Inversion of the dielectric constant from the co-polarized ratio and the co-polarized discrimination ratio of the scattering coefficient

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***Abstract* –Based on the co-polarized ratio and the co-polarized discrimination ratio of the back scattering coefficient, the inversion of the dielectric constant is discussed in this paper. The value of the co-polarized ratio and the co-polarized discrimination ratio are calculated for minimum effect of surface roughness for the retrieval of the dielectric constant. The ratio of the co-polarized and co-polarized discrimination of the SSA method extends the scope of the application of the ratio method. Combined with the minimum squares techniques and the GA method, two different ratios of the backscattering coefficients were proposed to retrieve the dielectric constant. The retrieved results of the real parts and the imaginary parts are in good agreement with the original results, which proves the validity of the theory primely. What is most significative is that it shows an alternative method of great help for us to retrieve the surface dielectric constant in a wide range by the ratio of the SSA method.**

I. INTRODUCTION

The research for the electromagnetic scattering and inverse scattering plays an extremely important role in the inversion of surface physical parameters and geometric parameters in the microwave remote sensing. The surface parameters inversion has been developed for multi-band multi-polarization, multi-angle surface inversion. Soil moisture retrieval is one of the important applications of microwave remote sensing in recent decades. Soil moisture is a key parameter to the environment, climate change and the grown status of the agriculture. Retrieval of the soil moisture is possible if the dielectric constant of the rough surface is known.

A lot of work has been done in the retrieval of the dielectric constant, and different retrieval algorithms have been designed. Some empirical model and semi-empirical models for the inversion of the bare soil roughness parameters and the dielectric constant are derived, such as the Oh mode [1], Dubois model [2] and Shi model [3], but these models are limited to know a lot of the priori information, and some ignores the retrieval of the correlation length. In addition, some optimization algorithms combined with the electromagnetic scattering theoretical model are used to retrieve the rough surface parameters. Yuequan Wang and Ya-Qiu Jin used the measured data of the backscattering coefficient combined with the Genetic Algorithm to simultaneously retrieve the soil roughness and dielectric constant, and the retrieved results are in good agreements with the measured data[4]. Ceraldi using the polarization ratio of the Bistatic scattering coefficient combined with the genetic algorithm to retrieve the dielectric rough surface information, but the inversion requires a lot of times to get the true value of the dielectric constant[5]. Based on the measured scattering coefficients of different types of soil, R. Prakash established an empirical formula of C-band polarization ratio, and the inversion results are better than the Kirchhoff approximation, on which the soil composition have a less effect[6]. 2011, Mingquan Jia uses an AIEM method combined with the neural network to retrieve the parameters of the soil surface , with the measured data of the L / S / C / X-band of the soil surface, single frequency dual-polarization and dual-frequency single co-polarization backscattering coefficient, the retrieved results and experimental data are in good agreement, but the process of the retrieval is complex and time costing [7].

The present paper shows the co-polarized ratio and the co-polarized discrimination ratio are alternative methods to retrieve the surface dielectric constant fast and precisely. The co-polarized ratio and the co-polarized discrimination ratio are calculated to minimize the surface roughness effect, so the number of the unknowns are reduced. Additional, the comparison between the two ratio equations are analyzed. The retrieval and the original data are in great agreements, confirming the Correctness of the methods.

II THEORY FORMULAS

A. The equations of the co-polarized ratio and the co-polarized discrimination ratio

In the theory of the rough surface scattering, the SPM, KA, and SSA are the most common and accuracy methods. Based on these methods, the co-polarized ratio and the co-polarized discrimination ratio will be derived in this section.

1) The small perturbation method (SPM)

The SPM method can predicte the scattering coefficient of the Small fluctuation rough surface well, in the condition of which the height standard of the rough surface is much smaller than the incident wavelength and the slope of the rough surface is not high. The scattering coefficient of the SPM can be written as follows:

$$\sigma_{pq}^{0}=8\left(k^{2}\cos\theta_{s}\cos\theta_{i}\left|\alpha_{pq}\right|\right)^{2}W\left(q_{x},q_{y}\right) \quad (1)$$

The co-polarized ratio in the direction of the backscattering is:

$$f_{SPM}^{C}\left(\theta,\varepsilon\right)=\frac{\sigma_{hh}^{0}}{\sigma_{vv}^{0}}=\left|\frac{R_{hh}}{R'_{vv}}\right|^{2} \quad (2)$$

The co-polarized discrimination ratio in the backscattering direction can be expressed as:

$$f_{SPM}^{D}\left(\theta,\varepsilon\right)=\frac{\sigma_{vv}^{0}-\sigma_{hh}^{0}}{\sigma_{vv}^{0}+\sigma_{hh}^{0}}=\frac{\left(R'^{2}_{vv}-R_{hh}^{2}\right)}{\left(R'^{2}_{vv}+R_{hh}^{2}\right)} \quad (3)$$

where

$$R'_{vv}=\frac{(\varepsilon-1)\left(\sin^{2}\theta-\varepsilon(1+\sin^{2}\theta)\right)}{\left(\varepsilon\cos\theta+\sqrt{\varepsilon-\sin^{2}\theta}\right)^{2}} \quad (4)$$

$$R_{hh}=\frac{\left(\cos\theta-\sqrt{\varepsilon-\sin^{2}\theta}\right)}{\left(\cos\theta+\sqrt{\varepsilon-\sin^{2}\theta}\right)} \quad (5)$$

2） The Scalar approximation Kirchhoff equations

When the radius of the curvature is much great than the incident length, the rough surface scattering plane can be treated as an infinite plane tangent. In this case, the method of the Scalar Kirchhoff approximation can be used to calculate the scattering coefficient of the rough surface.

Supposing the rough surface satisfy the Gaussian random process and Gaussian correlation, the scattering coefficient can be written as three components:

$$\sigma_{pqc}^{r}=\pi k_{1}^{2}\left|a_{0}\right|^{2}\delta\left(q_{x}\right)\delta\left(q_{y}\right)e^{-q_{z}^{2}\sigma^{2}} \quad (6)$$

$$\sigma_{pqn}^{r}=\left(\left|a_{0}\right|k_{1}l/2\right)^{2}\exp(-q_{z}^{2}\sigma^{2}) \cdot\sum_{n=1}^{\infty}\frac{(q_{z}^{2}\sigma^{2})^{n}}{(n!n)}\exp\left(-\frac{(q_{x}^{2}+q_{y}^{2})l^{2}}{4n}\right) \quad (7)$$

$$\sigma_{pqs}^{r}=-(k_{1}\sigma l)^{2}\left(q_{z}/2\right)\exp(-q_{z}^{2}\sigma^{2}) \cdot\mathrm{Re}\left\{a_{0}\left(q_{x}a_{1}^{*}+q_{y}a_{2}^{*}\right)\right\}\times\sum_{n=1}^{\infty}\frac{\left(q_{z}^{2}\sigma^{2}\right)^{n-1}}{n!n} \cdot\exp\left(-\frac{(q_{x}^{2}+q_{y}^{2})l^{2}}{4n}\right) \quad (8)$$

The co-polarization ratio in the backscattering direction is:

$$f_{KA}^{C}\left(\theta,\varepsilon\right)=\frac{\sigma_{hh}^{0}}{\sigma_{vv}^{0}}=\left|\frac{R_{hh}}{R_{vv}}\right|^{2} \quad (9)$$

Which ignores the incoherent scattering component σ_{pqs}^{r}.

The co-polarization discrimination ratio in the backscattering direction can be expressed as:

$$f_{KA}^{D}\left(\theta,\varepsilon\right)=\frac{\sigma_{vv}^{0}-\sigma_{hh}^{0}}{\sigma_{vv}^{0}+\sigma_{hh}^{0}}=\frac{\left(R_{vv}^{2}-R_{hh}^{2}\right)}{\left(R_{vv}^{2}+R_{hh}^{2}\right)} \quad (10)$$

where the value of R_{hh} can refer to the equations (5) and the value of R_{vv} can be written as:

$$R_{vv}=\frac{\varepsilon\cos\theta-\sqrt{\varepsilon-\sin^{2}\theta}}{\varepsilon\cos\theta+\sqrt{\varepsilon-\sin^{2}\theta}} \quad (11)$$

3）The method of the Small slope approximation

The height standard of the rough surface is not limited in the SSA method. But it requires that the rms slope of the rough surface is low. The SSA method can respectively degenerate to the Kirchhoff approximation and the small perturbation method. With SSA-1, the bistatic scattering coefficient can be expressed as:

$$\Sigma(\vec{k},\vec{k}_{0})=\frac{1}{\pi}\left(\left|\frac{2qq_{0}}{q+q_{0}}B(\vec{k},\vec{k}_{0})\right|\right)^{2} \cdot\int\left\langle\exp\left[j\left(q+q_{0}\right)\cdot(z_{2}-z_{1})\right]\right\rangle\exp\left[-j(\vec{k}-\vec{k}_{0})\cdot\vec{r}\right]d\vec{r} \quad (12)$$

where $W(\vec{r})$ is the Autocorrelation function and $W(0)=\sigma^{2}$. $\left(k_{0},k\right)$ are the horizontal projections of the incident wave vector and the scattering wave vector. $\left(q_{0},q\right)$ are the vertical components of the incident wave vector and the scattering wave vector. $B(\vec{k},\vec{k}_{0})$ is associated with the dielectric constant and the incident vector wave.

Where

$$B_{vv}(\vec{k},\vec{k}_{0})=\frac{\varepsilon-1}{(\varepsilon q^{(1)}+q^{(2)})(\varepsilon q_{0}^{(1)}+q_{0}^{(2)})}\left(q^{(2)}q_{0}^{(2)}\frac{\vec{k}\cdot\vec{k}_{0}}{kk_{0}}-\varepsilon kk_{0}\right) \quad (13)$$

$$B_{hh}(\vec{k},\vec{k}_{0})=-\frac{\varepsilon-1}{(q^{(1)}+q^{(2)})(q_{0}^{(1)}+q_{0}^{(2)})}\frac{\omega^{2}}{c^{2}}\frac{\vec{k}\cdot\vec{k}_{0}}{kk_{0}} \quad (14)$$

So the co-polarization ratio in the backscattering direction is:

$$f_{SSA}^{C}\left(\theta,\varepsilon\right)=\frac{\sigma_{hh}^{0}}{\sigma_{vv}^{0}}=\left|\frac{B_{hh}}{B_{vv}}\right|^{2} \quad (15)$$

The co-polarization discrimination ratio in the backscattering direction can be expressed as:

$$f_{SSA}^{D}\left(\theta,\varepsilon\right)=\frac{\sigma_{vv}^{0}-\sigma_{hh}^{0}}{\sigma_{vv}^{0}+\sigma_{hh}^{0}}=\frac{B_{vv}^{2}-B_{hh}^{2}}{B_{vv}^{2}+B_{hh}^{2}} \quad (16)$$

III RETRIEVAL ALGORITHM

According to the above co-polarization ratio and the co-polarization discrimination ratio in the backscattering direction, the retrieval of the dielectric constant can be conducted based on the minimum squares theory. The evaluation function of the inversion can be written as

$$g\left(\varepsilon',\varepsilon''\right)=\sum_{i}\left|f\left(\varepsilon',\varepsilon'',\theta_{i}\right)-\hat{f}\left(\theta_{i}\right)\right|^{2} \quad (17)$$

Where $\hat{f}(\theta_i)$ is the experimental data, and $f(\varepsilon', \varepsilon'', \theta_i)$ is the theoretical results of the co-polarization ratio or the co-polarization discrimination ratio in the backscattering direction, and $\varepsilon', \varepsilon''$ refer to the real and imaginary parts of the dielectric constant. The Genetic Algorithm is used to find the minimization of the function(17). The co-polarization ratio and the co-polarization discrimination ratio in the backscattering direction can be used to retrieve the dielectric constant in a wider range of the rough surface roughness.

IV NUMERICAL RESULTS AND DISCUSSIONS

A For the small fluctuant rough surface

For the soil surface, as shown in the Fig. 1, the SPM method can give a good description of the small fluctuant rough surface, with the dielectric constant of the soil surface (15.3,-3.7). Now we can use the co-polarized ratio and the co-polarized discrimination ratio of the backscattering coefficient to retrieve the dielectric constant of the soil minimizing the surface roughness.

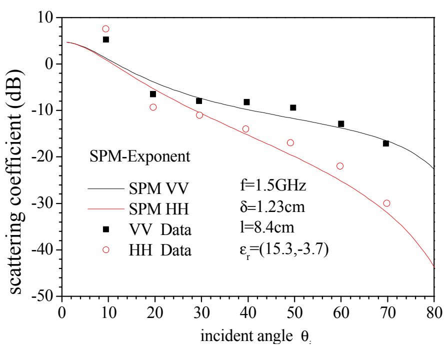

Figure 1. The backscattering coefficient of soil surface via the incident angle with SPM method

The co-polarized ratio formula and the measured data were put together in a least-squares equation with GA method yields the retrieved value of the dielectric constant. In the inversion process, the population size of each generation is 6000.The maximum number of iterations is 10000. The chromosome mutation probability is 2%, and the crossover probability is 90%. Finally, the dielectric constant of the retrieved results is (15.29,-3.70), and the evaluation value is 5.03e-14. As shown in the Fig. 2, the predicted and actual values of the ratio are in good agreements.

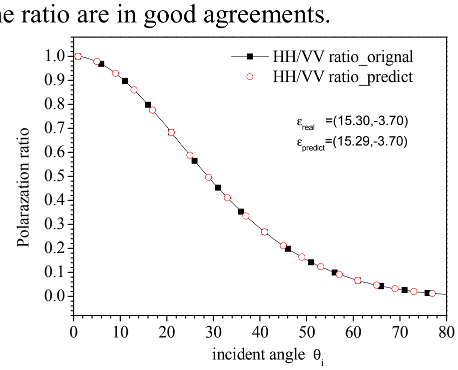

Figure 2. The co-polarized ratio via the incident angle

For the co-polarized discrimination ratio, the same method is applied for the retrieval of the dielectric of the soil surface. It can also minimize the effect of the roughness of the soil surface. The parameters of the soil surface are shown in the Fig. 1. The retrieved results of the dielectric is (15.29, -3.71), the value of evaluation is 1.747e-13. The co-polarized discrimination ratio of the original and the retrieved were compared in Fig. 3. The good agreements show the correctness of the method.

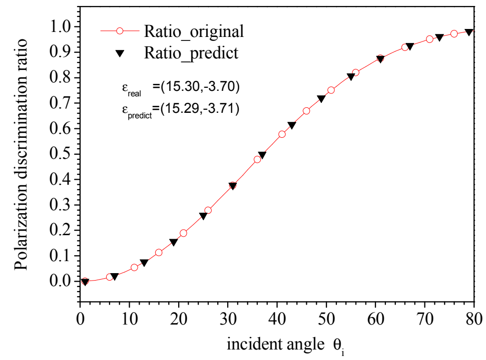

Figure 3. The co-polarized discrimination ratio via the incident angle

B. For the moderate fluctuant rough surface

The Small slope approximation method had a wide range of the surface roughness. It can predict the scattering coefficient effectively and precisely. As shown in the Fig. 4, the SSA method is used.

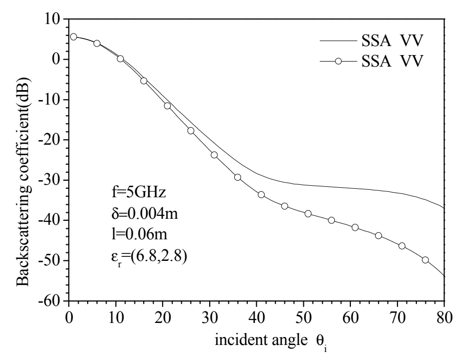

Figure 4. The backscattering coefficient of surface via the incident angle with SSA method

Referring to the equation (15), (16) and (17), the co-polarized ratio and the co-polarized discrimination ratio for the SSA method are used to retrieve the dielectric of the rough surface in the same way, but the equations of the ratio are different from that of the SPM. The retrieved results of the dielectric constant, using the co-polarized ratio, are shown in the Fig. 5. The retrieved values of the dielectric constant are (6.79,-2.80), and the evaluation value is 4.17e-12. In the same way, the retrieved results of the dielectric constant, using the co-polarized discrimination ratio, is shown in the Fig. 6. The retrieved value of the dielectric constant is (6.80,-2.80), and the evaluation value is 2.56e-15.The good agreement proves the validity of the theory primely.

To demonstrate this method is very effective in retrieving the dielectric constant, more dielectric constant values will be analyzed and retrieved. The real parts and imaginary parts of the retrieval results are shown in Fig. 7.

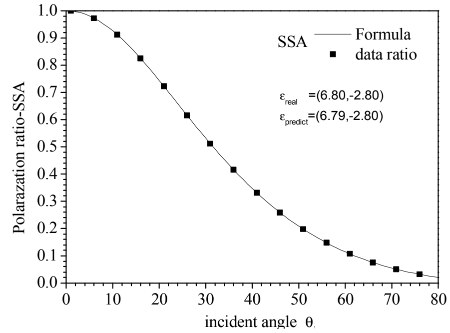

Figure 5. The co-polarized ratio of the SSA method via the incident angle

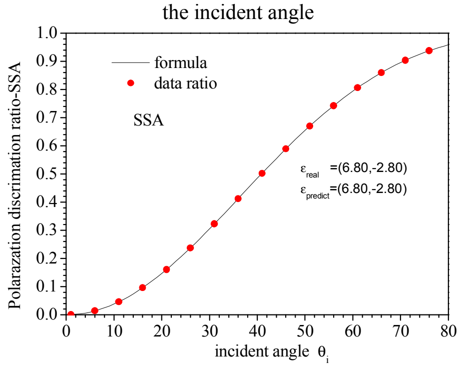

Figure 6. The co-polarized discrimination ratio of the SSA method via the incident angle

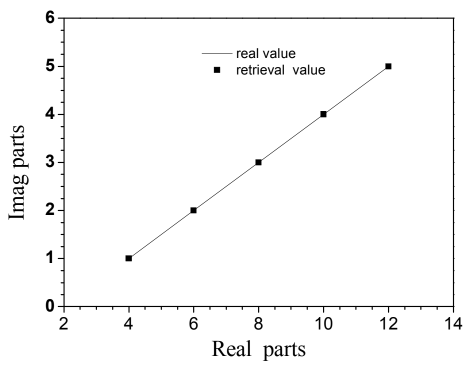

Figure 7. The original and the retrieved dielectric constant

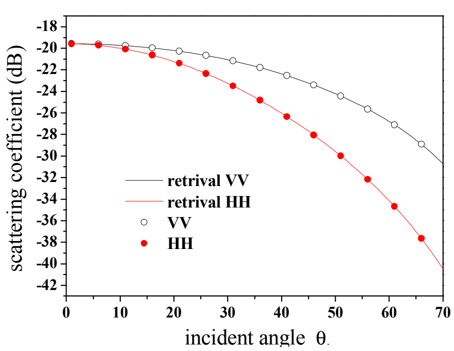

Figure 8. The comparison between the retrieval results and the original results with SPM method

In the Fig. 7, the inverse results are very accurate using the co-polarization ratio equations and the GA method. Put one of the inversion results, (3.99998, -1.00005), into the SPM method and calculate the backscattering coefficients, as shown in the Fig. 8, the calculation results are in good agreement with the original data of the backscattering coefficients, which the true dielectric constant is (4.0, -1.0).

V. CONCLUSION

In this paper, the ratio of the co-polarized and co-polarized discrimination of the SSA method extends the scope of the application of the ratio method. Combined with the minimum squares techniques and the GA method, two different ratios of the backscattering coefficients were proposed to retrieve the dielectric constant in this paper, with the method of SPM, KA and SSA. The ratio of the co-polarized and the co-polarized discrimination of the backscattering coefficients minimized the roughness of the surface and it was only related to the dielectric constant. In a wide range of the roughness, the ratio exists, for the method of the SPM, KA, and SSA. The great agreements on the retrieved results and the original results of the dielectric constant, confirm the present retrieved method. It is a alternative method of great help for us retrieve the surface dielectric constant precisely.

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REFERENCES

[1] Y. Oh, K. Sarabandi, and F. T.Ulaby, "An empirical model and an inversion technique for radar scattering from bare soil surfaces " *IEEE Transactions on Geoscience and Remote Sensing,* vol. 30, pp. 370-381, 1992.

[2] P. C. Dubois, J. Van Zyl, and T. Engman, "Measuring soil moisture with imaging radars," *Geoscience and Remote Sensing, IEEE Transactions on,* vol. 33, pp. 915-926, 1995.

[3] J. Shi, J. Wang, A. Y. Hsu, *et al.*, "Estimation of bare surface soil moisture and surface roughness parameter using L-band SAR image data," *Geoscience and Remote Sensing, IEEE Transactions on,* vol. 35, pp. 1254-1266, 1997.

[4] Yuequan Wang, Yaqiu Jin," The roughness and the soil moisture of the land surface simultaneously are retrieved with the Genetic Aglorithm," Journal of Remote Sensing, vol. 4, 2000.

[5] D. Singh, "Polarization discrimination ratio approach to retrieve bare soil moisture at X-band ", 2005.

[6] R.Prakash, S. D, and N.P.Pathak, "The Effect of Soil Texture in Soil Moisture Retrieval for Specular Scattering at C-Band," *PIER,* vol. 108, pp. 177-204, 2010.

[7] J. Mingquan, C. Yan, T. Ling, *et al.*, "Land-Based Scatterometer Measurements and Retrieval of Surface Parameters Using Neural Networks," in *Education Technology and Training, 2008. and 2008 International Workshop on Geoscience and Remote Sensing. ETT and GRS 2008. International Workshop on*, 2008, pp. 448-452.