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TREATMENT OF ANISOTROPIC PROPERTY IN CONDENSED NODE SPATIAL NETWORK FOR VECTOR POTENTIAL

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1. Introduction

Recently, for many kinds of complicated engineering problems, numerical analysis methods have become very useful with the remarkable development of the digital computer, especially the super-computer. The full-wave or vector analysis is essential for the three-dimensional problems in the time domain. For this purpose, some numerical methods such as the Finite-Difference Time-Domain method (FD-TD) and the Transmission Line Matrix method (TLM) have been proposed. I have recently proposed a new method for the vector analysis of the electromagnetic field, which is called as the Spatial Network Method (SNM) [1], and showed that this SNM can be expanded to the vector potential field by using not only the magnetic vector potential but also the electric vector potential [2-3]. In these formulation, the utilization of both the current continuity law including polarization vector itself and the conservation law of generalized momentum including vector potential fields can introduce simpler expression for dispersive properties than that by electromagnetic field variables [4]. But in all of above-mentioned methods, each field component is located at different spatial lattice points arranged to satisfy the relation among every components. This treatment can be called as the "expanded node" expression, and brings about some difficulties in treating not only fine spatial structure of analyzed objects or regions but also anisotropic properties which are produced by combination among different field components. In the expanded node SNM, these anisotropic properties can be formulated by using the equivalent mutual inductance inserted in series at each line at the nodes where coupled field components exist as equivalent currents [5]. But this formulation results in complicated expressions including many variables for voltage drops and coupling terms in four inductances at each node.

On the other hand, the "condensed node" formulation, in which all field components exist at each spatial lattice point, have been proposed for the TLM [6-7]. This condensed node treatment can be easily applied to the SNM [8]. The resultant condensed node SNM has the advantage in treating gyro-anisotropic properties. Since all field components exist as equivalent voltage variables at each internal node, the combination of field components can be formulated by using direct coupling between equivalent mutual capacitance [9].

In this paper, after explaining the resultant expression of condensed node SNM, the improvement in treating anisotropic property is shown by using the dielectic medium with uniaxial anisotropy and validity of the treatment is shown by effective dielectric constant of the strip line on this substrate.

2. Condensed Node Spatial Network

The used characteristic equations for vector potential are defined as follows;

$$\nabla \times A = P^{\bullet} + \sigma^{*} S + \mu_0 \frac{\partial S}{\partial t} \qquad (\equiv B) \qquad (1a)$$

$$\nabla \times S = {}^{\cdot}P - \sigma A - \varepsilon_0 \frac{\partial A}{\partial t} \qquad (\equiv D) \qquad (1b)$$

here, A and S are the magnetic and electric vector potentials, respectively. P and $P^{\bullet}$ are the electric and magnetic polarization vectors. σ and σ^{*} are the conductances for currents and hypothetical magnetic currents.

In the condensed node, each node for conventional expanded node to which each field component of the vector potential is assigned respectively as shown in Table I, is constituted as the hypothetical internal node with satisfying the relations given by the above fundamental characteristic equations. The connection between the adjacent nodes is expressed as the following "transmission quantity" evaluated by both equivalent voltage and current variables for each port of the condensed node shown in Fig. 1 at each time step "t".

$$A1^{t} \equiv 2(V_y - z_0 V_x^{\bullet}) - A2^{t-\Delta t} \qquad (2a)$$
$$A2^{t} \equiv 2(V_y + z_0 V_x^{\bullet}) - A1^{t-\Delta t} \qquad (2b)$$
$$A3^{t} \equiv 2(V_x + z_0 V_y^{\bullet}) - A4^{t-\Delta t} \qquad (2c)$$
$$A4^{t} \equiv 2(V_x - z_0 V_y^{\bullet}) - A3^{t-\Delta t} \qquad (2d)$$
$$A5^{t} \equiv 2(V_y + z_0 V_z^{\bullet}) - A6^{t-\Delta t} \qquad (2e)$$
$$A6^{t} \equiv 2(V_y - z_0 V_z^{\bullet}) - A5^{t-\Delta t} \qquad (2f)$$
$$A7^{t} \equiv 2(V_z - z_0 V_y^{\bullet}) - A8^{t-\Delta t} \qquad (2g)$$
$$A8^{t} \equiv 2(V_z + z_0 V_y^{\bullet}) - A7^{t-\Delta t} \qquad (2h)$$
$$A9^{t} \equiv 2(V_x - z_0 V_z^{\bullet}) - A10^{t-\Delta t} \qquad (2i)$$
$$A10^{t} \equiv 2(V_x + z_0 V_z^{\bullet}) - A9^{t-\Delta t} \qquad (2j)$$
$$A11^{t} \equiv 2(V_z + z_0 V_x^{\bullet}) - A12^{t-\Delta t} \qquad (2k)$$
$$A12^{t} \equiv 2(V_z - z_0 V_x^{\bullet}) - A11^{t-\Delta t} \qquad (2l)$$

,where, A1–A12 are the port numbers, z_0 is the characteristic impedance of the free space, and $t-\Delta t$ denote that the values are evaluated at the previous time step. Furthermore, the quantity in the parentheses in each equation corresponds to the Poynting vector by supposing each $V^{\bullet}$ to be a current term. The equivalent voltages, which correspond to the components of vector potential are calculated as follows by using current conservation law in each hypothetical internal node.

For voltages corresponding to magnetic vector potentials;

$$\text{At node } D_n: \quad V_x = \frac{A_3 + A_4 + A_9 + A_{10}}{4} \qquad (3a)$$

$$\text{node } A_n: \quad V_y = \frac{A_1 + A_2 + A_3 + A_4}{4} \qquad (3b)$$

$$\text{node } E_n: \quad V_z = \frac{A_7 + A_8 + A_{11} + A_{12}}{4} \qquad (3c)$$

For voltages corresponding to electric vector potentials;

$$\text{At node } C_n: \quad V_x^{*} = \frac{A_1 - A_2 - A_{11} + A_{12}}{4} \qquad (3d)$$

$$\text{node } F_n: \quad V_y^{*} = \frac{-A_3 + A_4 + A_7 - A_8}{4} \qquad (3e)$$

$$\text{node } B_n: \quad V_z^{*} = \frac{-A_5 + A_6 + A_9 - A_{10}}{4} \qquad (3f)$$

3. Treatment of Anisotropic Property

To present the advantage of the condensed node SNM, a problem with the uniaxial anisotropic medium is analyzed. In the expanded node, the connection among different components presented as the tensor dielectric constant in the steady state is formulated by using the mutual inductance as shown in Fig. 2. Therefore, the formulation becomes complicated because of using the voltage drops in each inductance and using four inductances at each node. On the other hand, in the condensed node formulation, this connection is expressed by the direct coupling in the mutual capacitance as shown in Fig. 3. The each self- and mutual capacitance corresponds to each element in the used tensor dielectric constant as follows;

$$\begin{vmatrix} 2C_0 + \Delta C_{xx} & \Delta C_{xy} & 0 \\ \Delta C_{yx} & 2C_0 + \Delta C_{yy} & 0 \\ 0 & 0 & 2C_0 + \Delta C_{zz} \end{vmatrix} \equiv \varepsilon_0 \begin{vmatrix} 1 + \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & 1 + \chi_{yy} & 0 \\ 0 & 0 & 1 + \chi_{zz} \end{vmatrix} \qquad (4)$$

The simplicity of the latter expression is clear and the decrease of numbers of used variables is listed in Table II. The validity of the treatment for the stripline is shown in Fig. 4 by the effective dielectric constant as the function of tilted angle of optical axis of the uniaxial dielectric substrate in the x-y plane to be perpendicular to the propagation direction z. The white squares show good coincidence with the black squares shown in [10].

4. Conclusion

In this paper, the fundamental expression of the condensed node SNM and the advantage in treating the anisotropic property by the equivalent mutual capacitance are presented.

The ability of the condensed node especially that all field components exist at each spatial lattice point will be shown in more complicated medium properties such as chirality.

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Hypothetical Electric Node			Hypothetical Magnetic Node		
	Constituent Equations *	Variables		Constituent Equations	Variables
A.	$\frac{\partial S_x}{\partial z}-\frac{\partial S_z}{\partial x}=-\varepsilon_0\frac{\partial A_y}{\partial t}$	$V_y=A_y$	F.	$\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}=\mu_0\frac{\partial S_y}{\partial t}$	$V_y^*=S_y$
	$-\frac{\partial A_y}{\partial z}=\mu_0\frac{\partial S_x}{\partial t}$	$I_x=S_x$		$-\frac{\partial S_y}{\partial z}=-\varepsilon_0\frac{\partial A_x}{\partial t}$	$I_x^*=-A_x$
	$\frac{\partial A_y}{\partial x}=\mu_0\frac{\partial S_z}{\partial t}$	$I_z=-S_z$		$\frac{\partial S_y}{\partial x}=-\varepsilon_0\frac{\partial A_z}{\partial t}$	$I_z^*=A_z$
D.	$\frac{\partial S_z}{\partial y}-\frac{\partial S_y}{\partial z}=-\varepsilon_0\frac{\partial A_x}{\partial t}$	$V_x=A_x$	B.	$\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}=\mu_0\frac{\partial S_z}{\partial t}$	$V_z^*=S_z$
	$\frac{\partial A_x}{\partial z}=\mu_0\frac{\partial S_y}{\partial t}$	$I_z=-S_y$		$\frac{\partial S_z}{\partial y}=-\varepsilon_0\frac{\partial A_x}{\partial t}$	$I_y^*=A_x$
	$-\frac{\partial A_x}{\partial y}=\mu_0\frac{\partial S_z}{\partial t}$	$I_y=S_z$		$-\frac{\partial S_z}{\partial x}=-\varepsilon_0\frac{\partial A_y}{\partial t}$	$I_x^*=-A_y$
E.	$\frac{\partial S_y}{\partial x}-\frac{\partial S_x}{\partial y}=-\varepsilon_0\frac{\partial A_z}{\partial t}$	$V_z=A_z$	C.	$\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}=\mu_0\frac{\partial S_x}{\partial t}$	$V_x^*=S_x$
	$\frac{\partial A_z}{\partial y}=\mu_0\frac{\partial S_x}{\partial t}$	$I_y=-S_x$		$\frac{\partial S_x}{\partial z}=-\varepsilon_0\frac{\partial A_y}{\partial t}$	$I_z^*=A_y$
	$-\frac{\partial A_z}{\partial x}=\mu_0\frac{\partial S_y}{\partial t}$	$I_x=S_y$		$-\frac{\partial S_x}{\partial y}=-\varepsilon_0\frac{\partial A_z}{\partial t}$	$I_y^*=-A_z$
Capacitance	C_0	$=\varepsilon_0/2$	Capacitance	C_0^*	$=\mu_0/2$
Inductance	L_0	$=\mu_0/2$	Inductance	L_0^*	$=\varepsilon_0/2$
Polarization	ΔC	$=\varepsilon_0\chi_e/2\cdot\Delta l$	Magnetization	ΔC^*	$=\mu_0\chi_m/2\cdot\Delta l$
Conductivity	G	$=\sigma/2\cdot\Delta l$	Conductivity (For magnetic Current)	G^*	$=\sigma^*/2\cdot\Delta l$

Table I Assignment of Components to Hypothetical Internal Nodes

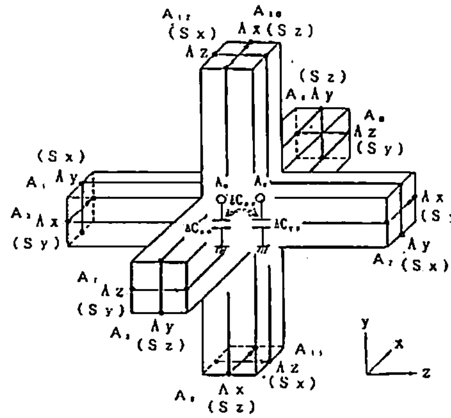

Fig. 1 Expression of Condensed Node

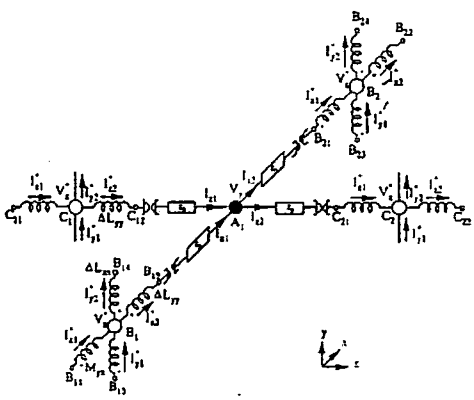

Fig. 2 Equivalent Circuit for Anisotropy in Expanded Node Expression

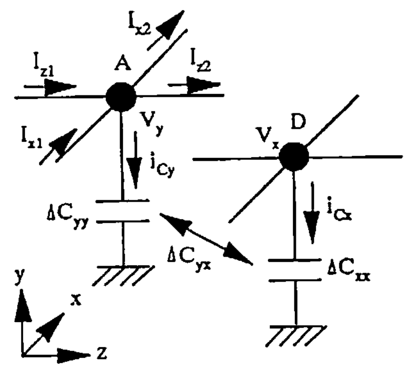

Fig. 3 Equivalent Circuit for Anisotropy in Condensed Node Expression

Numbers	Expanded Node	Condensed Node
For Basic Equation	3 (1)	2 (1)
Variables for Present values+Previous ones	8 (2)	4 (2)
per node	1 4 (8)	8 (4)

( ) : Increment number by the medium property

Table II Decrease of Numbers of Used Variables in Both Expressions

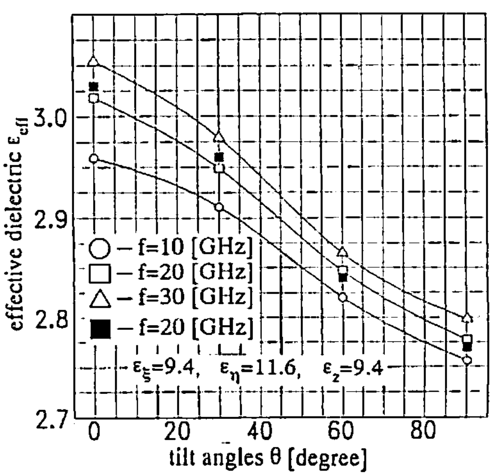

Fig. 4 Effective Permittivity for Stripline as Function of Tilted Angle of Optical Axis in Uniaxial Dielectric Substrate