# ANALYSIS AND DESIGN OF FREQUENCY SCANNED MICROSTRIP ANTENNA ARRAYS WITH INTEGRAL FEED NETWORKS

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## ABSTRACT

A linear array of microstrip elements connected in series by microstrip feed networks lying on the same substrate has been analyzed. The frequency scanning for obtaining steered beam has been considered. Microstrip array has been designed with a center frequency $f_o = 6 GHz$, 50 MHz bandwidth, 10 dB gain, 3 dB H-plane beamwidth as $75° \pm 5°$ and E-plane beamwidth as $30° \pm 5°$ with an efficiency of 85 percent. Numerical results have been presented.

## I. INTRODUCTION

A microstrip series array consists of a linear array of microstrip elements connected in series by microstrip feed networks lying on the same substrate. This array configuration tends to minimize the feed line radiation and dissipation losses, thus increasing the array efficiency. In the mid 1960's, the array technology was dominated by open-ended waveguides and dipole arrays fed by waveguide or coaxial transmission lines [1]. With the need for light weight missile antennas, stripline transmission circuits and stripline slot elements assumed an increasingly large share of the conformal missile antenna development. Extensive efforts have resulted in distinct paths of development that provide the array designer with variety of microstrip array and element types to satisfy various application needs [2],[3]. Features such as circular or variable polarization, wide angle coverage, dual frequencies and moderate power capability are now available in narrow-band low profile array configurations. This paper presents the analysis and design of frequency scanned microstrip array of rectangular patch elements.

## II. NETWORK ANALYSIS AND FREQUENCY SCANNING

The single elements in the antenna array are microstrip transmission resonators with a common ground plane as shown in Fig. 1. Each resonator radiates from its two ends. By using the modal-expansion technique the input impedance of the radiation is given by

$$Z_{in}(\omega) = jX_L - \frac{j(\omega/C_{10})}{\omega^2 - \omega_{10}^2(1 + j/Q)} \tag{1}$$

where $C_{10} = \frac{\epsilon W L}{2h}\cos^{-2}(\pi y_o/L)$ with $Q$ being the quality factor for $TM_{10}$ mode, $\omega_{10}$ the radiation frequency at resonance, $y_o$ the feed point location, $X_L$ the series reactance due to higher order modes and $W, L$ are the microstrip resonator physical dimensions. At resonance ($\omega = \omega_o$), the input impedance becomes,

$$Z_{in}(\omega_{10}) = jX_L + R_{in}$$
$$or \quad Y_{in} = 1/Z_{in} = G - jY \tag{2}$$

where $R_{in} = (Q/\omega_{10}C_{10})$ is the input resistance ranging from 100-200 ohms, $G = 1/R_{in}$ and $Y_L = X_L G_{in}^2$.
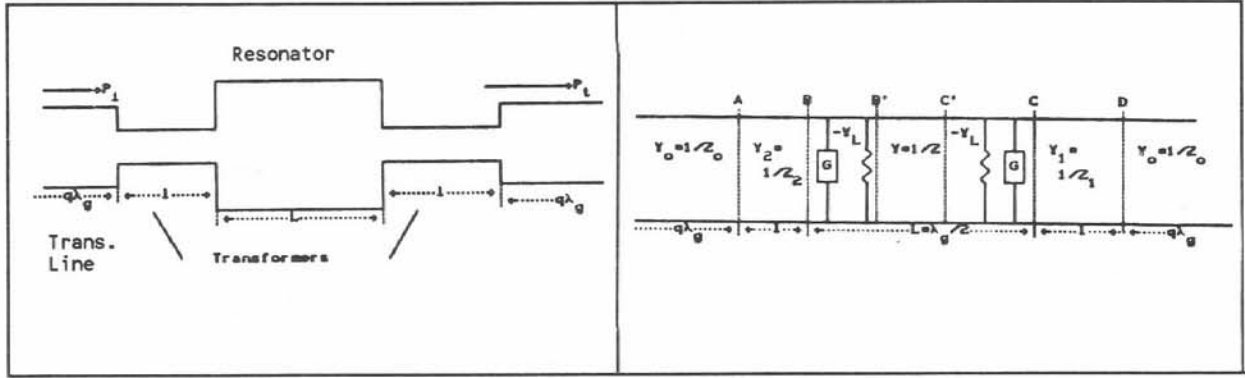
The coupling to the elements is performed by transformers of length $l$ at the input and output. At the reference planes C, B and A, the line admittances for $\beta = 2\pi/\lambda g$ are obtained as, [4],

$$Y_C = G - jY_L + Y_1 \frac{Y_o + jY_1 \tan\beta l}{Y_1 + jY_o \tan\beta l} \tag{3}$$

$$Y_B = G - jY_L + Y \frac{Y_C + jY \tan\beta l}{Y + jY_C \tan\beta l} = G - jY_L + Y_C \qquad (for\ L = \lambda g/2) \tag{4}$$

$$Y_A = Y_2 \frac{Y_B + jY_2 \tan\beta l}{Y_2 + jY \tan\beta l} \tag{5}$$
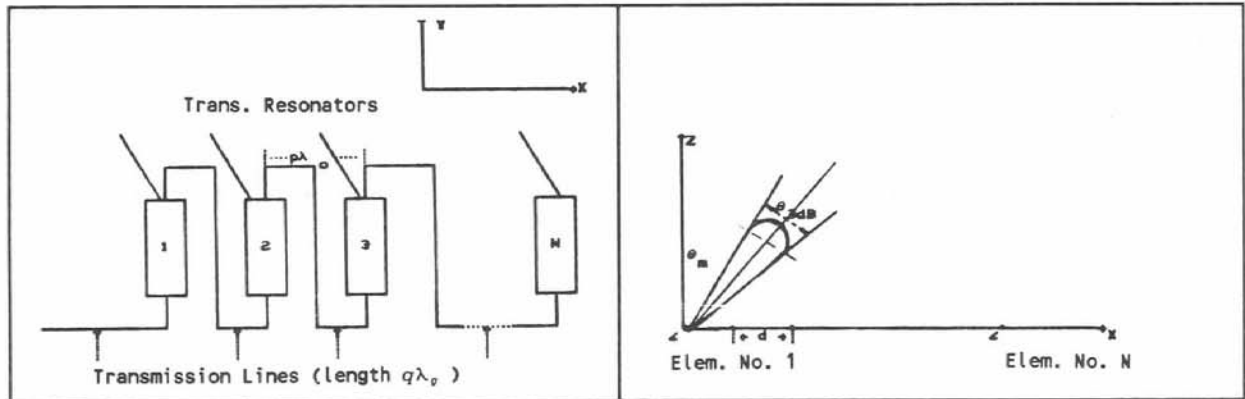
where $Y_o = 1/Z_o = 0.02\,(mho)$.



(a) resonator configuration        (b) equivalent network

Fig. 1. Transmission microstrip resonator and its equivalent network



(a) structure of antenna array        (b) characteristics parameters

Fig. 2. Antenna array structure and its characteristics parameters

The characteristic admittances of the transformers $Y_1$, $Y_2$ and of the resonator $Y$ are determined from matching conditions. The division of the input power in the radiated and transmitted power regulates the excitations of the elements and array radiation pattern. The power $P_i$ and $P_t$ in the incident wave at $B$ and output wave at $C$, respectively are found as the product of respective transformed transmission line characteristic admittances and the electric field squared. Thus, the power transmission coefficient is defined as,

$$T = \frac{P_t}{P_i} = \left[ \frac{y_2^2(1 + \tau^2)E_t^2}{y_2^2 + \tau^2} \right] \Big/ \left[ \frac{y_1^2(1 + \tau^2)E_i^2}{y_1^2 + \tau^2} \right] \tag{6}$$

where $y_1 = Y_1/y_o$, $y_2 = Y_2/Y_o$, $g = G/Y_o$ and $y_L = Y_L/Y_o$ with $E_i$ and $E_t$ as incident and transmitted electric field intensities.

A frequency scanned array is formed by $N$ elements spaced $p$ free space wavelengths and cascade-coupled by transmission lines of length $q\lambda g$ as shown in Fig. 2(a). The characteristics of antenna are described on the basis of main beam scanning angle $\theta_m$, 3-dB beamwidth $\theta_{3dB}$

( E-plane) the relative efficiency $\eta/\eta_e$ and the sidelobe level $v_{max}$. An array suitable for frequency-scanning characteristics is the one with identical elements, i.e., equal $T$ for the elements, resulting in exponential decay of the excitation. An exception is the last element in the array which is matched and hence has a different width. For exponential variation of the excitation, the $\theta_{3dB}$ has an optimal value for $N$ exceeding a certain value and is obtained by $T$ and $\gamma = 10^{-\alpha q \lambda g/20}$ where $\gamma$ is the element-to-element field attenuation factor and $\alpha$ is the attenuation probability in dB/m. Anticipating a power input to the first element as one (1) watt, the power input to the mth element is $(T\gamma^2)^{m-1}$. The fraction $(1 - T)$ of this power is radiated away. The Nth element matches the array at the end. The radiation pattern with equispaced elements on the X-axis as shown in Fig. 2(b) is,

$$G(v) = \sum_{m=1}^{N} I_m \exp[-j(m-1)v] \tag{7}$$

where $I_m$ is the excitation of mth element and

$$v = \delta + kd\sin\theta \tag{8}$$

with $k$ as propagation factor, $d = p\lambda_o$ and $\delta$ are the element-to-element distance and phase shift respectively and $\Theta$ is the angle between beam and broad side direction. In the optimal case, where the number of elements $N \gg 1/\gamma\sqrt{T}$, we find

$$\theta_{3dB} \approx (1 - \gamma\sqrt{T})/(\pi\rho) \tag{9}$$

and the scanning angle is found to be

$$\theta_m = arc\sin(-\delta/2\pi\rho) \tag{10}$$

For maximum beam scanning angle $\theta_m = -\theta_{max}$, the $\delta_{max}$ is obtained as,

$$\delta_{max} = 2\pi p \sin\theta_{max} \tag{11}$$

Furthermore, the overall efficiency given by,

$$\eta = \eta_e(1-T)/(1-T\gamma^2) \tag{12}$$

has to be taken into consideration to determine $T$ and $\gamma$.

### III. DESIGN OF ANTENNA ARRAY

The microstrip antenna array under consideration has the following requirements to be met.

operating center frequency $f_{ro} = 6GHZ$
VSWR < 2:1 for the desired BW of 50 MHZ
Minimum gain required, 10 dB
radiation pattern:

linear, broadside
3-dB H-plane beamswidth $\theta_{BH} = 75° \pm 5°$
max$^m$. scanning angle of the beam $\theta_m = 40°$
variation of the frequency $\pm 25$ MHz
3-dB E-plane beamwidth $\theta_{3dB} = 30° \pm 5°$
efficiency $\eta/\eta_e = 0.85$

The substrate selected is Epsilam-6 with $\epsilon_r = 6$, thickness $h = 0.03" = 0.762mm$, $\tan\delta = 0.001$ and conductor conductivity $\sigma = 5.7 \times 10^7\ mho/m$. The equations derived in Section II are used for synthesizing a linear array if the design requirements such as $\theta_m$, $\theta_{3dB}$, $\eta/\eta_e$ and $f_{ro}$ are known. With these values the element spacing $d = p\lambda o$, transmission line length $q\lambda g$, $K$ and $T$ values are obtained. Once $K$ and $T$ are specified then reflection coefficients $\rho_1$, $\rho_2$ and quality factor $Q$ are determined. By referring to the design curves in [4] a specific point of W/L for h = 0.03" is located. Since L should be $\lambda_g/2$ which in turn is calculated from W, both W and L are determined for each resonator. With $\rho_1$, $\rho_2$ and calculated $Y_L$ value, the characteristic admittances $Y_1$ and $Y_2$

and finally the widths of the transformers are obtained. With $\epsilon_r$, $f_{ro}$, $h$, $\sigma$ and $\tan\delta$ being known, the preliminary design information like $\lambda_o$, $X_L$, $Q_\epsilon$, $Q_{cu}$, $\alpha$, $W$, $\epsilon_e$ and directivities of single element as well as array are determined. After these steps the array element and transformer design is undertaken. In this case $v_{max}$ is determined as 200°, p = 0.33 and the element spacing $d = p\lambda_o = 1.65$ cm. The VSWR(S) for the desired bandwidth of 50 MHz is obtained from:

$$BW = \frac{S^{-1}}{Q\sqrt{S}} \qquad or \quad S = 1.92 \tag{13}$$

From the quality factor Q the ratio W/L is obtained. In order to obtain W and L for each element the following equation is solved.

$$L = [c/(2f_{ro}\sqrt{\epsilon_e})] - 2\Delta L \tag{14}$$

where $\epsilon_e$ and $\Delta L$ are functions of W. For the present design W = 2.87 cm, L = 0.98 cm. Once $\rho_1$, $\rho_2$, $Y$ and $Y_L$ are known, the transformer length, characteristic admittances and widths of the strips are determined.

## IV. NUMERICAL RESULTS

The performance parameters of the linear array are summarized in Table I, and the dependance of array directional gain on the element numbers is shown in Fig. 3. It has been observed that among the parameters in Table I, the Q factor and bandwidth are largely independent of the array application, while the directivity, beamwidths and gain are very sensitive. A beam scan of ± 40° with E-plane 3-dB beamwidth of 30° is obtained using a frequency sweep interval of ± 25 MHz.

| | |
|---|---|
| Substrate | Epsilam-6, $\epsilon_v$ =6, h=0.762mm |
| Frequency | 5.86 GHz |
| Number of Patches | 10 |
| $Z_o$ of transmission lines | 50 Ω |
| Radiation Efficiency | 70% |
| Gain of Array | 16 dB |
| Bandwidth (for VSWR=1.93) | 50 MHz |
| Maximum Patch Width | 28.7 mm |
| Maximum Scanning Angle | 40° (at H-plane) |
| by variation of | ±25 MHz |
| 3-dB H-plane Beamwidth | 30° |
| 3-dB E-plane Beamwidth | 78.6° |
| Radiation Pattern | Linear, Broadside |

Table I



Fig. 3. Dependence of array directional gain on element N.

## V. CONCLUSION

A theory of linear microstrip arrays with emphasis on analysis and practical design technique is presented. A network method based on conditions of matching, phase shift and power division for the analysis of frequency scanned series-fed microstrip arrays plus their feed networks has been discussed. The design formulas for array input impedance, 3-dB beamwidth and maximum scanning angle have been derived. The theory has been verified by a design example of a linear microstrip array for mobile satellite systems. Microstrip arrays are ideally suited to many applications requiring narrow bandwidth, low power and extreme lightweight. However, the substrate dielectric constant tolerance control, rigorous solutions for radiating wall admittances and large class of microstrip element configurations are very important considerations for improved array designs.

## REFERENCES

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