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Synchronization without correlation

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Abstract—In this work, we address two fundamental questions in the field of delay-coupled oscillators: Does the synchronization between coupled oscillators necessarily imply correlations? What can be inferred from the absence of correlations in networks of coupled nonlinear dynamical elements about their connectivity? We show that for a realistic configuration of delay-coupled dynamical elements negligible correlation or mutual information are observed, although the elements are synchronized and determine each others' behaviors completely. We employ for these results delay-coupled Mackey-Glass oscillators, presenting experimental results on the emergence of identically synchronized behavior between distant elements mediated via a signal with negligible correlations but synchronized in the generalized sense.

1. Introduction

The seminal paper of Pecora and Carroll [1] created a strong interest in the study of the synchronization of coupled chaotic dynamical systems. This field started in the early 1990's and has been of great relevance ever since. The first experimental demonstrations of chaos synchronization were performed in coupled electronic circuits [1, 2], starting a long-standing tradition of synchronization studies exploiting the versatility of electronic circuit implementations.

Electronic circuits also allow for the experimental study of delay-coupled nonlinear oscillators. We are referring here to the cases in which delay is responsible for the generation of deterministic chaos, i.e. the nonlinear oscillators are stable in the absence of delay. Examples of such a scenario can be found in [3, 4], in which two electronic circuits are rendered chaotic by means of bidirectional delay-coupling and/or delayed self-feedback.

Here, we extend these previous studies and check experimentally the synchronization and correlation properties of networks with a significantly larger number of delay-coupled oscillators. By doing so, we intend to question the concept of relating correlation and synchronization in a generalized sense.

2. Experimental implementation of a single oscillator

The nonlinear oscillator of our choice is an electronic analog of a Mackey-Glass oscillator [3, 5], whose circuit diagram is shown in Figure 1. The nonlinearity of such a circuit is produced by two coupled complementary junction field-effect transistors (JFET), i.e., a p-channel and a n-channel JFETs, and a resistor (R1 in Fig. 1). Its nonlinear voltage response function can be seen in Fig. 2, in which the output voltage is a nonlinear function of the input voltage. The value of the RC constant, $RC = 0.47$ ms (R4 and C1 in Fig. 1), limits the dynamical bandwidth of the electronic analog used for demonstration purposes. Nevertheless, electronic circuits can reach higher bandwidths if needed.

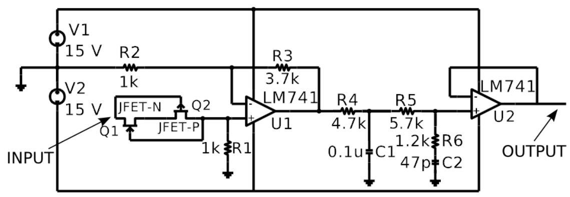

Figure 1: Circuit diagram equivalent of an individual Mackey-Glass electronic oscillator.

The relevant parameters of the oscillator can be extracted by fitting the nonlinear transfer function depicted in Fig. 2 to the following Mackey-Glass-type nonlinearity:

$$X_{out} = \frac{aX_{in}}{1 + (bX_{in})^c}. \tag{1}$$

The fit of the experimentally recorded nonlinear transfer function using the equation above yields the following parameters: $a = 2.1$, $b = 1/3$ and $c = 9.9$. We have checked that, in practice, different hardware implementations of the same electronic circuit diagram yield a parameter mismatch of a few percent ($< 5\%$) around the previous values due to the tolerance range in standard electronic components.

3. Experimental results with delay-coupled oscillators

In the experiments, we have coupled up to ten Mackey-Glass oscillators (MGOs) in a unidirectional manner, as

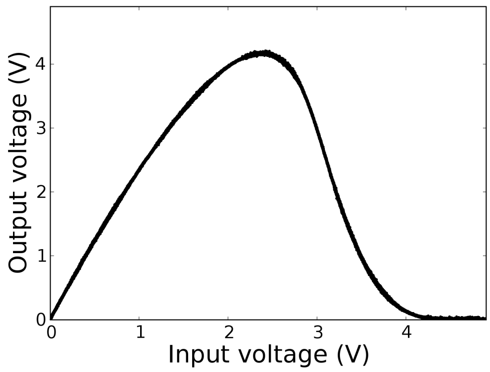

Figure 2: Experimentally recorded nonlinear transfer function of a Mackey-Glass oscillator.

shown in Fig. 3 (solid lines). We will show that this configuration does not present an identically synchronized state when the oscillators are driven into the chaotic regime. However, this does not prevent the oscillators in the ring to be synchronized in the generalizd sense.

It was numerically predicted in Ref. [6] that the ring configuration of unidirectionally delay-coupled oscillators can produce a chaotic output signal, whose temporal and spectral dynamics show a vanishing signature of the delay time for an increasing number of delay-coupled elements. In this configuration, the cross-correlation between distant elements, e.g. elements A and B in Fig. 3, decays strongly for an increasing number of elements in the coupling paths between A and B. Our experimental implementation includes a maximum of 4 MGOs in each of the coupling paths between A and B, i.e. a ring of 10 unidirectionally delay-coupled oscillators. In this configuration of delay-coupled oscillators, the delay does not need to be equally distributed in each coupling link, but the total delay can be located in a single coupling link. Apart from the corresponding time-shifts, this configuration and the configuration with distributed delay yield the same results. Here, we set the total time delay to a fixed value of 30 ms.

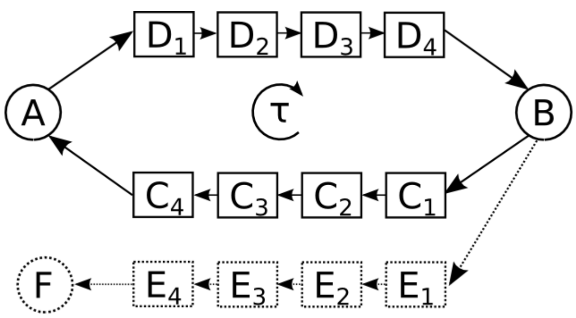

Figure 3: Schematic representation of two delay-coupled Mackey-Glass oscillators (A and B) with 4 oscillators in each of the coupling paths C and D (solid lines). In addition, element B is unidirectionally coupled to element F via coupling path E (dashed lines).

The temporal response of each individual Mackey-Glass oscillator in the delay-coupled configuration depicted in Fig. 3 shows chaotic fluctuations of the output voltage. In particular, the temporal dynamics of elements A and B are shown in Figure 4 (a) and (b). We focus on the correlation properties of elements A and B since the cross-correlation between elements which are far apart in Fig. 3 decays to zero for an increasing number of mediating elements [6]. We show in Figure 4 (c) the cross-correlation between elements A and B (see Appendix A for a definition of the cross-correlation quantifier). The maximum value of the correlation between these two elements is 0.04, which would decay even further if there were more elements in the coupling paths C and D [7].

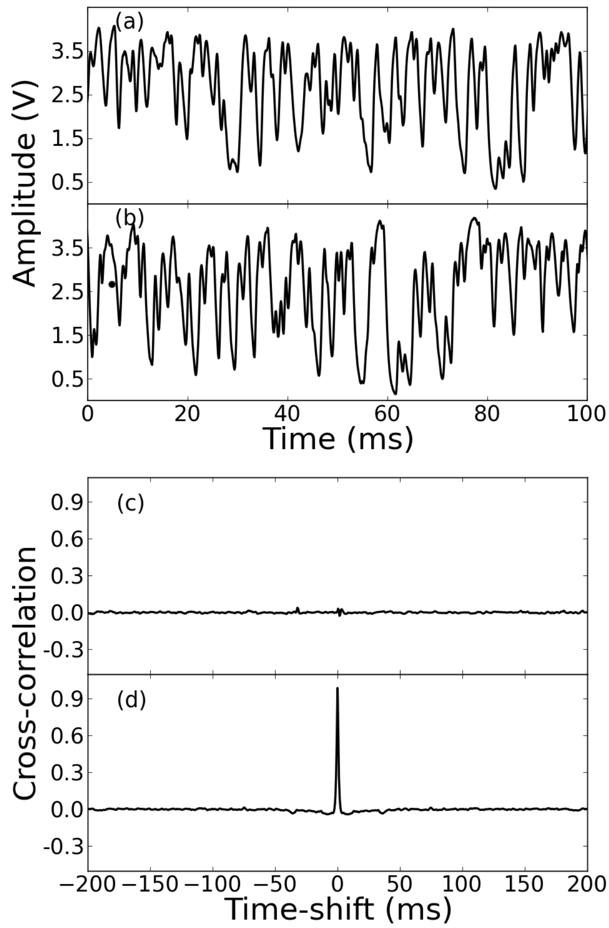

Figure 4: Experimentally recorded time-trace of the chaotic dynamics elements (a) A and (b) B in the configuration shown in Fig. 3, with a total coupling delay of 30 ms. Cross-correlation of the chaotic dynamics of elements (c) A and B and (d) A and F.

In the following, we show that it is possible to attain identical synchronization between elements of a ring of 10 elements (4 elements in coupling paths C and D) and a chain of 5 oscillators (4 elements in coupling path E) by unidirectional coupling. The experimental setup is shown schematically in Fig. 3. The symmetry properties of the ring and attached chain allow for the existence of an identically synchronized state, in which individual elements of the ring are identically synchronized to individual elements of the chain. Our experimental results show that this identically synchronized state indeed exists and it is robust against fluctuations. Figure 4 (d) shows the correlation between elements A and F in our setup. The cross-correlation between these two elements reaches a maximum value of 0.986, which is a proof of stable identical synchronization. Interestingly, A and F are perfectly correlated while elements A and B share almost no correlation as shown in Fig. 4 (c) and (d). For a perfect synchronization between elements of the ring and the chain, nearly identical oscillators must be placed in equivalent positions in the coupling paths C and E. This requires a strict selection and tuning of the electronic components employed in the hardware implementation.

In addition, we show in Fig. 5(solid line) that the mutual information between elements A and B in the ring also decays strongly when the number of elements in the coupling paths C and D is increased (see Appendix A for a definition of the delayed mutual information quantifier). As presented in Fig. 5, this decay in the mutual information between A and B (solid line) does not prevent elements A and F (dashed line) to share a nearly constant mutual information for an increasing number of elements in the coupling paths.

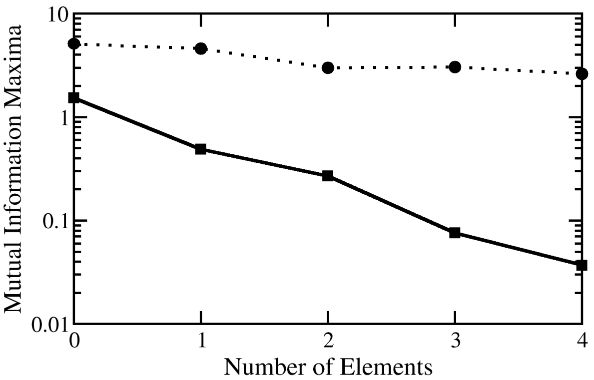

Figure 5: Maxima of the delayed mutual information between elements A-B (solid line) and A-F (dashed line) in the configuration shown in Fig. 3 as a function of the number of elements in the coupling paths C, D and E.

The existence of an identically synchronized state between elements A and F requires the existence of a generally synchronized state between all elements in the ring. The former statement is a consequence of our experimental arrangement in Fig. 3, which has been designed to apply the auxiliary system approach to detect generalized synchronization [9]. As shown in Fig. 4 (c), elements A and B share almost no correlation. Nevertheless, we find, via the auxiliary system approach, that they are generally synchronized.

4. Discussion and conclusions

The novelty of the results we present resides on the generalized synchronization between delay-coupled elements whose dynamics show negligible correlations. We test the existence of a generally synchronized state by using the auxiliary system approach and finding an identically synchronized state between elements in the ring and the auxiliary system. It is evident that two distant elements can only be identically synchronized provided that the mediating element, i.e. element B in Fig. 3, is synchronized in the generalized sense to both of them. We illustrate by example that generalized synchronization is robust against fluctuations and small parameter mismatches with an experimental realization using delay-coupled Mackey-Glass electronic circuits. We show that this is possible using the synchronization between a ring of 10 unidirectionally delay-coupled nonlinear oscillators and a chain of 5 elements. These results can be extended and complemented with numerical simulations, showing that the correlation and the mutual information between distant elements can be arbitrarily low by increasing the number of oscillators in the coupling paths [7]. In particular, Ref. [7] discusses the existence of a generally synchronized state with negligible correlations in simple network motifs of delay-coupled oscillators.

Acknowledgments

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A. Time-series analysis

In the data analysis, we have employed two different statistical quantifiers, namely the cross-correlation and the delayed mutual information. The normalized cross-correlation ($Xcorr$) is a measure of the similarity between two waveforms as a function of a time-lag applied to one of them, and it is defined as

$$Xcorr(s) = \frac{\langle [x(t) - \langle x(t)\rangle] \, [y(t-s) - \langle y(t-s)\rangle]\rangle}{\sqrt{[\langle x(t) - \langle x(t)\rangle\rangle]^2 \, [\langle y(t-s) - \langle y(t-s)\rangle\rangle]^2}}. \quad (2)$$

The delayed mutual information (DMI) measures the information on a given variable, i.e. x(t), that can be obtained by observing the time-lagged version of another variable, i.e. y(t-s), and it is defined as

$$DMI = \sum_{x(t), y(t-s)} p(x(t), y(t-s)) \log_2 \frac{p(x(t), y(t-s))}{p(x(t))p(y(t-s)))}, \quad (3)$$

where p stands for probability distribution function. In order to process the time series, we have used an implementation based on the algorithm presented in [8].

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