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Spontaneous exchange of leader-laggard relationship in mutually-coupled semiconductor lasers

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ABSTRACT– We investigate instantaneous behaviors of the leader-laggard relationship of temporal waveforms in mutually-coupled two semiconductor lasers in the open-loop configuration. The temporal waveform of one laser output is delayed with respect to that of the other laser output by the propagation delay time, and the leader-laggard relationship can be determined by measuring cross correlation between the two temporal waveforms with time delay. The leader can be identified on average when the optical carrier frequency of the leader laser is higher than the other laser. However, when short-term cross-correlation is measured and the local structure of the leader-laggard relationship is investigated, spontaneous exchange between the leader and laggard lasers can be observed in time, even for fixed initial optical frequency detuning.

1. Introduction

Coupled nonlinear dynamical systems show various synchronized phenomena, such as phase synchronization and chaos synchronization [1]. These phenomena have been observed in many nonlinear dynamical systems. Chaos synchronization in coupled semiconductor lasers has also been investigated intensively for recent years [2]. The applications of coupled semiconductor lasers to optical secure communication [3] and secure key distribution [4] have been proposed and demonstrated. Chaos synchronization in semiconductor lasers can be achieved by injecting light from one laser to another laser. The control of the optical frequency detuning between the two coupled semiconductor lasers is crucial in order to achieve chaos synchronization. For unidirectionally coupled semiconductor lasers, synchronization can be achieved when the optical carrier frequencies of the two lasers are matched by injection locking.

Chaos synchronization in mutually-coupled semiconductor lasers has also been investigated intensively. The leader-laggard relationship between two coupled lasers with time delay in the open-loop configuration (without self-feedback) has been studied [5-7], where the temporal waveform of one laser output follows that of the other laser output with the time shift by the propagation delay time. It has been reported that one laser with higher initial optical frequency becomes the leader in coupled semiconductor lasers [5]. For the closed-loop configuration (with self-feedback), zero-lag synchronization has been observed and the leader-laggard relationship disappears. In addition, bubbling events have been observed in the mutually-coupled semiconductor lasers [8, 9], where bubbling occurs in the regime of low frequency fluctuations (LFFs). The bubbling occurs instantaneously, however, the leader-laggard relationship under these instantaneous phenomena has not been well investigated yet.

In this study, we investigate instantaneous behaviors of the leader-laggard relationship in mutually-coupled semiconductor lasers. We change the initial optical frequency detuning and find spontaneous exchange of the leader-laggard relationship.

2. Numerical Model

We used the Lang-Kobayashi (LK) equations in our numerical simulations. The LK equations are often used as a model for a semiconductor laser with optical feedback.

$$\frac{dE_{1,2}(t)}{dt} = \frac{1+i\alpha}{2}\left[\frac{g(N_{1,2}(t)-N_0)}{1+\varepsilon\left|E_{1,2}(t)\right|^2} - \frac{1}{\tau_p}\right]E_{1,2}(t) + \kappa E_{2,1}(t-\tau)\exp(i\theta_{1,2}(t)) \quad (1)$$

$$\frac{dN_{1,2}(t)}{dt} = J - \frac{N(t)}{\tau_s} - \frac{g(N_{1,2}(t)-N_0)}{1+\varepsilon\left|E_{1,2}(t)\right|^2}\left|E_{1,2}(t)\right|^2 \quad (2)$$

$$\theta_{1,2}(t) = \Delta\omega t - \omega_{2,1}\tau \quad (3)$$

where E is the complex electric field and N is the carrier density. The subscripts 1 and 2 represent laser 1 and 2, respectively. The linewidth enhancement factor is $\alpha = 3.0$, and the carrier density at transparency is $N_0 = 1.40 \times 10^{24}$ m^{-3}. The photon lifetime is τ_p = 1.927 ps, the carrier lifetime is τ_s = 2.04 ns, the gain coefficient is $g = 8.40 \times 10^{-13}$ $\mathrm{m^3s^{-1}}$, the coupling strength between two lasers is κ = 46.6 ns^{-1}, and the injection current is $J = 1.3J_{th}$, where J_{th} is the injection current at the threshold for laser oscillation. The gain saturation parameter is $\varepsilon = 3.0 \times 10^{-23}$ and the propagation time of light between two lasers is τ = 4.0 ns. The initial optical frequency detuning $\Delta f = f_1 - f_2$ is changed in our simulations ($f_{1,2}$ are the optical frequency of the laser 1 and 2). ω is the optical angular frequency. $\Delta\omega = 2\pi\Delta f$ is the initial optical angular frequency detuning.

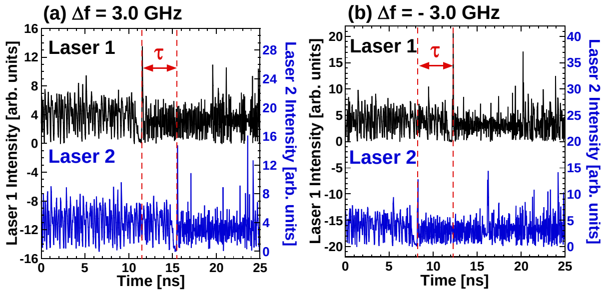

Fig. 1 Temporal waveforms of Laser 1 and 2. (a) The optical frequency detuning is positive ($\Delta f = 3.0$ GHz), and Laser 1 is the leader. (b) The optical frequency detuning is negative ($\Delta f = -3.0$ GHz), and Laser 2 is the leader.

3. Leader-laggard relationship

Figure 1 shows the temporal waveforms of Laser 1 and 2 when the initial optical frequency detuning is set to be a positive or negative value. When the initial optical frequency detuning is positive ($\Delta f = 3.0$ GHz), the temporal waveform of Laser 2 output is delayed with respect to that of Laser 1 output by the propagation delay time τ (Fig. 1(a)). Therefore, Laser 1 is the leader and Laser 2 is the laggard. On the contrary, when the initial optical frequency detuning is negative ($\Delta f = -3.0$ GHz), the output of Laser 1 is delayed with respect to that of Laser 2, and Laser 2 is the leader (Fig. 1(b)).

To determine which laser becomes the leader, we calculated the cross correlation value between Laser 1 and 2 (denoted as $C_{1,2}$), between the time-delayed Laser 1 and Laser 2 (denoted as $C_{1\tau,2}$), and between Laser 1 and time-delayed Laser 2 (denoted as $C_{1,2\tau}$) by shifting the propagation delay time τ as follows,

$$C_{1,2} = \frac{\left\langle \left(I_1(t) - \overline{I_1}\right)\left(I_2(t) - \overline{I_2}\right)\right\rangle}{\sigma_1 \sigma_2} \qquad (4)$$

$$C_{1\tau,2} = \frac{\left\langle \left(I_1(t-\tau) - \overline{I_1}\right)\left(I_2(t) - \overline{I_2}\right)\right\rangle}{\sigma_1 \sigma_2} \qquad (5)$$

$$C_{1,2\tau} = \frac{\left\langle \left(I_1(t) - \overline{I_1}\right)\left(I_2(t-\tau) - \overline{I_2}\right)\right\rangle}{\sigma_1 \sigma_2} \qquad (6)$$

where, I_1 and I_2 are temporal waveform of laser 1 and laser 2 intensity. $\overline{I}_1$ and $\overline{I}_2$ are the mean values of I_1 and I_2. σ_1 and σ_2 are the standard deviations of I_1 and I_2. We also calculated the average optical frequency detuning under mutual coupling as follows.

$$\Delta f_{ave} = \Delta f + \frac{1}{2\pi}\left(\frac{d\phi_1(t)}{dt} - \frac{d\phi_2(t)}{dt}\right) \qquad (7)$$

$$\phi(t) = \arctan\left(\frac{E_{im}(t)}{E_{re}(t)}\right) \qquad (8)$$

The average optical frequency detuning Δf_{ave} under mutual coupling is obtained from the optical phase ϕ, and Δf_{ave} is shifted from the initial optical frequency detuning Δf ($\phi_{1,2}$ is the optical phase of Laser 1 or 2). ϕ can be calculated from the real part E_{re} and the imaginary part E_{im} of the electric field E. The peak value of the optical frequency detuning (Δf_{peak}) is also calculated from the distribution of the instantaneous optical frequency detunings.

The temporal waveforms with the length of 10 μs are used for the calculation of cross-correlation. Figure 2(a) shows the cross-correlation values of $C_{1,2}$, $C_{1\tau,2}$, and $C_{1,2\tau}$ when the initial optical frequency detuning Δf is changed. When Δf is positive, $C_{1\tau,2}$ is larger than $C_{1,2\tau}$, indicating that Laser 1 is leader (see also Fig. 1(a)). On the contrary, when Δf is negative, $C_{1,2\tau}$ is larger than $C_{1\tau,2}$, indicating that Laser 2 is the leader (see also Fig. 1(b)). When the cross-correlation value is calculated without time shift, no correlation is observed (i.e., $C_{1,2} \approx 0$). Therefore, the leader-laggard relationship exists between the two mutually-coupled semiconductor lasers. For Fig. 2(a), we found that Laser 1 is the leader when the initial optical frequency detuning is positive, whereas Laser 2 is the leader when the detuning is negative. It is worth noting that this result is obtained from the long-term average of the cross-correlation value. Figure 2(b) shows the corresponding optical frequency detuning under mutual coupling. The average detuning Δf_{ave} slightly changes as Δf is changed. However, the peak value Δf_{peak} of the detuning is almost constant in the range within ±13 GHz. This result indicates that the two optical frequencies interact to each other and almost matches within this range.

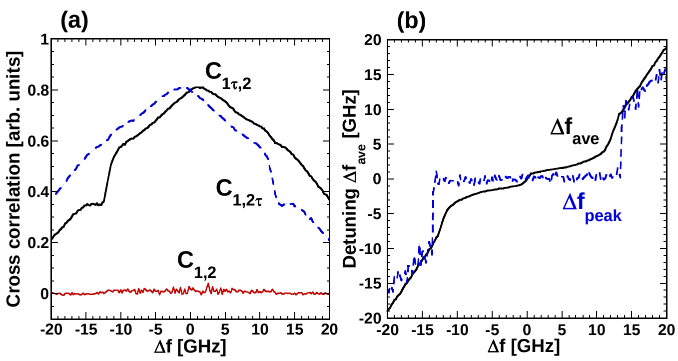

Fig. 2 (a) Cross-correlation between Laser 1 and 2. $C_{1\tau,2}$: Laser 1 is delayed. $C_{1,2\tau}$: Laser 2 is delayed. When $\Delta f > 0$, $C_{1\tau,2} > C_{1,2\tau}$ (Laser 1 is the leader). When $\Delta f < 0$, $C_{1\tau,2} < C_{1,2\tau}$ (Laser 2 is the leader). (b) Optical frequency detuning. Δf_{ave} is the average optical frequency detuning. Δf_{peak} is the peak of the distribution of the instantaneous optical frequency detunings.

4. Leader-laggard relationship in short-term range

Next we evaluate short-term leader-laggard relationship. We calculated short-term cross correlation between a temporal waveform of one laser and a time-delayed waveform of the other laser with a calculation window size of 4.0 ns. Figure 3 shows an example of temporal waveforms and short-term correlation plots at different parts of the waveforms. The initial optical frequency detuning is fixed at 3 GHz. For Fig. 3(a), the chaotic oscillation of Laser 2 is delayed with respect to the oscillation of Laser 1 by the delay time τ at the duration of 7.5~15.5 ns (see that a_1 and a_2 look similar in Fig. 3(a)). On the other hand, the chaotic oscillation of Laser 2 is advanced with respect to the oscillation of Laser 1 at the duration of 20~28 ns (d_1 and d_2 look similar). Figures 3(b)-3(e) show the short-term correlation plots for the regions of a_1-a_2, b_1-b_2, c_1-c_2, and d_1-d_2 in Fig. 3(a). A better correlation value for a_1-a_2 (Fig. 3(b)) is obtained than that for b_1-b_2 (Fig. 3(c)), indicating that Laser 1 is the leader and Laser 2 is the laggard ($C_{1\tau,2} > C_{1,2\tau}$). On the contrary, the correlation of d_1-d_2 (Fig. 3(e)) is larger than that of c_1-c_2 (Fig. 3(d)), indicating that Laser 2 is the leader and Laser 1 is the laggard ($C_{1\tau,2} < C_{1,2\tau}$). It is found that spontaneous switching between the leader and the laggard lasers occurs for short term and the leader-laggard relationship changes in time.

We investigate instantaneous behaviors of the leader-laggard relationship and the averafe optical frequency detuning in the low-frequency fluctuations (LFFs) regime. Figure 4 shows the short-term cross correlation between one laser and the other laser with time delay, and the short-term average optical frequency detuning. The time window for the calculation is set to 4.0 ns. First, the short-term optical frequency detuning increases from 0 to 20 GHz, and the correlation value of $C_{1,2\tau}$ starts decreasing. After the delay time τ, the correlation value of $C_{1\tau,2}$ decreases, indicating that Laser 1 is the leader in this range ($C_{1,2\tau} < C_{1\tau,2}$), where the short-term optical frequency detuning is positive. The peak value of the short-term optical frequency detuning is ~20 GHz and this frequency shift can be estimated as $\Delta f = \alpha\kappa / 2\pi$ [10]. When the short-term optical frequency detuning approaches 0 GHz and becomes a negative value, $C_{1,2\tau}$ is larger than $C_{1\tau,2}$, indicating that Laser 2 becomes the leader. Therefore, spontaneous exchange of the leader-laggard relationship occurs in mutually coupled semiconductor lasers in the LFF regimes.

Short-term behaviors of $C_{1\tau,2}$, $C_{1,2\tau}$, and Δf_{ave} are re-plotted in Fig. 5(a). To visualize which laser is the leader or the laggard for short term, we calculated the difference in the two cross correlation values $C_{1\tau,2}$ and $C_{1,2\tau}$. We set the threshold $C_{th} = 0.0$ to determine the leader-laggard relationship, i.e., Laser 1 is the leader for $C_{1\tau,2} - C_{1,2\tau} > C_{th}$ or Laser 2 is the leader for $C_{1\tau,2} - C_{1,2\tau} < C_{th}$.

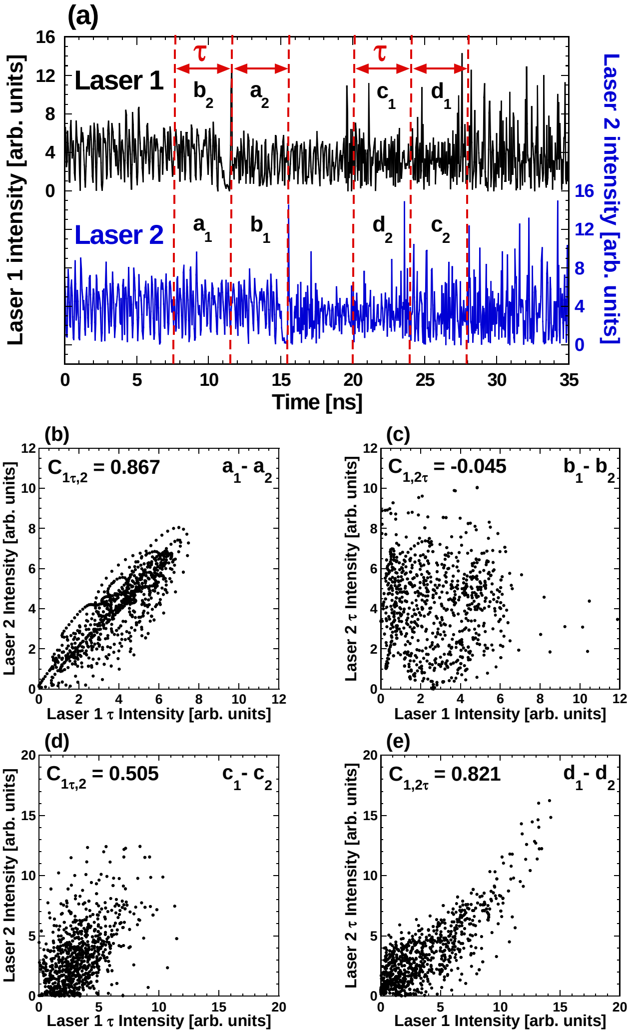

Fig. 3 (a) Temporal waveforms of Laser 1 and 2 at Δf = 3.0 GHz. (b), (c) Short-term correlation plots at the duration of 7.5~15.5 ns ($a_1 - a_2$ and $b_1 - b_2$ in (a)). $C_{1\tau,2} > C_{1,2\tau}$ and Laser 1 is the leader. (d), (e) Short-term correlation plot at the duration of 20~28 ns ($c_1 - c_2$ and $d_1 - d_2$ in (a)). $C_{1\tau,2} < C_{1,2\tau}$ and Laser 2 is the leader.

Figure 5(b) shows instantaneous temporal behavior of the leader-laggard relationship and the average optical frequency detuning. After power dropout occurs in the LFF regime, Laser 1 becomes the leader (black points in Fig. 5 (b)). When the temporal dynamics recover from the dropout, the leader-laggard relationship is switched and Laser 2 becomes the leader (blue points in Fig. 5 (b)). It is worth noting that the dynamics of the short-term optical frequency detuning corresponds to the leader-laggard relationship, i.e., Laser 1 is the leader for positive detuning and Laser 2 is the leader for negative detuning even for short term. The oscillation of the optical frequency detuning occurs due to the dropout effect, which results in spontaneous exchange of the leader-laggard relationship.

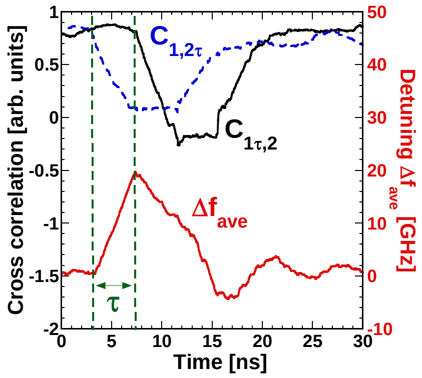

Fig. 4. Short-term cross-correlation and short-term optical frequency detuning. The calculation window size is 4.0 ns. Black curve: $C_{1\tau,2}$, blue curve: $C_{1,2\tau}$, red curve: Δf_{ave}. The initial optical frequency detuning Δf = 10.0 GHz. Spontaneous exchange of the leader-laggard relationship is found after power dropout in the LFF regime.

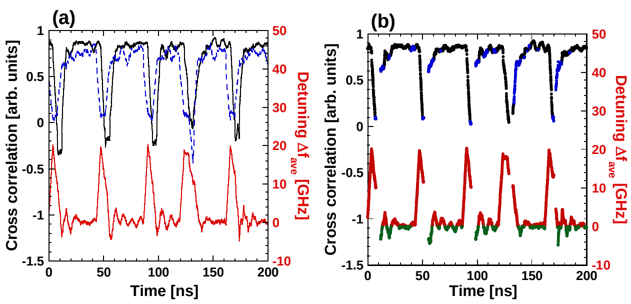

Fig. 5. Instantaneous temporal behaviors of leader-laggard relationship. (a) Short-term cross-correlation and optical frequency detuning. The initial detuning is Δf = 10.0 GHz (b) Visualization of leader-laggard relationship by calculating $C_{1\tau,2} - C_{1,2\tau}$ and optical frequency detuning. Back: Laser 1 is the leader, blue: Laser 2 is the leader, red: positive detuning, green: negative detuning.

5. Probability of leader and laggard roles

Next we calculated the probability of being the leader or laggard for the two lasers under spontaneous exchange of the leader-laggard relationship. An ensemble of the short-term cross-correlation (4 ns) is calculated from two million data points in both lasers and the probability of being the leader is calculated from the relationship of $C_{1\tau,2} - C_{1,2\tau} > C_{th}$ (Laser 1 is the leader) or $C_{1\tau,2} - C_{1,2\tau} < C_{th}$ (Laser 2 is the leader) when the initial optical frequency detuning is changed. Figure 6 shows the probability of the leader for the two lasers for two threshold values of correlation (C_{th}). For C_{th} = 0.0 in Fig. 6(a), when the initial optical frequency detuning is positive, the probability of the leader for Laser 1 is larger than that for Laser 2. However, Laser 2 can be the leader with a small probability even for positive Δf. It is worth noting that there is a small peak of the probability of the leader for Laser 2 at Δf ~12 GHz, which indicates that spontaneous exchange of the leader-laggard relationship occurs more frequently near the boundary of optical frequency matching (see Fig. 2(b)). This peak still remains when the threshold for the correlation is increased to C_{th} = 0.2. This indicates that spontaneous exchange of the leader-laggard relationship exists for some duration of temporal dynamics.

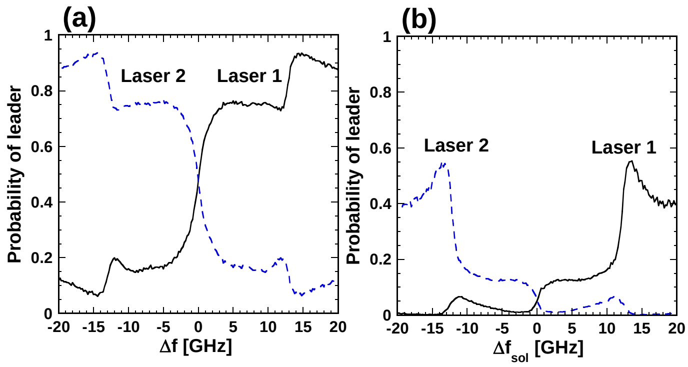

Fig. 6 Probabilities of being the leader for the two lasers as a function of the initial optical frequency detuning. Laser 1 is the leader when $C_{1\tau,2} - C_{1,2\tau} > C_{th}$, whereas Laser 2 is the leader when $C_{1\tau,2} - C_{1,2\tau} < C_{th}$,. (a) C_{th} = 0.0 and (b) C_{th} = 0.2.

6. Conclusion

We have investigated instantaneous behaviors of the leader-laggard relationship of temporal waveforms in mutually-coupled two semiconductor lasers in the open-loop configuration. The temporal waveform of one laser output is delayed with respect to that of the other laser output by the propagation delay time, and the leader-laggard relationship can be determined by measuring cross correlation between the two temporal waveforms with time delay. When short-term cross-correlation is measured and the local structure of the leader-laggard relationship is investigated, spontaneous exchange between the leader and laggard lasers can be observed in time. The leader can be identified when the optical frequency of the leader laser is higher than the other laser even for short-term behaviors.

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