# The Fast and Wideband MoM Based on GPU and Two-Path AFS Acceleration

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Abstract—In this paper, a General Purpose Unit (GPU) accelerated full-wave method of moment (MoM) is combined with a two-path adaptive frequency sampling (AFS) approach to analyze the wideband characteristic of the body-wire structures. An equivalent principle is employed to treat the wire as surface so that the model which is analyzed based on the electric-field integral equation (EFIE) could be purely discretized by triangles, avoiding adopting three different basis functions. Numerical results for a monopole mounted on the square ground plane show the efficiency and accuracy of the proposed methods.

## Index Terms -- MoM, body-wire, GPU, two-path AFS

## I. INTRODUCTION

In the computational electromagnetics (CEM), the full-wave method of moment has been used extensively because of its accuracy, especially for structures composed of wires and bodies. When solving a MoM problem, one must select suitable basis functions. As to traditional body-wire, it usually employs three different basis functions to represent the wires, bodies and body-wire junctions [1]. It was introduced in [2] that the wire could be treated as surface using an equivalent principle. In this way all elements could be discretized by triangles, so that only one kind of basis function is needed, which will improve the consistency to the numerical implementation. RWG [3] is one of the most widely used basis function to model arbitrary surfaces. However, special attention should be paid to the extra T-junction problem [2] when the triangle belongs to different structures.

Although MoM is an accurate and robust method, it is of very high computation complexity. Parallel computing is an effect way to accelerate MoM computation. Much time could be reduced by using the General Purpose Unit (GPU), which disperses the task for one thread on CPU to thousands of threads on GPU. In fact, many problems in CEM have exploited the powerful computing ability of GPU to get better efficiency, such as FDTD. At the same time, the GPU was engaged to MoM first in [4] using a stream programming language Brook, which poses a programming challenge for researchers. Since the CUDA being proposed by Nvidia at 2006, it has promoted the GPU application in CEM. In [5-6], GPU is adopted to tackle MoM problem with CUDA and high speedup ratios were achieved.

Antenna simulation over a wideband demands a large number of MoM calculation at the sample frequencies. Adaptive frequency sampling (AFS) [7] is a technique to reduce the frequency points when obtaining the broadband performance of antennas. For traditional AFS method, it's necessary to inverse a matrix to get the coefficients of the targeted rational interpolation function. However, when the sampling points increase, it's nontrivial to select linearly independent points to avoid ill condition of the matrix. In [8], the Stoer-Bulirsch AFS (S-B AFS) algorithm has been introduced to avoid the matrix inversion, but it needs to get the EM value at every adapting step and utilizes only one adaptive interpolation model. In this paper, we will extend that method to a two-path AFS approach to get the broadband characteristic of the target model. This improving method could be used to simulate more complex targeted model and faster convergency rate could be achieved.

This paper is organized as follows. Theory of MoM as well as RWG basis functions is presented in Section II. Two methods have been taken to reduce the running time by employing GPU acceleration and AFS method in Section III. At last, the numerical results for a monopole antenna mounted on a conducted square plane are given in Section IV.

#### II. MOM

The MoM utilized to analyze the body-wire model in this paper is based on the electric-field integral equation (1).

$$\boldsymbol{E}_{tan}^{i} = (j\omega \boldsymbol{A}_{S} + \nabla \boldsymbol{\Phi}_{S})_{tan} \quad r \text{ in } S \quad , \tag{1}$$

Following the MoM procedure, the structure should be first divided into triangles, applied to handle arbitrary shaped objects [3]. As mentioned above, the wire is equivalent to a rectangle surface on condition that the width of the square-W and the radius of the wire-R meet the relationship of R=0.25W. Their height keeps equal. Thus, all elements could be represented by RWG basis function. It is important to notice the T-junction problem in Fig.1caused by body-wire junction. In the programming, a trick to assign different indexes to structures different could remove the T-junctions. Subsequently, the current on the conducted surface may be approximated in terms of  $f_n$  as

$$\boldsymbol{J}_{s} = \sum_{n=1}^{N} \boldsymbol{I}_{n} \boldsymbol{f}_{n}$$
(2)



Figure 1. T-junction caused by body-wire junction

 $f_n$  is just the RWG basis function which is defined as

$$f_{n} = \begin{cases} \frac{l_{n}}{2A_{n}^{+}} \rho_{n}^{+} & r \text{ in } T_{n}^{+} \\ \frac{l_{n}}{2A_{n}^{-}} \rho_{n}^{-} & r \text{ in } T_{n}^{-} \\ 0 & \text{ otherwise} \end{cases}$$
(3)

Then, choosing the  $f_n$  as the weight function according to the Galerkin method, we can get (4)

$$\langle \boldsymbol{E}^{i}, \boldsymbol{f}_{m} \rangle = j\omega \langle \boldsymbol{A}_{s}, \boldsymbol{f}_{m} \rangle + \langle \nabla \Phi_{s}, \boldsymbol{f}_{m} \rangle \quad , \qquad (4)$$

Substituting the current expansion (2) into (4) yields a matrix form equation in (5). The concrete computing methods for elements in the matrix could refer to [3]. Once obtaining the current on the structure, we can get other performance of radiating/scattering objects.

$$Z V = I \tag{5}$$

#### **III. IMPROVING METHODS**

## A. GPU Acceleration

As mentioned above, GPU is a powerful hardware accelerating tool for CEM problems. Together with CUDA, it's much convenient to parallelize MoM in C++ version. In addition, it was also much more practical and flexible than traditional versions implemented in Fortran and Matlab.

Due to the advantage of the GPU architecture, it's much more suitable to handle with tasks which are more independent and less logical. In addition, the CPU tends to deal with what is more relevant and logical. Therefore, in our programming, the subdivision of model, structure parameters and EM parameters were calculated on the host. At the same time, the device was in charge of matrix filling and matrix equation solving.

Note that the executing time for GPU includes data transfer between GPU and CPU, as well as kernel launching [9]. In order to avoid the bottleneck of data transfer, we could turn to mapped memory, which maps the address on the host to the device address, reducing data transfer latency. Since the accessing time for every kind of memory on the device is different, another problem for GPU implementation is how to select reasonable memory type for every variable. In addition, the register accessing time is the fastest on device, but its amount is limited per block. Therefore, if the block size is too large, the registers for every thread will be insufficient and if the block size is too small, it will lower the level of parallelism. The experiment results show that the 16x8 block size is an appropriate scheme.

In this study, we employed the CULA library to solve matrix equation. It provides two functions – culaDeviceZgesv() and culaZgesv() [10]. The former needs variables allocated on the device and the latter needs variables allocated on the host. Both of them adopt the LU factorization. Thereby the solution has high accuracy.

## B. Adaptive Frequency Sampling

When getting the broadband performance of radiating /scattering objects, it always tends to calculating at substantial frequency points, which will consume a lot of time. Due to the AFS strategy, we will obtain the quality of interest by constructing a rational function R(f) in (5), so as to reduce the frequency points computed by EM method. Reference [11] has employed a S-B AFS streme to get R(f), which will avoid inversing a matrix when getting the coefficients of R(f) in traditional AFS technique. In this study, it extends that method to a two-path AFS approach, which can simulate more complex objective function and converge much faster

$$R(f) = \frac{N_0 + N_1 f + \dots + N_n f^n}{1 + D_1 f + \dots + D_d f^d},$$
(6)

Unlikely [11], the two-path AFS constructs the rational function R(f) from two directions, that is to say using two efficient recursive tabular algorithms in (7) and (8) in according to [12].

$$R_{j,k}^{I} = \frac{(f - f_{j})R_{j+l,k-l}^{I} + (f_{j+k} - f)R_{j,k-l}^{I}}{f_{j+k} - f_{j}}$$
(7)

$$R_{j,k}^{2} = R_{j+1,k-2}^{2} + \frac{f_{j+k} - f_{j}}{\frac{f - f_{j}}{R_{j+1,k-1}^{2} - R_{j+1,k-2}^{2}} + \frac{f_{i+k} - f}{R_{j,k-1}^{2} - R_{j+1,k-2}^{2}}}$$
(8)

The procedure for two-path AFS algorithm is illustrated in Fig. 2. It is assumed the simulated frequency band ranges from  $f_L$  to  $f_H$ . Before the recursive procedure, two initial values calculated by MoM are needed for (7) and (8). The *eps* is the convergence error for the process. After the construction process is terminated, we can use the rational functions (6) to calculate the value at any other frequency points, instead of the time-consuming MoM. Therefore, it will improve the simulation efficiency greatly.



Figure 2 Flowchart of two-path AFS algorithm

### IV. RESULT

The experiment for this paper is carried on Dell-Precision T7600 Workstation where the hardware detail of CPU is Xeon E5-2607 with 2.4GHz Frequency and 64G RAM and the GPU type is Tesla C2075. The code is implemented in C++ on Ubuntu 12.04 LTS with double precision to achieve high accuracy.

The benchmark structure considered in this study is a monopole antenna mounted on a square ground as shown in Fig.3. The antenna's width and height is 0.04m and 1m respectively. The ground' edge is 2m.

At first, Fig.4 has displayed the result by the MoM used in this paper for return loss (S11) of the antenna in the frequency



Figure 3 A monopole antenna mounted on a square ground plane with antenna height 1m, width 0.04m and ground edge 2m as the benchmark for this study.



Figure 4 The result of return loss computed by the MoM for 476 frequency points, compared with the result from HFS S.

Table I Executing details on CI O versus OI O (T frequency point)	Table	1 Executing	details on	1 CPU	versus GPU	(1	freque	ncy	point)
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Element	Executing Ti	Speedup Ratio		
Nulliber	On CPU	On GPU		
519	12.02	0.56	21.46	
687	26.54	0.91	29.16	
879	53.51	1.64	32.62	
1059	111.81	2.46	45.45	
1335	222.53	3.47	64.13	
1887	617.78	6.52	94.75	



Figure 5 The return loss for the benchmark in the study only 123 frequency points need to be computed by MoM with two-path AFS method getting the same performance effect for 476 frequency points without two-path AFS

range from 25MHz to 500 MHz. As compared, the result from HFSS is also given. Good agreement can be observed from results.

Then, table 1 gives the executing details for the calculation of return loss for one frequency point by MoM. It includes the total running time and speedup ratio on CPU versus GPU with the matrix size increasing. Note that the CUDA timing includes the data transfer between CPU and GPU.

Finally, Fig.5 displays the antenna's S11 performance with two-path AFS method versus without it executed on GPU. We need to compute 476 frequency points for the given frequency range without the Two-Path AFS. However, after taking this technique, only 123 points are needed to get the intact wideband characteristic by MoM. Therefore, it is proved that this method offers a noticeable improvement for the efficiency of calculating antenna's broadband performance.

## V. CONCLUSION

This paper has utilized an equivalent principle to model the body-wire structure by purely triangles, avoiding employing three different basis functions. A trick was developed in programming to remove the T-junction caused by body-wire junction. Then two methods were used to improve the executing efficiency by employing GPU acceleration and adaptive frequency sampling method. The experiment results for a monopole mounted against a square ground plan indicate all these methods could achieve good effect.

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