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TRANSIENT ANALYSIS OF ELECTROMAGNETIC WAVES USING PRINCIPLES OF WAVE DIGITAL FILTERS

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1. Introduction

Recently, a new method for numerically integrating partial differential equations (PDEs) that describe physical systems has been proposed so far [1,2]. The method is based on certain principles for multidimensional (MD) digital signal processing. The resulting algorithms have important advantages of which some of the major ones are preservation of passivity, preservation of massive parallelism, easy way to accomodate arbitrarily varying parameters as well as arbitrary boundary shapes and conditions.

Problems are analysed from a multidimensional point of view. First, we introduce usually four-dimensional manifold by adding time axis to three dimensional space. Then, sampling is done along the time axis similar to the usual digital signal processing. Therefore, this approach is suitable for the time-dependent or transient phenomena, since spatial and physical variables are calculated every sampling time. This paper aims at giving transient analysis of electromagnetic or light wave propagation by directly integrating Maxwell's equations in place of wave equation. The original system of Maxwell's equations is transformed into a multidimensional Kirchhoff circuit, which is later applied by the principles of multidimensional wave digital filter theory.

2. Transformation of Maxwell's equations

Maxwell's equations are given by

$$\nabla \times H = j + \frac{\partial(\varepsilon E)}{\partial t}, \tag{1}$$

$$\nabla \times E = -\frac{\partial(\mu H)}{\partial t}. \tag{2}$$

We have written t_1, t_2 and t_3 for the spatial variables x, y and z, respectively. And t stands for the time, which is transformed into t_4. We assume t_4 to have the dimension of a spatial variable and define it by means of a function v_4 equal to the derivative,

$$v_4 = \frac{dt_4}{dt}, \qquad v_4 > 0. \tag{3}$$

Physical variables are, in general, functions of vector $t = (t_1, t_2, t_3, t_4)^T$, but that v_4 depends at most on t_4.

Partial derivatives with respect to t_i are designated by D_i and a corresponding vector by D, thus

$$\mathrm{D} = (\mathrm{D}_1, \mathrm{D}_2, \mathrm{D}_3, \mathrm{D}_4); \qquad \mathrm{D}_i = \frac{\partial}{\partial t_i}. \qquad (i = 1 \text{ to } 4) \tag{4}$$

Introducing a coordinate transformation from t to t' expressed by a matrix H, we have

$$\mathrm{D}' = H^T\mathrm{D}, \quad \mathrm{D}' = (\mathrm{D}'_1, \mathrm{D}'_2, \mathrm{D}'_3, \mathrm{D}'_4)^T, \quad \mathrm{D}'_i = \frac{\partial}{\partial t'_i}, \qquad (i = 1 \text{ to } 4) \tag{5}$$

where H is an orthogonal Hadamard matrix as follows:

$$H = H^T = \frac{1}{2}\begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}. \tag{6}$$

By embedding four-dimensional space of t in higher seven-dimensional one of t'' with the aid of supplementary operators [2],

$$\mathrm{D}'' = (\mathrm{D}''_1, \mathrm{D}''_2, \mathrm{D}''_3, \mathrm{D}''_4, \mathrm{D}''_5, \mathrm{D}''_6, \mathrm{D}''_7), \tag{7}$$

we have transformed Maxwell's equations as follows:

$$\mathrm{D}''_7(\varepsilon' E_1) + (\mathrm{D}''_3 - \mathrm{D}''_6)E_5 + (\mathrm{D}''_5 - \mathrm{D}''_2)E_6 + \sigma' E_1 = 0, \tag{8a}$$
$$\mathrm{D}''_7(\varepsilon' E_2) + (\mathrm{D}''_1 - \mathrm{D}''_4)E_6 + (\mathrm{D}''_6 - \mathrm{D}''_3)E_4 + \sigma' E_2 = 0, \tag{8b}$$
$$\mathrm{D}''_7(\varepsilon' E_3) + (\mathrm{D}''_2 - \mathrm{D}''_5)E_4 + (\mathrm{D}''_4 - \mathrm{D}''_1)E_5 + \sigma' E_3 = 0, \tag{8c}$$
$$\mathrm{D}''_7(\varepsilon'' E_4) + (\mathrm{D}''_6 - \mathrm{D}''_3)E_2 + (\mathrm{D}''_2 - \mathrm{D}''_5)E_3 = 0, \tag{8d}$$
$$\mathrm{D}''_7(\varepsilon'' E_5) + (\mathrm{D}''_4 - \mathrm{D}''_1)E_3 + (\mathrm{D}''_3 - \mathrm{D}''_6)E_1 = 0, \tag{8e}$$
$$\mathrm{D}''_7(\varepsilon'' E_6) + (\mathrm{D}''_5 - \mathrm{D}''_2)E_1 + (\mathrm{D}''_1 - \mathrm{D}''_4)E_2 = 0, \tag{8f}$$

where ε', ε'' and E_i (i = 4 to 6) are respectively expressed by

$$\varepsilon' = 2r_0 v_4 \varepsilon_0, \quad \varepsilon'' = 2v_4\mu_0/r_0, \quad E_{i+3} = r_0 H_i, \qquad (i = 1 \text{ to } 3) \tag{9a,b,c}$$

and r_0 denotes intrinsic impedance [2].

3. Equivalent MD-passive Kirchhoff circuit

Eqs. (8a)~(8f) are transformed into a Kirchhoff circuit by replacing differential operators with equivalent inductances. Fig. 1 shows a Kirchhoff circuit representing Maxwell's equations, where $E_1 \sim E_6$ correspond to currents in usual electric circuits [2]. Furthermore, the function v_4 is limited by the condition that the equivalent inductance has to be positive, as follows [2]:

$$v_4 \geq \frac{2}{\sqrt{\varepsilon_{min}\mu_{min}}}. \tag{10}$$

4. Derivation of the WDF algorithm

We adopt so-called power waves instead of the voltage waves usually preferred in wave digital filtering. The WDF elements are connected by series adapters. The desired WDF algorithm can be immediately obtained by applying methods as known from the theory of MD-WDFs [3], which results in a wave-flow diagram shown in Fig. 2.

5. Experimental results

Numerical experiments are done for diffraction phenomenon of light wave in two dimensional space. A light wave with plane wavefront travels at a velocity of c from left-hand side and dashes perpendicularly a conductive screen with two apertures that are a wavelength wide and separated from each other by two wavelengths, as shown in Fig. 3. Here, calculations are carried out under the conditions that the wavelength λ of the wave is 0.6 μm and the direction of its polarization is parallel to the x-axis.

Figs. 4a ~ 4e show that the wavefront of the wave and the diffraction pattern change gradually with time, where the intensity of electric field of the wave is expressed by shade of each dot.

It is found that each figure is drawn like animation.

6. Conclusion

A transient analysis of electromagnetic wave propagation has been presented by directly integrating Maxwell's equations using principles of wave digital filters. Since actual physical space is treated as four-dimensional space-time manifold by adding time axis, the spatial and physical variables are calculated every sampling time. Therefore, this approach is suitable for the time-dependent or transient phenomena. In fact, it is foud that calculated results are drawn like animation every sampling time.

References

1. A. Fettweis and G. Nitsche, J. VLSI Signal Processing, 3, (1991) 7.
2. A. Fettweis and G. Nitsche, Multidimensional Systems and Signal Processing, 2 (1991) 127.
3. A. Fettweis, Proc. IEEE, 74 (1986) 276.

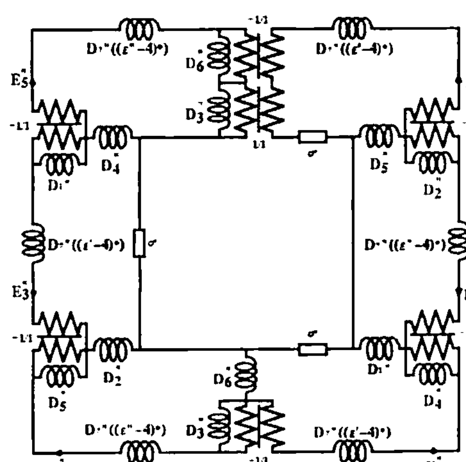

Fig. 1 MD Kirchhoff circuit representing Maxwell's equations.

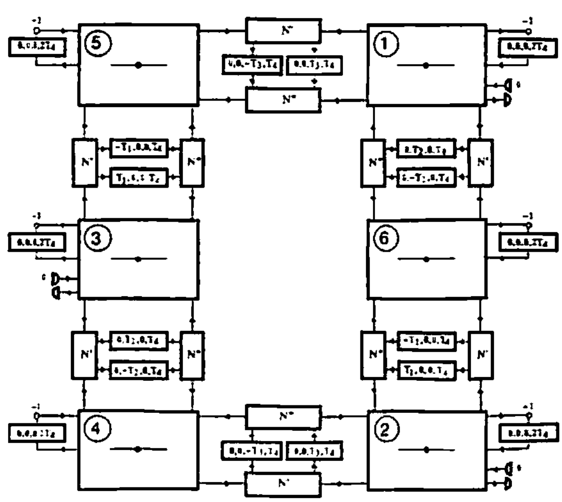

Fig. 2 Wave-flow diagram for Maxwell's equations.

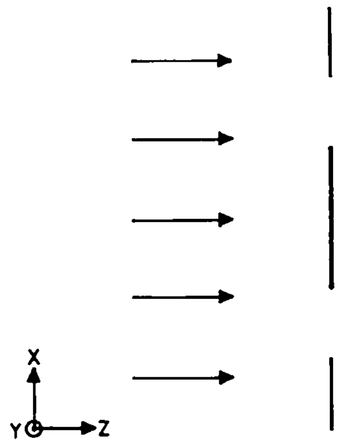

Fig. 3 Coordinate system and screen with two apertures.

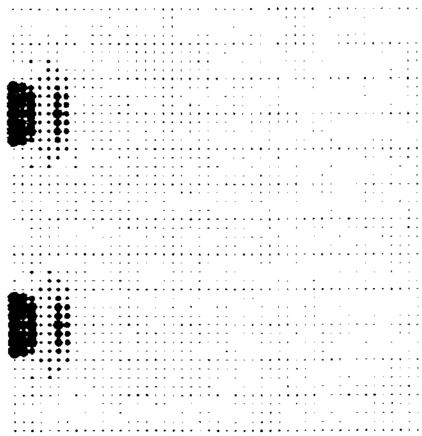

Fig. 4a Wavefront and diffraction pattern for $t = 0.833\lambda/c$.

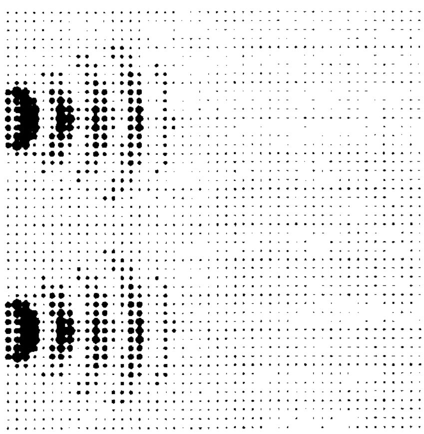

Fig. 4b Wavefront and diffraction pattern for $t = 2.417\lambda/c$.

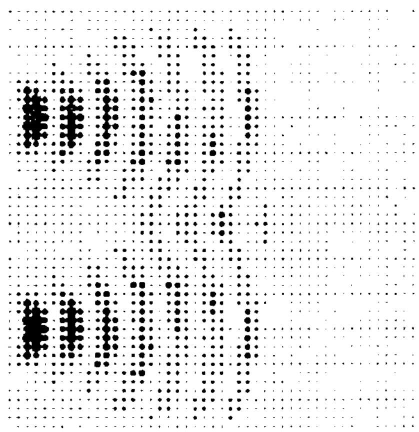

Fig. 4c Wavefront and diffraction pattern for $t = 4.000\lambda/c$.

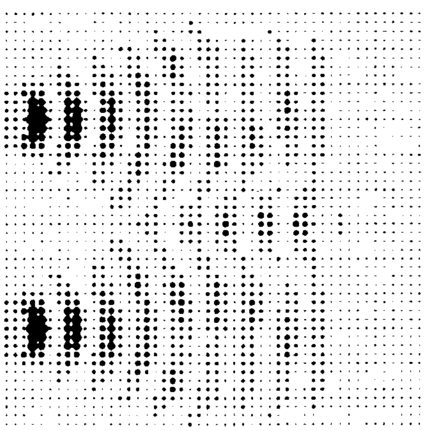

Fig. 4d Wavefront and diffraction pattern for $t = 5.083\lambda/c$.

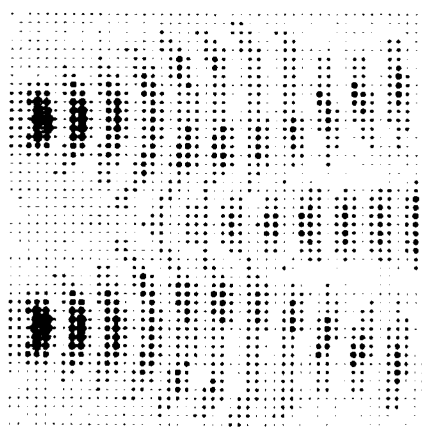

Fig. 4e Wavefront and diffraction pattern for $t = 6.583\lambda/c$.