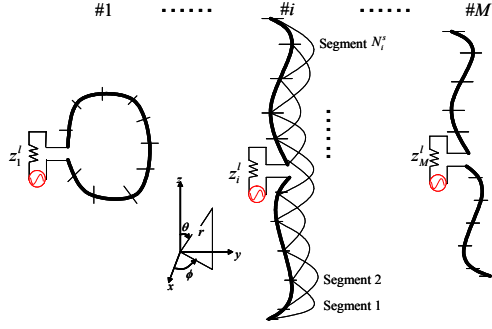


Fig. 1. Array antenna with arbitrary geometry

The  $i$ th element of the array antenna shown in Fig. 1 is divided into  $N_i^s$  segments and the following matrix equation can be obtained by the MoM

$$[V^{inc}] = [Z][I] \quad (3)$$

where  $[I]$  is the unknown current vector with dimension of  $N$ , and  $[Z]$  is the  $N \times N$  impedance matrix.  $[V^{inc}]$  is the  $N$  voltage vector representing the inner product of the weighting functions and the incident electric field.  $N$  equals to  $\sum_{i=1}^M N_i^s$  representing the total number of the segments.

The current coefficients at the terminals of the antenna elements can be obtained by

$$[I^{ter}] = [Y^{ter}][V^{inc}] \quad (4)$$

where  $[I^{ter}]$  is the  $M$  current vector representing the currents at the antenna terminals and  $[Y^{ter}]$  with dimension of  $M \times N$  is the part of admittance matrix  $[Y]$  corresponding to the mutual admittances.

Assuming that the terminal of the  $i$ th element is loaded by an impedance of  $z_i^l$ , the received voltages which are the elements of the USV are obtained by

$$[A^u(\theta, \phi, p)] = [z^l][Y^{ter}][V^{inc}] \quad (5)$$

### III. PERFORMANCES OF AAA IN THE PRESENCE OF MUTUAL COUPLING

#### A. Input SINR and Output SINR

Using the LMS algorithm to estimate the optimum weights for  $M$ -element array illuminated by one desired signal and  $K$  undesired signals, the steady state optimum weight vector  $[W]$  can be given by

$$[W] = [R_{vv}]^{-1}[r_{vr}] \quad (6)$$

where  $[R_{vv}]$  and  $[r_{vr}]$  are the covariance matrix of the received voltages and the reference correlation vector, respectively, and given by

$$[R_{vv}] = E([V][V]^H), [r_{vr}] = E(r(t)[V]^*) \quad (7)$$

where  $E(\cdot)$  denotes the statistical expectation, superscript asterisk and  $H$  denote the complex conjugate and the transpose conjugate, respectively.  $r(t)$  represents the reference signal.

It should be noted that the received voltage vector  $[V]$  in (7) is different from that in (2) because there are  $k+1$  incident signals in (7).  $[V]$  in (7) should be

$$[V] = [V^d] + [V^u] + [V^n] \quad (8)$$

where  $[V^d]$  is the voltages generated by the desired wave  $s_d$  and  $[V^u]$  is the summation of the voltages generated by  $K$  undesired waves which can be given by

$$[V^d] = [A(\theta_d, \phi_d, p_d)]s_d, [V^u] = \sum_{i=1}^K [A(\theta_u^i, \phi_u^i, p_u^i)]s_u^i \quad (9)$$

The input SINR of  $M$ -element array is given by

$$SINR^{in} = \frac{\sum_{i=1}^M |v_i^d|^2}{\sum_{i=1}^M |v_i^u|^2 + \sum_{i=1}^M |v_i^n|^2} \quad (10)$$

while the output SINR can be obtained by using the optimum weight  $[W]$  as

$$SINR^{out} = \frac{\sum_{i=1}^M |w_i^* v_i^d|^2}{\sum_{i=1}^M |w_i^* v_i^u|^2 + \sum_{i=1}^M |w_i^* v_i^n|^2} \quad (11)$$

The SINR of 6-element dipole array shown in Fig.2(a) are calculated when only one desired wave is considered as done in [1]. The incident SINR is defined as the ratio of the incident signal power to the thermal noise power. Each element of the array is loaded by the conjugate of its self-impedance when the other elements are open. Only the output SINR was shown in [1], however, the input SINR are also given in Fig. 2(b). The results are compared with those without the mutual coupling. It is found that both of the output and the input SINR depend on DOA of desired signal due to the mutual coupling.

Moreover, it should be noted that the mutual coupling does not always degrade the output SINR and the input SINR. It should be also mentioned that the

output  $SINR$  is always greater than the input  $SINR$  by 7.8dB which is equivalent to  $10\log(M)$ , independently of the DOA and the presence of the mutual coupling. It means that mutual coupling does not affect the enhancement of  $SINR$  by the adaptive process. Therefore, the mutual coupling fundamentally affects the input  $SINR$ , rather than the output  $SINR$  which is obtained by adding the enhancement of  $SINR$  to the input  $SINR$ .

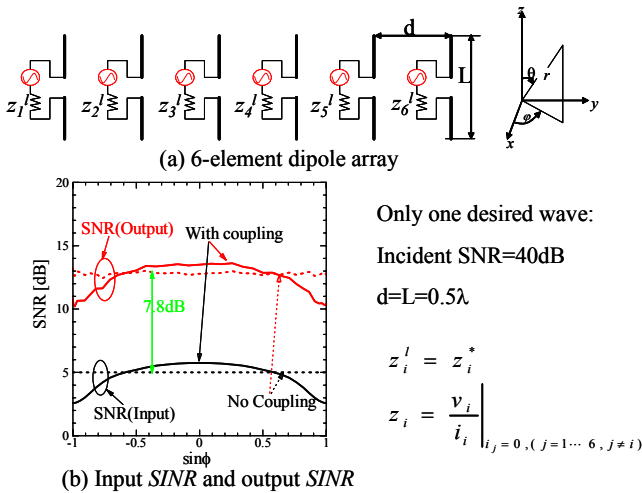


Fig. 2. Input and output  $SINR$  for dipole array

### B. Convergence of LMS Algorithm

It is well-known that the convergence of LMS algorithm becomes fast when the ratio of the maximum eigenvalue to the minimum eigenvalue of the covariance matrix decreases. For  $M$ -element array, there are  $M$  eigenvalues which are associated with the power of the desired and undesired signals, and also the noise power. When the undesired signals are absent, the maximum eigenvalue can be given by [5]

$$\lambda_1 = \lambda_{\max} = \sigma^2(1 + SINR^{out}) \quad (12)$$

Since only one desired wave is considered, the remaining eigenvalues  $\lambda_i = \sigma^2 (i=1, \dots, M)$  associated with the noise power  $\sigma^2$  are the same.

The convergence of mean squared error (MSE) of LMS for the 6-element dipole array antenna with changing the spacing  $d$  is shown in Fig. 3(a) comparing with that in the absence of the mutual coupling effect where the DOA of the desired signal is  $(\theta=90^\circ, \phi=0^\circ)$ . Although the mutual coupling becomes stronger when  $d$  is reduced from  $0.5\lambda$  to  $0.1\lambda$ , the convergence performances fast. It can be explained from the eigenvalues which become small when the spacing  $d$  is reduced as shown in Fig. 3(b).

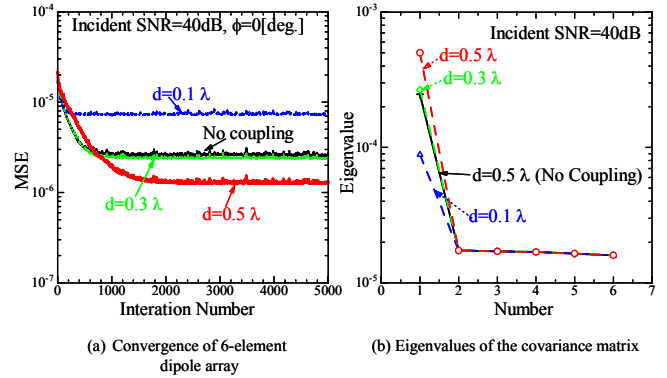


Fig. 3. Convergence and eigenvalues of dipole array

From the above simulation examples, it can be said that the mutual coupling makes the convergence of the LMS algorithm faster in some cases, but makes the convergence slower in another cases.

### C. Synthesized pattern

The synthesized radiation pattern  $f(\theta, \phi, p)$  of the array should be calculated by using the USV in the presence of the mutual coupling as shown as

$$f(\theta, \phi, p) = [W]^H [A(\theta, \phi, p)] \quad (13)$$

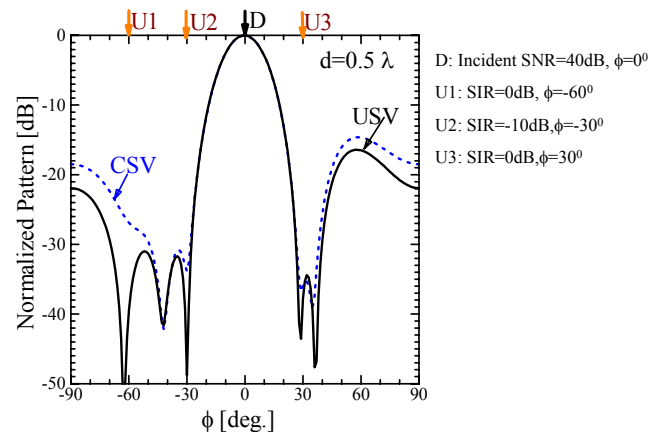


Fig. 4. Synthesized pattern for 6-element dipole array

The synthesized pattern for 6-element dipole array is illustrated in Fig. 4 where the desired signal comes from  $\phi=0^\circ$ , and three undesired signals come from  $\phi=-60^\circ, \phi=-30^\circ, \phi=30^\circ$  in  $xy$  plane, respectively. The incident  $SINR$  and signal-to-interference ( $SIR$ ) of three undesired waves are 5dB, 0dB, -10dB, 0dB respectively.

The synthesized pattern for 2-element monopole array mounted on a mobile handset is illustrated in Fig. 5 where the desired signal comes from  $\phi=0^\circ$ , and the

undesired signals comes from  $\phi=50^\circ$  in  $xy$  plane. The input  $SINR$  and  $SIR$  of the undesired wave are 20dB, 0dB respectively.

It is found that the pattern can be accurately synthesized even in the presence of the mutual coupling by using USV, but not by using CSV.

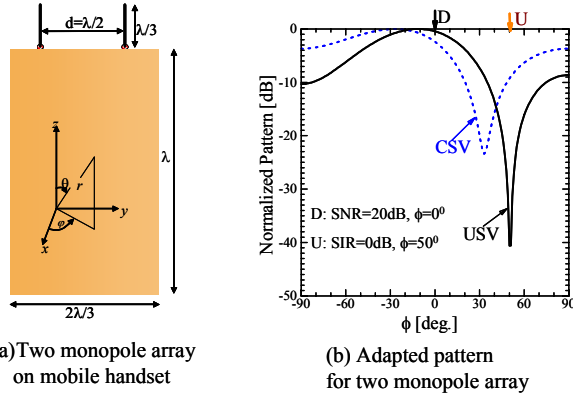


Fig. 5. Synthesized pattern for 2-element monopole array on mobile handset

#### IV. EFFECT OF COMPENSATION MATRIX ON OUTPUT $SINR$

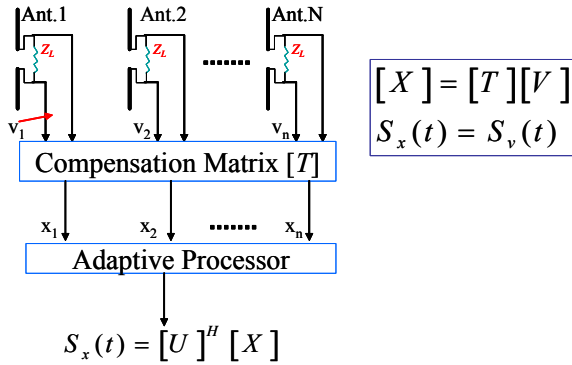


Fig. 6. Adaptive array antenna using  $[T]$

If the received voltage vector  $[V]$  is compensated by the invertible matrix  $[T]$  inserted between the array terminals and the adaptive processor as shown in Fig.6 to remove the coupling mutual effect, the compensated signals  $[X]$  can be obtained by

$$[X] = [T][V] \quad (14)$$

The steady state weight vector  $[U]$  by LMS algorithm for  $[X]$  can be obtained similar to (6) with calculating the covariance matrix  $[R_{xx}]$  and the reference correlation vector  $[r_{xr}]$ . Since the matrix  $[T]$  is assumed to be invertible, the following equation can be obtained.

$$S_x = [U]^H [X] = [W]^H [V] = S_v \quad (15)$$

which means that inserting a linear invertible compensation matrix  $[T]$  between the array terminal and the adaptive processor brings no effect on the output signal or the output  $SINR$ .

#### V. CONCLUSIONS

The property of the AAA has been numerical analyzed by using the received voltage at array terminals. It has been found that the mutual coupling affects the input  $SINR$  rather than the output  $SINR$ , but does not always degrade the output  $SINR$  and reduce the convergence of the adaptive algorithm. And it has been also found that the pattern using the weights obtained in the presence of the mutual coupling effect can be accurately synthesized to make the main beam to the DOA of desired wave and the nulls to the DOAs of undesired waves. Finally, it has been shown that any linear invertible compensation matrix inserted between the array terminal and the adaptive processor can not be expected to improve the output  $SINR$  of AAA.

#### ACKNOWLEDGEMENTS

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