RECENT ADVANCES IN FAST SOLVERS FOR COMPUTATIONAL ELECTROMAGNETICS

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Introduction

Due to the rapid advances in computer technology, increasing amount of electromagnetic analysis is performed using computational electromagnetics. However, traditional computational algorithms are slow and memory inefficient. Our Center has focused on developing fast solvers which are both memory and CPU efficient These fast solvers permit the solution of problems of unprecedented sizes using existing computers. Method of moments problems with about 10 million unknowns have been solved as a result [1]. More recently, these fast solvers have been parallelized, and generalized to very low frequency applications [2,3]. This talk will discuss these advances.

Large Scale Computing with MLFMA

The MLFMA (multilevel fast multipole algorithm) for electrodynamics is developed at the University of Illinois for large scale computing of scattering by large electromagnetic targets [4-7]. Its working principle is based on telephone network communication. When an integral equation is solved by the method of moments, all current elements on the scatterer communicate to each other via a matrix-vector multiplication. A straight-forward way of performing a matrix-vector product is similar to connecting N telephones together via direct links. The number of links is proportional to N^2. Therefore, it is necessary to reduce the number of links via a multilevel approach, as in done in a telephone network.

Figure 1. The analogy of a matrix-vector multiplication to a telephone network. The left picture shows a straightforward matrix-vector product, where the number of links is enormous. The right shows a two-level matrix vector product where the number of links is greatly reduced.

In the two-level approach, the scattering elements are organized into groups according to their physical proximity. The information from nearby sources are aggregated into the hub at the center of the group. Then, the information is transferred from group to group via the hubs. Evidently, the number of links are reduced. In order to facilitate the two-stage communication of information, a matrix element has to be factorized as

$$A_{ij} = \overline{V}^t_{il} \bullet \overline{\overline{T}}_{ll'} \bullet \overline{V}_{l'j}$$

where $\overline{\overline{T}}_{ll'}$, the translation matrix, is a diagonal matrix. The diagonalization of the translation matrix is first achieved by Rokhlin [8]. In this manner, the computational complexity of a matrix-vector multiplication is reduced to $N^{1.5}$.
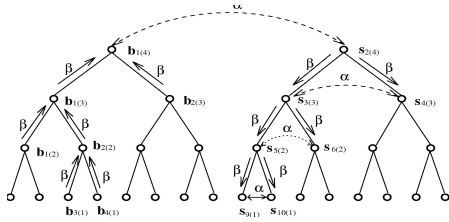
Figure 2. A multilevel tree structure for transferring information between current elements in a scatterer. As a consequence, the information can be transferred in N log N operations.

The idea of a two-level algorithm can be generalized to a multilevel approach where a matrix element has to be factorized as:

$$A_{ij} = \overline{V}_{il_1}^t \bullet \overline{\overline{\beta}}_{l_1 l_2} \bullet \overline{\overline{\beta}}_{l_2 l_3} \cdots \overline{\overline{\beta}}_{l_{L-1} l_L} \bullet \overline{\overline{T}}_{ll'} \bullet \overline{\overline{\beta}}_{l_L l_{L-1}} \cdots \overline{\overline{\beta}}_{l_3 l_2} \bullet \overline{\overline{\beta}}_{l_2 l_1} \bullet \overline{V}_{l'j}$$

The workload at the higher-level can be reduced by interpolation and anterpolation. In this manner, the workload at each level can be kept to $O(N)$. Since there are $\log(N)$ levels in the tree, a matrix-vector product can be performed in $O(N \log N)$ operations.

Figure 3. In three dimension, the scatterer is enclosed in a box, which is recursively subdivided into smaller boxes until the smallest one contains only a few current elements. An oct-tree network is set up to facilitate information transfer in *N log N* operations.
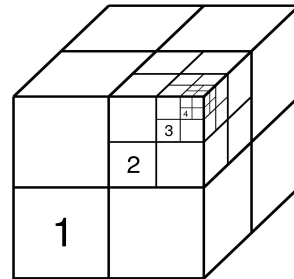
For three-dimensions, the scatterer is first embedded in an enclosing box. The box is recursively divided into eight smaller boxes until the smallest one contains about one or two subscattering elements. Then an abstract oct-tree communication network is establish within the computer to facilitate a matrix-vector product to be effected in $O(N \log N)$ operations.

As a consequence, the fast way of performing a matrix-vector product can be substituted into a iterative matrix solver such as the conjugate gradient (CG) solver. When CG is applied to solve a dense matrix system arising from the method of moments, the bottleneck of computation is the matrix-vector product. A traditional approach of invoking a matrix-vector product is of $O(N^2)$ in CPU time and memory. However, with the fast solvers, it can be invoked in $O(N \log N)$ operations. The method is also matrix free, bypassing the need to generate the matrix in the solution process. Consequently, the memory requirements of such an algorithm is greatly reduced. This phenomenal reduction in memory requirements and CPU time illustrates the cruelty of computational complexity. For large problems, the memory requirements and CPU time can be orders of magnitude larger as illustrated in Table 1.

Bistatic RCS of VFY218 at 3 GHz
8 processors of SGI Origin 2000
# of Unknowns N = 2 millions
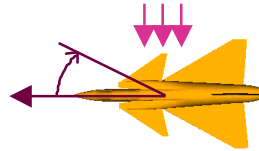
	Memory (GB)	Matrix Fill (days)	LUD (years)	One-RHS (hrs)
FISC	5	0.1		9
LUD	32,000	600.0	200	4
CG	32,000	600.0		500

Table 1. Resources needed for FISC (Fast Illinois Solver Code) for 2 million unknowns when solving the VFY218 full size aircraft at 3 GHz. This problem is practically unsolvable using conventional methods like LUD (lower and upper triangular decomposition) and CG (conjugate gradient). The problem was solved on 8 processors of SGI Origin 2000. (RHS-right hand side.)

Advanced Applications

With the availability of the geometry files, electromagnetic analysis now can be easily performed with these fast solvers. Figure 4 illustrates the computational of the electromagnetic interaction with the car.
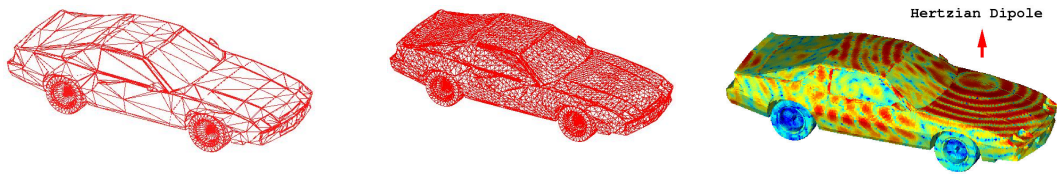
Figure 4. The left geometry file is from VRML (virtual reality modeling language). The middle geometry file has undergone automatic mesh refinement to be electromagnetically compatible. The right figure illustrates the calculation of the interaction of a Hertzian dipole with the car.
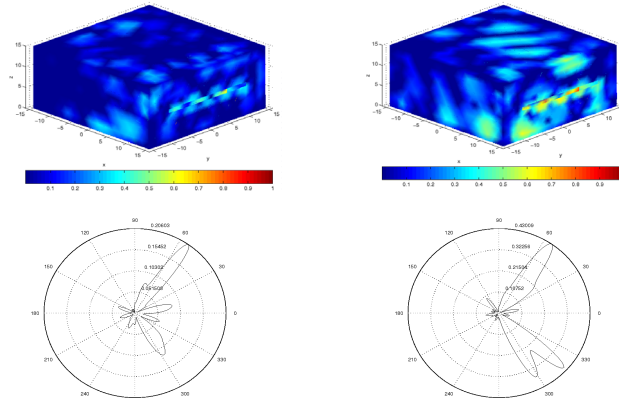

Figure 5. The use of fast solvers for EMC/EMI applications in studying the leakage of electromagnetic signals from computer chassis.

These solvers have also been applied for EMC/EMI studies to ascertain the leakage of electromagnetic signals from computer chassis as shown in Figure 5.
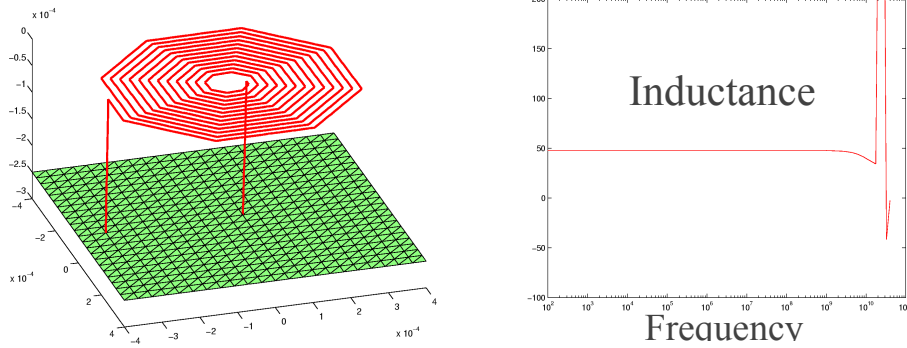

Figure 6. The application of fast solvers for circuits from very low frequencies (static) to microwave frequencies.
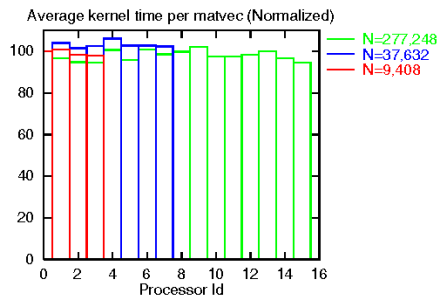
Figure 7. The load balancing of a parallelized version of MLFMA called ScaleME. The right picture shows the physical configuration of a Linux cluster that it runs on.

Other Algorithm Developments

Recently, we have developed a low-frequency MLFMA (LF-MLFMA) that is stable at very low frequencies as well as microwave frequencies. It has been used to model an inductor in a micro-circuit as illustrated in Figure 6.

The MLFMA has also been parallelized into a code called ScaleME. The code has been able to solve a problem up to 4 million unknowns on an SGI O2K, while it can solve a 600,000 unknown problem on a Linux cluster costing $15,000. Figure 7 shows the load balancing of ScaleME running on such a cluster. These algorithms have also been replicated in the time domain [9,10]. Moreover, higher order versions of these algorithms have been developed [11]. Also, fast algorithms have been developed for layered media [12,13].

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