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VLF RADIATION RESISTANCE OF A FINITE DIPOLE IN A UNIFORM WARM MAGNETOPLASMA

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A strong interest currently exists in the study of VLF antenna radiation characteristics in a magnetoplasma because of the application of this study to satellite-based wave-particle interaction experiments in the magnetosphere, to satellite communications systems and to ionospheric diagnostics. The bulk of the knowledge on the problem of antenna-plasma coupling can be found in a number of recent reports.[1-3] The purpose of the present paper is to study the VLF whistler mode radiation of a dipole antenna in a warm-electron, uniform VLF magnetoplasma of infinite extent. In conjunction with the plasma model assumed, we use the following basic equations governing the electromagnetic radiation from a time harmonic ($e^{j\omega t}$), small-amplitude source immersed in the magnetoplasma:

$$\nabla\times\vec{E} = -j\omega\mu_o\vec{H},\quad \nabla\times\vec{H} = j\omega\epsilon\vec{E} + \sum_s q_s N_s \vec{v}_s + \vec{J} + eN_e\vec{v}_e$$

$$jN_e\omega m_e\vec{v}_e = eN_e(\vec{E} + \vec{v}_e\times\vec{B}_o) - \nabla P_e \qquad (1)$$

$$j\omega m_s\vec{v}_s = q_s(\vec{E} + \vec{v}_s\times\vec{B}_o),\quad u^2 m_e N_o \nabla\cdot\vec{v}_e = -j\omega P_e$$

where ϵ, μ_o, ω, $\vec{B}_o$, and $\vec{J}$ stand for free-space permittivity, free-space permeability, the angular signal frequency, the static magnetic field and the source current density, respectively, and where $\vec{v}_s$, m_s, q_s, and N_s stand for the ordered velocity, the mass, the charge and the unperturbed density for the "s" species of particle, respectively, and where P_e is the perturbed electron pressure and u is the thermal speed of the electron gas.

Following an analysis similar to that given in Chapter 3 of Wang[2] or Wang and Bell[4] and assuming that $\vec{B}_o$ is in parallel to z axis it can be shown that the three principal components of the Fourier transformed electric field $\mathcal{E}_\nu(\vec{k})$ ($\nu = +1,-1,0$) are given by

$$\mathcal{E}_\nu(\vec{k}) = \frac{\beta^2 \mathcal{J}_\mu \delta_{\nu\mu} + (\alpha_\nu - 1)k_\nu \vec{k}\cdot\vec{\mathcal{J}} + K(\mu,\nu)}{j\omega\epsilon(k^2-\beta^2\epsilon_\nu)};\ \mu,\nu = +1,0,-1 \qquad (2)$$

$$K(\mu,\nu) = \frac{\alpha_\nu k_\nu \Pi(k)}{N(k)} \sum_\mu \frac{(\beta^2 k_{-\mu}\mathcal{J}_\mu + \vec{k}\cdot\vec{\mathcal{J}}(\alpha_\mu - 1)k_{-\mu}k_\mu)}{(k^2-\beta^2\epsilon_\mu)}$$

$$\Pi(k) = (k^2-\beta^2\epsilon_{+1})(k^2-\beta^2\epsilon_{-1})(k^2-\beta^2\epsilon_o)$$

$\delta_{\nu\mu} = 1$, when $\nu = \mu$, $= 0$, otherwise.

$$N(k) = C_1 k^6 + C_2\beta^2 k^4 + C_3\beta^4 k^2 + \beta^6 C_4$$

$$C_1 = \tilde{u}^2\left(\frac{\sin^2\theta}{1-Y^2} + \cos^2\theta\right)$$

$$C_2 = -\tilde{u}^2\left[\frac{(\epsilon_o+\epsilon_s-\epsilon_d Y)}{1-Y^2}\sin^2\theta + 2\epsilon_s\cos^2\theta\right] - a(\theta),$$

$$C_3 = \tilde{u}^2\left[\frac{\epsilon_o(\epsilon_s-\epsilon_d Y)}{1-Y^2}\sin^2\theta + \epsilon_{+1}\epsilon_{-1}\cos^2\theta\right] + \epsilon_s(2\epsilon_o + (a-\epsilon_o)\sin^2\theta)$$

$C_4 = -\epsilon_o\epsilon_{+1}\epsilon_{-1}$, $\beta = \omega/c$, $\tilde{u} = u/c$, $c = 3\times10^8$ m/sec

$Y = |e|B_o/\omega m_e$, θ = polar angle between $\vec{B}_o$ and $\vec{k}$

$a(\theta) = \epsilon_o\cos^2\theta + \epsilon_s\sin^2\theta$, $\epsilon_s = \frac{1}{2}(\epsilon_{+1}+\epsilon_{-1})$, $\epsilon_d = \frac{1}{2}(\epsilon_{+1}-\epsilon_{-1})$

$a = \epsilon_{+1}\epsilon_{-1}/\epsilon_s$, $\alpha_\nu = 1-\tilde{u}^2/(1-\nu Y)$, $\epsilon_\nu = 1 - \sum_s X_s/(1+\nu Y_s)$,

$X_s = \omega_{ps}^2/\omega^2$, $Y_s = q_s B_o/m_s\omega$, ω_{ps} = plasma frequency for "s" species.

In the case of T_e (electron temperature) $\to 0$, then $u \to 0$, and (2) reduces identically to Eq. (12) of Wang and Bell[4] for the case of a cold magnetoplasma. To derive a formal expression for the input impedance, $Z_{\parallel,\perp}$, of a short dipole oriented either parallel or perpendicular to $\vec{B}_o$, the dipole current is assumed to be a skin-triangular distribution.

Using (2)-(3), and the relation $Z = 2P/I_o^2$ (I_o = current at input terminals), where $2P = -(2\pi)^{-3}\int \sum_\nu \mathcal{E}_\nu(\vec{k})\mathcal{J}_\nu^*(k)d\vec{k}$ the formal expression of $Z_{\parallel,\perp}$ is given by

$$Z_{\parallel,\perp} = \frac{jZ_o(h\beta)^2}{8\pi^3}\int G_{\parallel,\perp}(n,\theta)\,d\vec{n} \qquad (3)$$

where:

$$G_\parallel(n,\theta) = \left\{\frac{1+(\alpha_o-1)n_z^2}{(n^2-\epsilon_o)}\left[1 + \frac{\alpha_o n_z^2(n^2-\epsilon_{+1})(n^2-\epsilon_{-1})}{N(n)}\right] + \frac{\alpha_o n_z^2 n_\perp^2}{N(n)}\times\right.$$
$$\left.\left[(\alpha_s-1)n^2 - \frac{1}{2}(\alpha_{+1}\epsilon_{-1} + \alpha_{-1}\epsilon_{+1}) + \epsilon_s\right]\right\} S_\parallel$$

$$G_\perp(n,\theta) = \left\{A_{+1} + A_{-1} + \frac{\Pi(n)}{2N(n)}\left[\frac{\alpha_{+1}n_{+1}}{n^2-\epsilon_{+1}} + \frac{\alpha_{-1}n_{-1}}{n^2-\epsilon_{-1}}\right]\times\right.$$
$$\left.\left[n_{-1}A_{+1} + n_{+1}A_{-1} + \sqrt{2}\,n_x n_z^2\frac{\alpha_o-1}{n^2-\epsilon_o}\right]\right\} S_\perp$$

$S_\parallel = [\sin(\lambda n_z)/(\lambda n_z)]^4 J_o^2(\beta r n_\perp)$, $S_\perp = [\sin(\lambda n_x)/(\lambda n_x)]^4$
$J_o^2(\beta r n_\perp')$, $\lambda = h\beta/2$

$A_{\pm1} = [1 + \sqrt{2}(\alpha_{\pm1}-1)n_x n_{\pm1}]/2(n^2-\epsilon_{\pm1})$, $\alpha_s = (\alpha_{+1} + \alpha_{-1})/2$,

$\vec{n} = \vec{k}/\beta$, $n_\perp = \sqrt{n_x^2 + n_y^2}$.

$n_\perp' = \sqrt{n_y^2 + n_z^2}$, $N(n) = \beta^{-6}N(k = \beta n)$, $\Pi(n) = \beta^{-6}\Pi(k)$, and

$n_{\pm1} = (n_x \pm jn_y)/\sqrt{2}$.

h, r = half length and radius of dipole antenna, respectively.

To derive the radiation resistance, $R^{w,t}$ due to the radiation of whistler mode and the electron thermal mode from (3), it is necessary to examine the roots of the dispersion equation, $N(n) = 0$. Since the thermal effects of the ions are ignored in this analysis we shall only consider $R^{w,t}$ for the frequencies between the electron-gyrofrequency (f_{He}) and the lower-hybrid-resonance frequency (f_{LHR}). Within this frequency range, the wave propagation characteristics are as summarized below:

Case A. $f_{He} > f > f_q$, f_o (plasma frequency) $> f_{He}$, ($T_e \lesssim 10^5$ °K).

1. There exists a unique positive root n_p^2 of $N(n) = 0$ for the wave normal angle range $0 < \theta \le \theta_1$, where θ_1 = arctan $\sqrt{Y^2-1}$, and further $n_p \simeq n_w$ for the angular range $0 \le \theta < \theta_2$ whereas in the range $\theta_2 < \theta \le \theta_1$, n_p^2 differs drastically from n_w^2, where n_w is the refractive index for whistler mode in a cold magnetoplasma (see Eq. (11) of Wang and Bell[4]) and θ_2 = arctan $\sqrt{(-\epsilon_o-2\epsilon_s\tilde{u}^2)/[\epsilon_s + \tilde{u}^2(\epsilon_o+\epsilon_s-\epsilon_d Y)/(1-Y^2)]}$.

2. In the range $\theta_2 < \theta \lesssim \pi/2$, there exists no real positive roots of $N(n) = 0$.

Case B. $f_q > f > f_{LHR}$, $f_o > f_{He}$ ($T_e \lesssim 10^5$ °K).

1. For the wave normal angles $0 \le \theta \le \theta_1$, there exists a unique positive root n_p^2 such that $n_p \simeq n_w$ and for the range $\theta_1 < \theta < \theta_2$, there are two positive roots (both differ drastically from n_w^2) from the dispersion equation $N(n) = 0$.

2. In the angular range $\theta_2 < \theta \le \pi/2$, there exists no real positive roots to the equation $N(n) = 0$.

In the above two cases, $\theta_{1,2}$ is the unique root of $C_{1,2}(\theta)=0$ and f_q is given by solving $\theta_1-\theta_2=0$. For a proton-electron plasma f_q is given approximately by $f_q \simeq [\eta(f_{He}^2/f_o^2 + 2\tilde{u}^2)]^{-\frac{1}{2}} f_{He}$, where η is the proton to electron mass ratio.

Using the above characteristics of the real positive roots and (3), a contour integration yields[4-6] formal (integral) solutions for the leading term of the radiation resistance, $R^{w,t}_{\parallel,\perp}$:

For $f_q > f \ge f_{LHR}$, $R^t_{\parallel,\perp}$ is given by

$$R^t_{\parallel} \simeq C \sum_{t=1}^{2} \int_{\theta_1}^{\theta_2} \left|F(n_t)\right| \frac{\sin^4(\lambda n_t \cos\theta)}{\cos\theta} J_o^2(V_{\parallel}) \tan\theta d\theta \qquad (4a)$$

$$R^t_{\perp} \simeq \frac{C}{\pi} \sum_{t=1}^{2} \int_o^{\pi/2} d\phi \int_{\theta_1}^{\theta_2} \left|F(n_t)\right| \frac{\sin^4(\lambda n_t \sin\theta\cos\phi)}{\sin\theta\cos^2\phi} J_o^2(V_{\perp}) d\theta \qquad (4b)$$

where: $C = 4Z_o/\pi h^2\beta^2$, $F(n_t) = (1-C_1 n_t^2)/\Delta n_t$, $n^2_{1,2} = (-B\pm\Delta)/2C_1$ ("+" for n_1^2 and "-" for n_2^2),

$B = (C_2C_3 - C_1C_4)/C_3$ and $\Delta = \left\{B^2 - 4C_1\left[C_3 - \frac{C_4}{C_3} \times (C_2 - C_1C_4/C_3)\right]\right\}^{1/2}$

$V_{\parallel} = \beta r n_t \sin\theta \qquad V_{\perp} = \beta r n_t \sqrt{1-\sin^2\theta\sin^2\phi}$.

For $f_{He} > f \gtrsim f_q$, $R^t_{\parallel,\perp}$ is given by (4) with the limit of θ integration being reversed and setting the term with $t = 2$ which equals zero. The leading term of $R^w_{\parallel,\perp}$ is given by

$$R^w_{\parallel} \simeq \frac{(h\beta)^2 Z_o}{4\pi} \int_o^{\theta_{1,2}} \frac{n_{\parallel}^2 n_{\perp}(n^2-\epsilon_{+1})(n^2-\epsilon_{-1})}{G(\theta)(n^2-\epsilon_o)} S^w_{\parallel} d\theta \qquad (5a)$$

$$R^w_{\perp} \simeq \frac{(h\beta)^2 Z_o}{2\pi^2} \int_o^{\pi/2} d\phi \int_o^{\theta_{1,2}} \frac{n_{\perp}^3(n^2-\epsilon_o)}{G(\theta)} \left[\cos^2\phi + \frac{\epsilon_d^2}{(n^2-\epsilon_{+1})(n^2-\epsilon_{-1})}\right] S^w_{\perp} d\theta \qquad (5b)$$

where: $S^w_{\perp} = \left(\frac{\sin\lambda n_x}{\lambda n_x}\right)^4 J_o^2(\beta r\sqrt{n^2-n_x^2})$, $S^w_{\parallel} = \left(\frac{\sin\lambda n_{\parallel}}{\lambda n_{\parallel}}\right)^4 J_o^2(\beta r n_{\perp})$

$n_{\parallel} = n\cos\theta$, $n_{\perp} = n\sin\theta$ $n_x = n\sin\theta\cos\phi$,

$n = n_w = [(\epsilon_{+1}\epsilon_{-1} - \epsilon_o\epsilon_s)\sin^2\theta + 2\epsilon_o\epsilon_s - G(\theta)]/2n(\theta)$

$G(\theta) = \sqrt{(\epsilon_o\epsilon_s - \epsilon_{+1}\epsilon_{-1})^2\sin^4\theta + 4\epsilon_o^2\epsilon_d^2\cos^2\theta}$.

Where the upper limit θ_2 for the case of frequencies $f_{He} \geq f \geq f_q$ whereas θ_1 for the range $f_q \geq f \geq f_{LHR}$.

Using plasma parameters modeled upon the inner magnetosphere, (4) and (5) have been integrated numerically for the frequencies $0.15 < f/f_{He} < 1$ and $f \simeq f_{LHR}$. These results are summarized in Figs. 1 and 2 and in Table 1. It can be shown that, in general, the conditions for heavy Landau or cyclotron damping (i.e., $\bar{u}n \cos\theta \simeq 1 - \nu Y$, $\nu = 0, \pm 1$) are satisfied in the angular range (θ_1, θ_2). Thus, it is reasonable to consider the portion of antenna power in excitation of the thermal mode as a loss. Using the results of Figs. 1 and 2 and Table 1 along with the definition $\eta^w = R^w/(R^w + R^t)$, the whistler mode radiation efficiency of the antenna has been summarized in Table 2.

f_o/f_{He}	f_{LHR}/f_{He}	$R^w_{\parallel}$(ohms)	$R^w_{\perp}$(ohms)	$R^t_{\parallel}$(ohms)	$R^t_{\perp}$(ohms)
10	0.0233	6.25×10^4	9×10^4	5.8×10^4	8.2×10^3
5	0.0228	1.25×10^4	2×10^4	2.1×10^4	7.4×10^2

TABLE 1. RADIATION RESISTANCE AT $f \cong f_{LHR}$, $h_o = 0.05$.

f_o/f_{He}	h_o	$\eta^w_{\parallel}$	$\eta^w_{\perp}$
5	0.05	70-80%	66-68%
	0.5	89-100%	95%
10	0.05	80-94%	95%
	0.5	99%	~ 100%

(a) $0.2f_{He} < f < 0.975f_{He}$

f_o/f_{He}	f/f_{He}	$\eta^w_{\parallel}$	$\eta^w_{\perp}$
5	0.022	~ 38%	~ 76%
10	0.023	~ 50%	~ 90%

(b) $f \simeq f_{LHR}$, $h_o = 0.05$

TABLE 2. SUMMARY OF THE WHISTLER MODE EFFICIENCY OF A DIPOLE ANTENNA IN THE FREQUENCY RANGE $f_{He} > f >> f_{LHR}$ AND $f = f_{LHR}$.

For the above numerical results, we are able to estimate the VLF ($f_{He} > f \geq f_{LHR}$) whistler mode radiation efficiency for an electric antenna in a warm magnetoplasma. From our numerical data (Table 2), it can be concluded that for a radiating dipole in the magnetospheric plasma, the whistler mode radiation efficiency η^w is usually greater than ~ 70%, although at $f \simeq f_{LHR}$, η^w may be as low as ~ 38%. In general η^w increases with an increase in any one of the factors: dipole length, plasma density and dipole orientation angle with respect to the earth's magnetic field. In most cases $\eta^w_{\perp} > \eta^w_{\parallel}$ by at least ~ 10%. It is important to note that in the present investigation, we have assumed that the plasma is entirely uniform and have therefore not considered the effects of an antenna sheath. The antenna-sheath problem is in general a nonlinear one[1], and it is likely that a portion of the dipole power will be lost in nonlinear mechanisms within the sheath. Thus for a more accurate calculation of η^w, the nonlinear antenna-sheath interaction should be one of the most important problems to investigate.

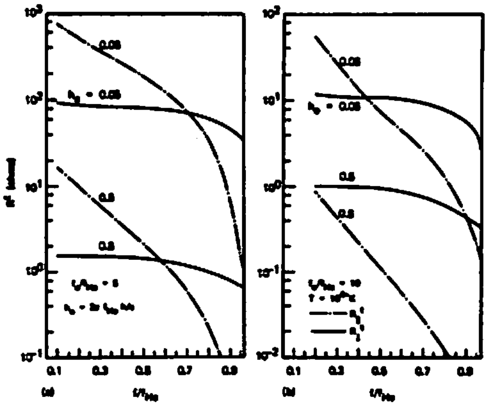

FIG. 1. VLF DIPOLE RADIATION RESISTANCE OF THE ELECTRON THERMAL MODE AS A FUNCTION OF NORMALIZED ANTENNA LENGTH, ANTENNA ORIENTATION, AND DRIVING FREQUENCY FOR $f_q << f < f_{He}$. TWO NORMALIZED DENSITIES ARE CONSIDERED: a) $f_o/f_{He} = 5$ and b) $f_o/f_{He} = 10$.

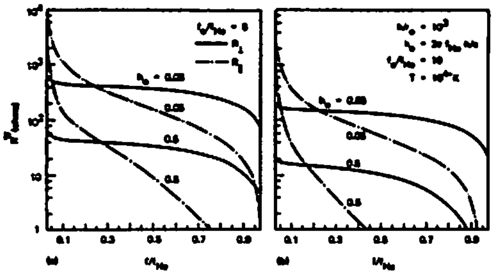

FIG. 2. VLF DIPOLE RADIATION RESISTANCE OF THE WHISTLER MODE AS A FUNCTION OF NORMALIZED ANTENNA LENGTH, ANTENNA ORIENTATION AND DRIVING FREQUENCY FOR $f_{LHR} < f < f_{He}$. TWO NORMALIZED DENSITIES ARE CONSIDERED: a) $f_o/f_{He} = 5$ and b) $f_o/f_{He} = 10$.

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