A—8—2 HALLEN TYPE INTEGRAL EQUATION FOR ANTENNA COMPOSED OF STRAIGHT WIRES.

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1. Introduction

A Hallen-type integral equation for an arbitrarily shaped wire antenna generally has double integral terms[1]. Recently, a simplified Hallen-type integral equation, obtained by the reduction of the double integral to the single integral, has been presented for the antenna composed of straight wires[2],[3].

A new simplified Hallen-type integral equation is also derived here with the extended boundary condition that the axial component of electric field on the axis of wire is taken to be zero. The simplified equation, which is given here, has a simple expression as excluded the waste part in comparison with the equations given previously.

As an example of application, an umbrella antenna is analyzed and the calculated current distribution are compared with the experimental values.

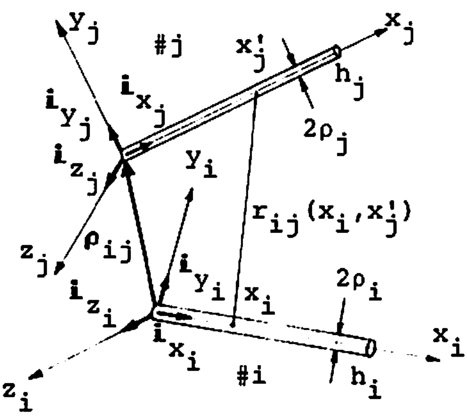

Fig. 1 Coordinate systems for arbitrarily located straight wires.

2. Integral equation

Fig. 1 shows two straight wire elements and their coordinate systems. We assume that the current flows on the surface and along the axis of wires. The electric field at the point x_i of element #i is given as follows.

$$\mathbf{E}(x_i) = \mathbf{E}^{inc}(x_i) - j\frac{\eta}{4\pi k}(\nabla\nabla\cdot + k^2)\sum_j \int_0^{h_j} I_j(x_j')\,\mathbf{i}_{x_j} G_{ij}(x_i, x_j')\,dx_j' \quad (1)$$

where $I_j(x_j')$ is the total current at x_j'.

Now, using the extended boundary condition that the axial component of electric field must vanish on the axis of element #i, we obtain

$$E_i^{inc}(x_i) - j\frac{\eta}{4\pi k}\sum_j \int_0^{h_j} I_j(x_j')\left[k^2(\mathbf{i}_{x_j}\cdot\mathbf{i}_{x_i}) - \frac{\partial^2}{\partial x_i \partial x_j'}\right]G_{ij}(x_i, x_j')\,dx_j' = 0 \quad (2)$$

$$G_{ij}(x_i, x_j') = \frac{e^{-jkr_{ij}(x_i, x_j')}}{r_{ij}(x_i, x_j')} \quad (3)$$

where k, η is the wavenumber and the intrinsic impedance in free space, respectively. The distance r_{ij} between the observation point x_i and the representative point x_j' on the surface of element #j is given by

$$r_{ij}(x_i,x_j')=[\{x_i-(\rho_{ij}\cdot\mathbf{i}_{x_i})-x_j'(\mathbf{i}_{x_j}\cdot\mathbf{i}_{x_i})\}^2+\{(\rho_{ij}\cdot\mathbf{i}_{y_i})+x_j'(\mathbf{i}_{x_j}\cdot\mathbf{i}_{y_i})\}^2$$
$$+\{(\rho_{ij}\cdot\mathbf{i}_{z_i})+x_j'(\mathbf{i}_{x_j}\cdot\mathbf{i}_{z_i})\}^2+\rho_j{}^2]^{1/2} \tag{4}$$

Now, $\Pi_{ij}(x_i,x_j')$ is defined by[4]

$$\Pi_{ij}(x_i,x_j')= f_1(x_j')\cos kx_i+ f_2(x_j')\sin kx_i$$
$$+\int_0^{x_i} dx_i'[\{k^2(\mathbf{i}_{x_j}\cdot\mathbf{i}_{x_i})-\frac{\partial^2}{\partial x_i'\partial x_j'}\}G_{ij}(x_i',x_j')]\frac{\sin k(x_i-x_i')}{k} \tag{5}$$

where $f_1(x_j')$, $f_2(x_j')$ are arbitrary functions. Then, we can obtain

$$[k^2(\mathbf{i}_{x_j}\cdot\mathbf{i}_{x_i})-\frac{\partial^2}{\partial x_i\partial x_j'}]G_{ij}(x_i,x_j')=(k^2+\frac{\partial^2}{\partial x_i{}^2})\Pi_{ij}(x_i,x_j') \tag{6}$$

Substituting eq.(6) to eq.(2), we obtain

$$(k^2+\frac{\partial^2}{\partial x_i{}^2})\sum_j\int_0^{h_j} I_j(x_j')\Pi_{ij}(x_i,x_j')dx_j'= -j\frac{4\pi k}{\eta}E_i^{inc}(x_i) \tag{7}$$

The solution gives the simplified Hallen-type integral equation, that is

$$\sum_j\int_0^{h_j} I_j(x_j')\Pi_{ij}(x_i,x_j')dx_j'= A_i\cos kx_i+ B_i\sin kx_i$$
$$-j\frac{4\pi}{\eta}\int_0^{x_i} dx_i'E_i^{inc}(x_i')\sin k(x_i-x_i') \tag{8}$$

To simplify the integral kernel Π_{ij} which is given by eq.(5), we note first that[5]

$$\int_0^{x_i} G_{ij}(x_i',x_j')\sin k(x_i-x_i')dx_i'$$
$$=\frac{1}{2j}[e^{jk\{x_i-f(x_j')\}}E_j(ku_1^+,ku_2^+)+ e^{-jk\{x_i-f(x_j')\}}E_j(ku_1^-,ku_2^-)] \tag{9}$$

where

$$E_j(ku_1,ku_2)= \int_{u_1}^{u_2}\frac{e^{-jku}}{u}du \tag{10}$$

$$\left.\begin{aligned} f(x_j')&= x_j'(\mathbf{i}_{x_j}\cdot\mathbf{i}_{x_i})+(\rho_{ij}\cdot\mathbf{i}_{x_i})\\ u_1^\pm&= r_{ij}(0,x_j')\mp f(x_j')\\ u_2^\pm&= r_{ij}(x_i,x_j')\pm\{x_i-f(x_j')\}\end{aligned}\right\} \tag{11}$$

Using eq.(9) and performing an integration by parts, we obtain

$$\int_0^{x_i} dx_i'[\{k^2(\mathbf{i}_{x_j}\cdot\mathbf{i}_{x_i})-\frac{\partial^2}{\partial x_i'\partial x_j'}\}G_{ij}(x_i',x_j')]\frac{\sin k(x_i-x_i')}{k}$$
$$=[\frac{1}{k}\frac{\partial}{\partial x_j'}G_{ij}(0,x_j')+\frac{j}{2}\{e^{-jkf(x_j')}\frac{e^{-jku_1^+}}{u_1^+}\frac{\partial u_1^+}{\partial x_j'}+e^{jkf(x_j')}\frac{e^{-jku_1^-}}{u_1^-}\frac{\partial u_1^-}{\partial x_j'}\}]$$
$$\times\sin kx_i+\frac{1}{2}[e^{-jkf(x_j')}\frac{e^{-jku_1^+}}{u_1^+}\frac{\partial u_1^+}{\partial x_j'}-e^{jkf(x_j')}\frac{e^{-jku_1^-}}{u_1^-}\frac{\partial u_1^-}{\partial x_j'}]\cos kx_i$$

$$-\frac{1}{2}\left[e^{jk\{x_i-f(x_j')\}}\frac{e^{-jku_2^+}}{u_2^+}\frac{\partial u_2^+}{\partial x_j'}-e^{-jk\{x_i-f(x_j')\}}\frac{e^{-jku_2^-}}{u_2^-}\frac{\partial u_2^-}{\partial x_j'}\right] \tag{12}$$

In the above equation, the terms which are multiplied by sin kx_i or cos kx_i do not depend on the observation point x_i. We use those terms with minus sign for arbitrary functions $f_1(x_j')$ and $f_2(x_j')$ of eq.(5). Then, we have

$$\Pi_{ij}(x_i,x_j')=\frac{g_2(x_i,x_j')}{g_1(x_j')}G_{ij}(x_i,x_j') \tag{13}$$

where

$$g_1(x_j')=\{(\rho_{ij}\cdot i_{y_i})+x_j'(i_{x_j}\cdot i_{y_i})\}^2+\{(\rho_{ij}\cdot i_{z_i})+x_j'(i_{x_j}\cdot i_{z_i})\}^2+\rho_j^2 \tag{14}$$

$$\begin{aligned}g_2(x_i,x_j')=\ & x_ix_j'\{1-(i_{x_j}\cdot i_{x_i})^2\}+x_i\{(\rho_{ij}\cdot i_{x_j})-(\rho_{ij}\cdot i_{x_i})(i_{x_j}\cdot i_{x_i})\}\\ & -x_j'\{(\rho_{ij}\cdot i_{x_i})-(\rho_{ij}\cdot i_{x_j})(i_{x_j}\cdot i_{x_i})\}-(\rho_{ij}\cdot i_{x_i})(\rho_{ij}\cdot i_{x_j})\\ & +(i_{x_j}\cdot i_{x_i})(\rho_{ij}\cdot\rho_{ij}+\rho_j^2)\end{aligned} \tag{15}$$

The scalar potential $\phi_i(x_i)$ at the point x_i is defined by

$$\phi_i(x_i)=\frac{\eta}{j4\pi k}\sum_j\int_0^{h_j}I_j(x_j')\frac{G_{ij}(x_i,x_j')}{x_j'}dx_j' \tag{16}$$

Using derivatives of equations (8) and (13) with respect to x_i, the above equation is written as follows.

$$\begin{aligned}\phi_i(x_i)=\ & -j\frac{\eta}{4\pi}A_i\sin kx_i+j\frac{\eta}{4\pi}B_i\cos kx_i\\ & +\int_0^{x_i}dx_i'E_i^{inc}(x_i')\cos k(x_i-x_i')-\frac{\eta}{4\pi}\sum_j\int_0^{h_j}I_j(x_j')\Gamma_{ij}(x_i,x_j')dx_j'\end{aligned} \tag{17}$$

$$\Gamma_{ij}(x_i,x_j')=\frac{g_3(x_j')}{g_1(x_j')}e^{-jkr_{ij}(x_i,x_j')} \tag{18}$$

$$\begin{aligned}g_3(x_j')=\ & x_j'\{(i_{x_j}\cdot i_{y_i})^2+(i_{x_j}\cdot i_{z_i})^2\}+(\rho_{ij}\cdot i_{y_i})(i_{y_i}\cdot i_{x_j})\\ & +(\rho_{ij}\cdot i_{z_i})(i_{z_i}\cdot i_{x_j})\end{aligned} \tag{19}$$

3. Numerical analysis and measurement

The umbrella antenna with two inclined elements is shown in Fig. 2. When the antenna is excited by a coaxial line at the origin of the vertical element, the incident field E_i^{inc} is given as the magnetic frill current. In the calculation of the current distribution, the point matching method is used with the equations (8) and (17). Fig. 3 shows the calculated and measured current distribution where $h_1=0.5141\lambda$, $h_2=0.485\lambda$, $\rho_1=0.001\lambda$, $\rho_2=0.001$ or 0.0001λ. (λ is the wavelength.) The calculated results agree well with the measured ones.

4. Conclusion

A new simplified Hallen-type integral equation for the antenna composed of straight wires is formulated using the extended boundary condition of the electric field. The integral kernel $\Pi_{ij}(x_i, x_j')$ may be considerable as the form as extended the integral kernel of the Hallen-type equation of a dipole. Thus, the simplified equations derived here have the form, extended naturally the Hallen-type integral equation of a dipole. The simplified equations (8) and (17) are useful for shortening of computing time in a numerical calculation of the antenna composed of straight wires.

References

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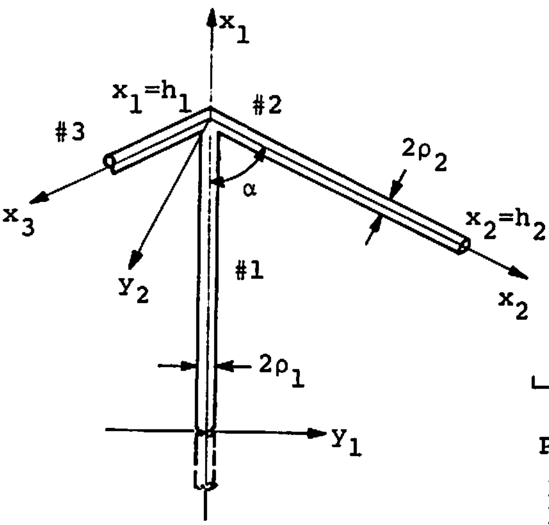

Fig. 2 Umbrella antenna

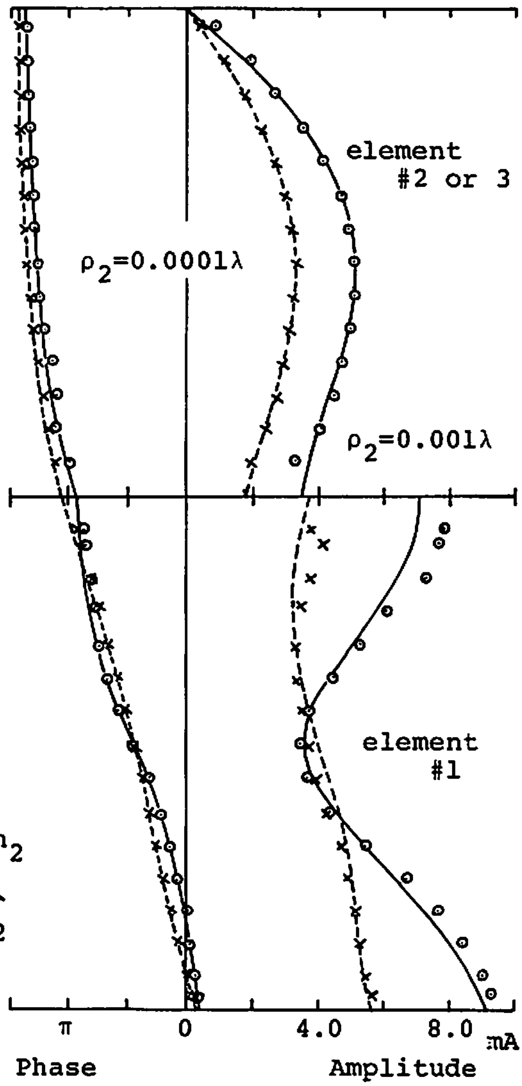

Fig.3 Current distribution
$h_1=0.5141\lambda$, $h_2=0.485\lambda$, $\alpha=\pi/6$

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