Direction of Arrival Estimation of the Maximum Incoming Wave for Number of Arrival Waves Exceeding Degree of Freedom

Naoki ITO, Hiroyuki ARAI

Depertment of Electrical and Computer Engineeringt, Yokohama National Univ.

79-5 Tokiwadai Hodogaya-ku Yokohama 240-8501 Japan, naoki@arailab.dnj.ynu.ac.jp

Abstract

Most of conventional methods for Direction of Arrival(DOA) estimation are based on assumption that the number of elements is more than that of incoming wave. This paper proposes the technique calculating the covariance matrix from the first eigenvector to find a single incoming wave with the maximum field strength under the number of wave exceeding degree of freedom. This paper presents that the DOA estimation error for the wave with the maximum field strength is improved remarkably by the proposed method.

1. INTRODUCTION

The Direction of Arrival(DOA) estimation using array antennas is required to improve the propagation environment of mobile communication systems. Many high-resolution DOA estimation techniques have been introduced. These methods use an eigenvalue decomposition of covariance matrix, for example MUltiple SIgnal Classification(MUSIC) and Min-Norm [1]. Most of these conventional methods are based on assumption that the number of elements is more than that of incoming waves [2]. The spatial smoothing technique is necessary for coherent signal incoming and the available number of elements are decreased. It is not cost effective to increase the number of elements, then the number of waves might exceed degree of freedom in many cases. Beamformer and cyclic MUSIC are free from this situation. However, beamformer may not find DOA with accuracy in severe condition, and cyclic MUSIC needs the information of the incoming wave priori [3].

This paper proposes the technique calculating the covariance matrix from the first eigenvector to find a single incoming wave with the maximum field strength under the number of wave exceeding degree of freedom.

2. DOA ESTIMATION USING EIGENVECTOR

Let us consider K-element array incoming Lwaves, and the DOA of the l-th source is denoted by θ_l. These elements have omni directional pattern, assuming no mutual coupling between elements. Then, the input of array elements can be expressed by the vector as,

$$\begin{aligned} \mathbf{X}(t) &= A\mathbf{F}(t) + \mathbf{N}(t) \\ A &= [\mathbf{a}(\theta_1),...,\mathbf{a}(\theta_l),...,\mathbf{a}(\theta_L)] \\ \mathbf{F} &= [F_1(t),...,F_L(t)] \\ \mathbf{N} &= [N_1(t),...,N_K(t)] \end{aligned} \tag{1}$$

where $F_l(t)$ is the complex envelope, $\mathbf{N}(t)$ is complex noise, and $\mathbf{a}(\theta_l)$ is the steering vector of source l with DOA θ_l. This covariance matrix is decomposed by the eigenvalue as follows.

$$\begin{aligned} R_{xx}\mathbf{e}_i &= (ASA^H + \sigma^2 I)e_i \\ &= (\lambda_i + \sigma^2)\mathbf{e}_i \qquad (i = 1,2,...,K) \end{aligned} \tag{2}$$

The smallest eigenvalue collesponds to a noise represented as eq. (3).

$$R_{xx}\mathbf{e}_i = \lambda_i \mathbf{e}_i = \sigma^2 \mathbf{e}_i \qquad (i = L+1,...,K) \tag{3}$$

When the matrix A and S are full rank, this eigenvector has the following relation. The eigenvector corresponding to noise and directional vector of incoming wave are orthogonal each other.

$$A^H \mathbf{e}_i = 0 \qquad (i = L+1,...,K) \tag{4}$$

The DOAs cannot be estimated by MUSIC spectrum in the absence of eigenvector corresponding to noise, which the number of waves exceed degree of freedom.

This paper proposes the technique calculating the covariance matrix derived by the first eigenvector to find a single incoming wave with the maximum field strength. Then we apply spatial smoothing method to this covariance matrix. This procedure is shown in Fig. 1.

The first eigenvector provides the following equation after the eigenvalue decomposition of R_{xx}.

$$\begin{aligned} R_{xx}\mathbf{e}_1 &= (ASA^H + \sigma^2 I)\mathbf{e}_1 \\ &= \lambda_1 \mathbf{e}_1 \end{aligned} \tag{5}$$

This eigenvector corresponding to the largest eigenvalue includes an incoming wave with the maximum field strength and strong-correlated waves, then the self-correlation matrix for this eigenvector is calculated as follows.

$$R'_{xx} = \mathbf{e}_1 \mathbf{e}_1^{\,H} \tag{6}$$

After spatial smoothing for eq. (6), the DOA estimation is carried out. This R'_{xx} is used as the same with R_{xx} in conventional method. The proposed method decreases

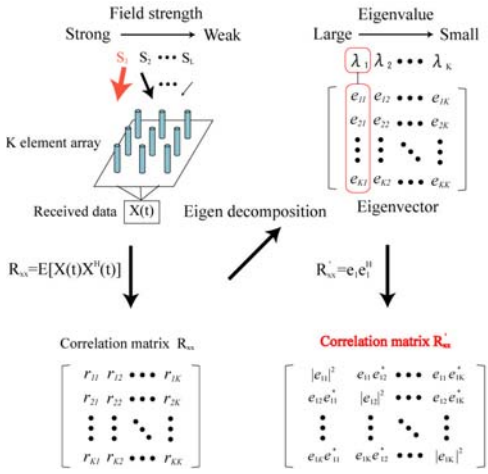


Fig. 1: Contours of the proposed method

information on other incoming waves by extracting the first eigenvector, and estimates DOA of Signal of Interest(SOI). In addition, due to rank degeneracy of covariance matrix because of coherence, a peak appeared in DOA of the wave correlated SOI.

3. SIMULATION AND EXPERIMENT

Simulations are presented to verify the accuracy of DOA estimation for the situation exceeding degree of freedom. The simulation parameters are shown in Table. 1, where λ denotes the signals' wavelength. Four waves are incident on the array sensor. An example of spectrum using MUSIC algorithm is shown in Fig. 2(a), and Capon algorithm is in Fig. 2(b). DOAs of incoming waves are denoted by the arrows and SOI is marked by a thick arrow. The color of arrows shows the correlation between signals. Though two or more peaks are appeared in the MUSIC spectrum, the peak of SOI is found by strength presumption as

$$S = (A^H A)^{-1} A^H (R_{xx} - \sigma^2 I) A (A^H A)^{-1}. \qquad (7)$$

Strength of incoming wave can be obtained by diagonal component of S, and the peak correspond to largest S is the peak of SOI. The peak of SOI is obtained as the maximum peak in the Capon spectrum. RMSE(Root Mean Square Error) of the DOA estimation by conventional method is 19.6° for MUSIC, and is 6.70° for Capon, while RMSE of the DOA estimation by proposed method is 6.37° for MUSIC, and is 4.69° for Capon. The accuracy of estimating DOA is deteriorates in conventional method. In addition, spectrum for the correlated SOI case are shown in Fig. 3. In this case, RMSE of the DOA estimation by conventional method is 14.1° and is 12.1° for Capon, while RMSE of the DOA estimation by proposed method is 1.03° for MUSIC and is 1.38° for Capon. As a result, the proposed method is effective for the correlated incoming wave. And a peak appeared in DOA of the correlated wave in fig. 3. RMSE of that is 1.30° for MUSIC and is 1.48° for Capon.

RMSE as function of SIR using proposed method and conventional method is shown in Fig. 5, for the incoming wave combination shown in Fig. 4 . RMSE is improved by the proposed method, especially for case 3 and case 4 because of the larger 1st eigenvalue includes the correlated SOI.

A general examination is given by changing the parameters of the correlation between signals, DOA, and strength at random as shown in Table 2. The averaged RMSE summarized in Table 3 shows that the proposed method improve the accuracy by 1.5° .

To demonstrate our proposed method, we made experiments in the wave anechoic chamber. Schematic view of anechoic chamber is shown in Fig. 6. There were 4-element linear array and 3-element subarray was used for spatial smoothing. The output signal by the receiver had been converted into the digital signal, and the DOA was calculated in offline by PC. Three waves were illuminated on the array that exceedes degree of freedom because of spatial smoothing. The signal of 2.001GHz from SG1(Signal Generator) was transmitted with two antennas as the coherent signals, and the signal of 2.0005GHz with 3dB in amplitude lower than that of SG1 was used for incoherence interference signal by SG2. The DOA of the wave from SG1 was estimated in these condition. Fig. 7 shows the result of experiment with 2000 times snapshot. These results shows that the coherent signals are successfully detected by using the proposed method. In this case, average of DOA estimation error by proposed method for MUSIC is 3.7° and that for the Capon is 3.75° . Finally, effectiveness of proposed method is confirmed by experiment.

4. CONCLUSION

This paper proposes the technique to find a single incoming wave with the maximum field strength under the number of wave exceeding degree of freedom by calculating the correlation matrix from the first eigenvector in DOA estimation. Our simulation and experiment show that the DOA estimation method improves the accuracy of SOI.

REFERENCES

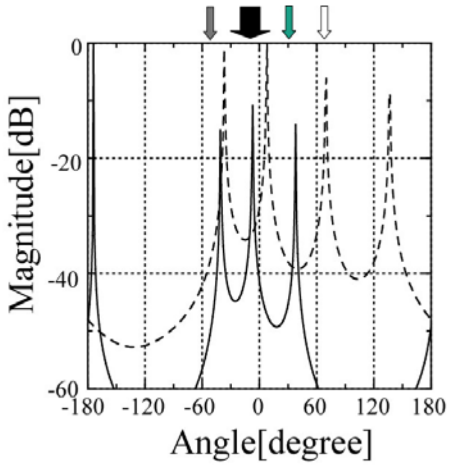
[1] R.O. Schmidt,, “Multiple emitter location and signal parameter estimation”, *IEEE Trans. Antennas Propaget.*, vol. AP-34 , no.3, pp.276-280, March 1986.

[2] N. Kikuma, “Adaptive Signal Processing with Array Antenna”, *Science and echnology Publishing Company*, Inc., 1999 (in Japanese).

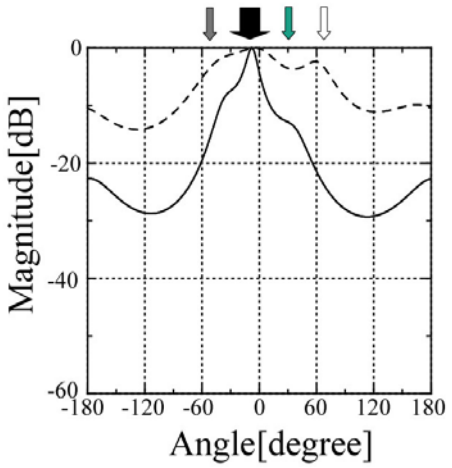
[3] H. Tsuji, Jingmin Xin, Shigetoshi Yoshimoto, and Akira Sano, “Detection of Direction of Number of Impinging Signals in Array Antenna Using Cyclostationarity”, *Technical report of IEICE, AP*, vol.J81-B2, pp. 19-28, January. 1998.

TABLE 1: DOA SIMULATION PARAMETER

Shapes of element	3×3
Shapes of subarray element	2×2
Distance between elements	0.4λ
Difference of amplitude between SOI and other	3dB
Snapshot	100times
SNR	20dB

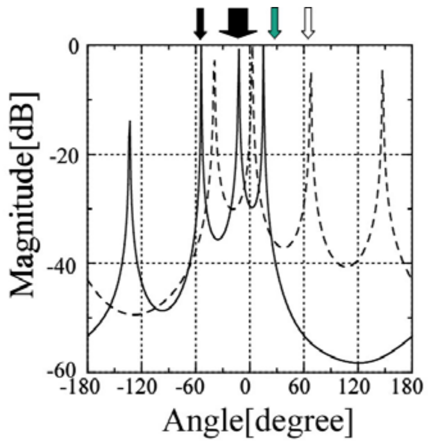


(a) MUSIC spectrum

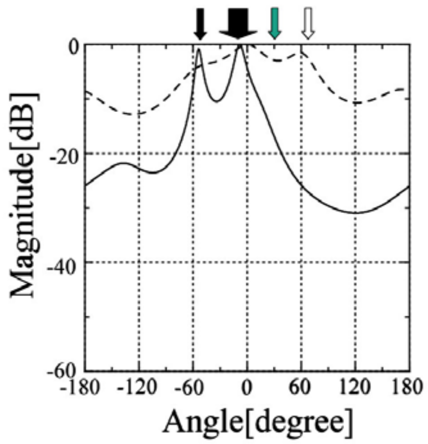


(b) Capon spectrum

Fig. 2: Comperison of proposed and conventional method. Incoming wave's interval is 40° . The solid line is proposed method while the dashed line is conventional method.



(a) MUSIC spectrum



(b) Capon spectrum

Fig. 3: Comperison of proposed and conventional method when the wave corresponding to SOI is arrived. Incoming wave's interval is 40° . The solid line is proposed method while the dashed line is conventional method

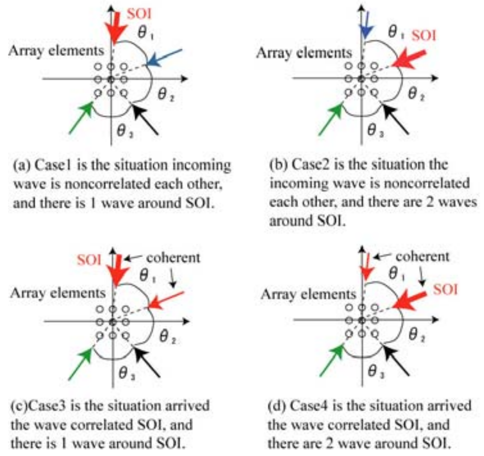


Fig. 4: Setting of situation for simulation

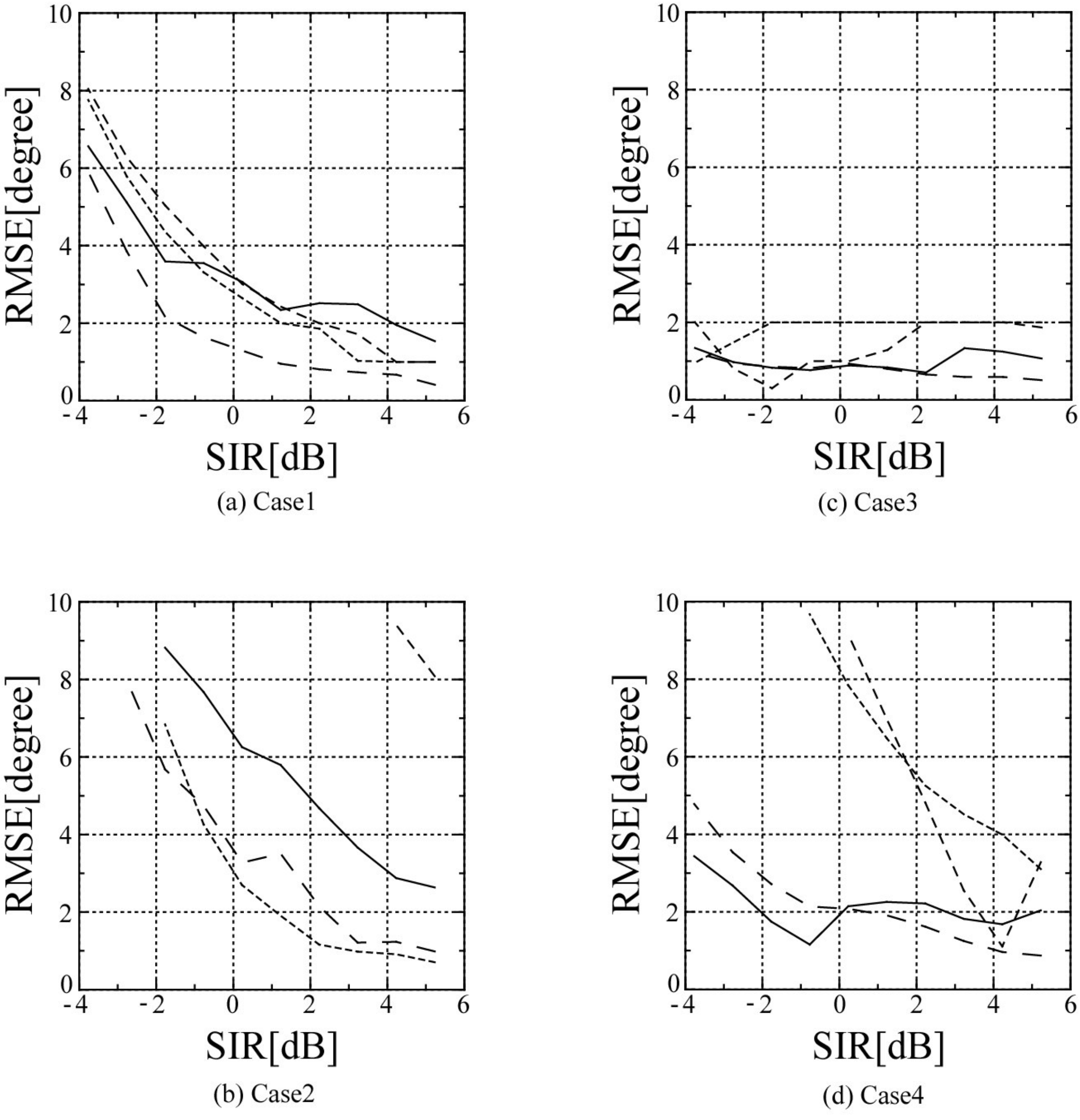


Fig. 5: RMSE as function of SIR is plotted. The line corresponds as follows.

Proposal MUSIC
Proposal Capon
Conventional MUSIC
Conventional Capon

TABLE 2: RANGE OF RANDOM WAVES

DOA	From −180° to 180°
Field strength of SOI	10dB
Field strength of other wave	From 0dB to 7dB
Correlation between each waves	Decorrelation or perfect correlation
Number of estimation	10000 times

TABLE 3: SIMURATION RESULT WITH RANDOM SITUATION

	RMSE[deg.]
Conventional MUSIC	4.36
Conventional Capon	3.75
Proposal MUSIC	2.68
Proposal Capon	2.37

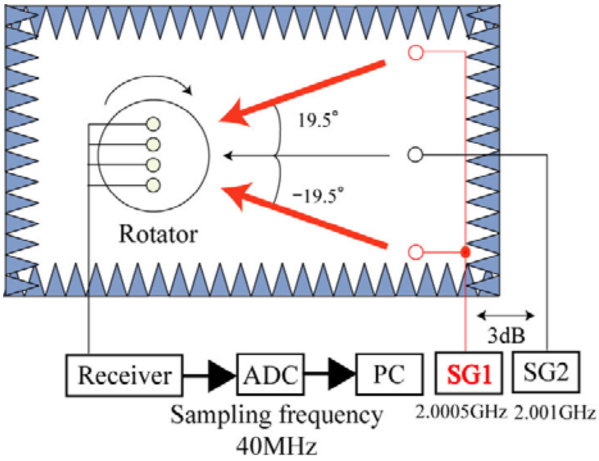


Fig. 6: Schematic view of experiment

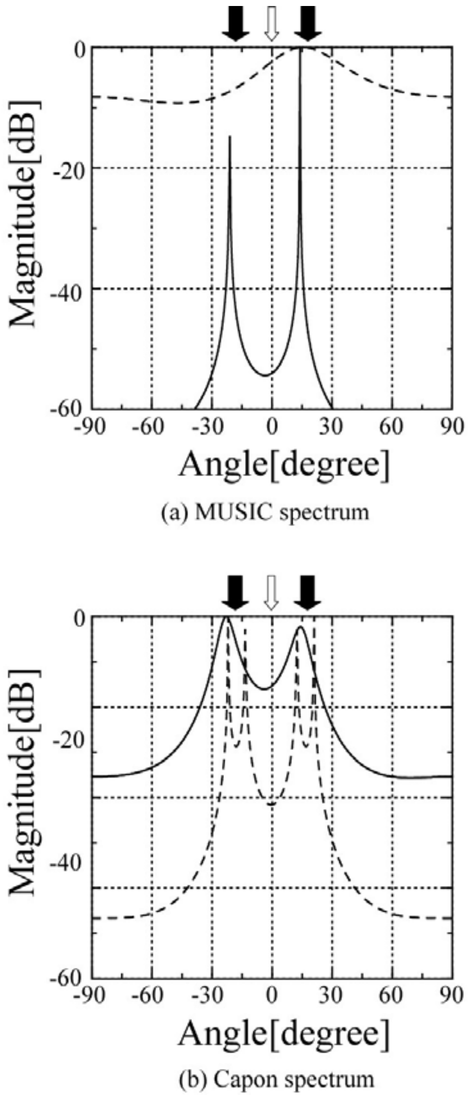


Fig. 7: Spectrum of experiment result. The wave arrived from 19.5° and −19.5° have correlation, which are SOIs, and from 0° is incoherent wave. The solid line is the proposed method, and the dashed line is the conventional method.