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THEORY OF PLASMA WAVE RADIATION FROM A LINEAR ANTENNA WITH A NEW BOUNDARY CONDITION

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1. Introduction

A number of boundary value problems related to the electron plasma waves in a compressible plasma have been studied by many investigators. In most of those theories "rigid boundary" or "absorptive boundary" is assumed [1] without theoretical foundation.

In this paper a new boundary condition for the electron velocity is proposed and its theoretical verification is made with some approximations. As an application, plasma wave radiation from a linear antenna is discussed with the new boundary condition.

2. A new boundary condition for the electron velocity

As a typical boundary of compressible plasma let us consider a one dimentional model shown in Fig. 1 where the electrons

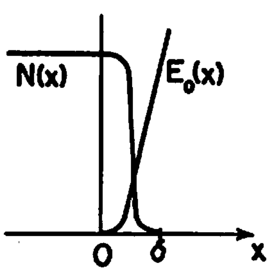

Fig. 1 model of boundary

of compressible plasma is bounded by static electric field $E_0(x)$. The effect of ion motion is neglected by assuming sufficiently high applied frequency. The electron dencity $N(x)$ varies continuously within the boundary region, but the thickness δ of the boundary region is assumed to be infinitesimally small.

Under these conditions the equation of macroscopical motion in the normal direction for the electrons which are in the boundary region can be transformed [2] as follows :

$$j\omega m \mathbf{V}\cdot\mathbf{n} = e\mathbf{E}\cdot\mathbf{n} - \frac{\alpha}{j\omega}\mathbf{V}\cdot\mathbf{n}, \qquad (1)$$

where

$\mathbf{V}$:macroscopic velocity of electrons,
$\mathbf{n}$:unit normal at the boundary surface,
e, m, :electron charge and mass,
ω :angular frequency of the applied field,
and α is a constant related to the stiffness of the boundary surface, and its approximate value is given by

$$\alpha \simeq -e\frac{\partial E_0}{\partial x}. \qquad (2)$$

Equation (1) is considered to be a new boundary condition which is more general than the conventional rigid boundary condition, $\mathbf{V}\cdot\mathbf{n} = 0$.

3. Plasma wave radiation from a linear antenna

Now, let us take an infinitely thin linear antenna which is covered with a thin ion sheath and immersed in a homogeneous compressible plasma. Then the electron velocity at the boundary surface between the ion sheath and the plasma must satisfy the boundary condition (1). At the same time the ordinary boundary conditions for the electromagnetic fields must also be satisfied.

Calculations have been made with these boundary conditions, and after some approximations a radiation formula for the electric field $\mathbf{E}^p(x', y', z')$ of the electron plasma wave has been derived [3] as follows :

$$E^{p}(x',y',z') \simeq \frac{-\alpha}{\{\alpha + m(\omega_p^2 - \omega^2)\}} \times \frac{1}{\left(1 - \frac{\omega^2}{\omega_p^2}\right)} \cdot \frac{\nabla'}{4\pi\varepsilon_0} \left[\int \frac{e^{-jk_p r}}{r} q \, dl \right] \quad (3)$$

where

ω_p :angular plasma frequency of the plasma,

k_p :propagation constant of the electron plasma wave,

q :charge density on the antenna wire per unit length,

r :distance from the integration point on the wire to the observation point.

In a special case where the boundary is rigid, α becomes infinity and eq. (3) reduces to the conventional and well-known formula [4] for a linear antenna which is immersed directly in a compressible plasma. The effect of α is shown in Fig. 2, where the values of $\frac{\alpha}{\alpha + m(\omega_p^2 - \omega^2)}$ are plotted varsus ω/ω_p with a parameter $A = \alpha/m\omega_p^2$.

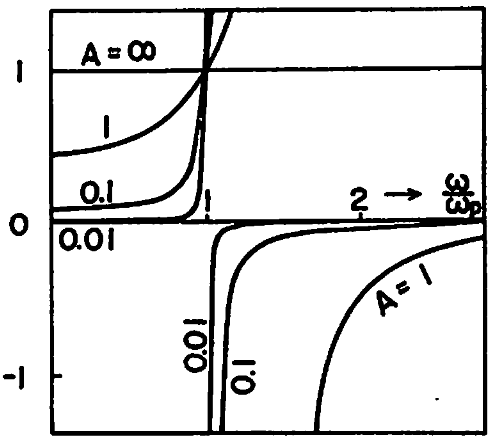

Fig. 2 Value of $\frac{\alpha}{\alpha + m(\omega_p^2 - \omega^2)}$ versus ω/ω_p with parameter $A = \frac{\alpha}{m\omega_p^2}$

A very rough estimation of A shows that there holds $A \ll 1$ for usual boundary, while $A = \infty$ for the rigid boundary.

Therefore, it is understood that the radiation of electron plasma wave from an antenna is, in general, much weaker than that expected from the conventional theory except for the case $\omega \simeq \omega_p$, where the effect of collision can not be neglected and the above theory should be revised.

4. Conclusion

Radiation of electron plasma wave is discussed with a newly proposed boundary condition for the electron velocity. Considered from the experience of the measurements [5] that we have made in the space chamber at Tohoku University, it seems that the results obtained here are more reasonable than those of the conventional theory, although some approximations and assumptions are involed in the present theory.

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