A Weighted OMP Algorithm for Doppler Super-resolution

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***Abstract*- Radar target speed resolution is related to the accumulation time. In order to obtain higher frequency resolution, we need longer coherent accumulation time. However, long accumulation will bring many problems. Since the targets are sparse in the Doppler domain, Doppler super-resolution can be implemented in shorter accumulation time. Doppler frequency of different targets may be correlative, as well as data obtained by single sample, which can be solved effectively by using compressed sensing theory. For the time-frequency transform dictionary generated from frequency resolution, a weighted OMP (Orthogonal matching pursuit) algorithm is proposed to reconstruct the sensing dictionary. This algorithm is superior to OMP etc. greedy algorithm. Especially in the case of the target Doppler frequency close, it shows the high resolution accuracy.**

***Index Terms*- Doppler super-resolution; compressed sensing; orthogonal matching pursuit; sensing dictionary**

I. INTRODUCTION

Doppler resolution of radar targets generally is implemented by virtue of the echo signals through matched filtering, distinguished by Doppler filter bank for different range unit. The length of coherent accumulation time directly affects the performance of Doppler resolution. In practice, to enhance the radar detection performance, more coherent accumulation cycle is needed. Because radar antenna beam has a certain width, M target echo pulses will be received when there exists targets in the beam and M detection pulses have been transmitted. If the targets move so fast that cross-range unit occurs, the coherent accumulation time will be greatly limited. How to use less accumulation time to achieve Doppler super-resolution is the research background in this paper.

For super-resolution problem, there have been many methods in the field of DOA (direction of arrival) estimation, but the target Doppler super-resolution method is relatively less studied.

The relationship between Doppler and DOA super-resolution has their similarities, but also differences.

Similarities: 1) both of them achieve high-precision estimation with fewer amount of data. Doppler super-resolution separates multiple targets radial velocity by fewer pulses, and DOA super-resolution uses fewer array elements to locate multiple targets spatial angle. 2) both of them can be distinguished by super-resolution method.

Differences: 1) Doppler estimation is for frequency domain, but DOA for spatial domain. 2) Doppler estimation usually uses single sample in the same range unit. Furthermore, the same target response is coherent and there exists partial correlation for similar speed targets. DOA generally is for multiple snapshots, and the correlation may exist between different targets.

Following the similarities and differences, several novel methods appeared for super-resolution by CS theory in recent years such as $\ell_1 - SVD$ [1], $ISL0$ [2], greedy algorithm etc. Particularly, greedy algorithm is widely applied due to its computational efficiency and similar performance with convex optimum in high SNR. When greedy algorithm is used in time-frequency domain transform dictionary (i.e. redundant Fourier matrix), OMP[3] algorithm is superior with respect to the other greedy algorithms, such as StOMP (stagewise orthogonal matching pursuit)[4], ROMP (regularized orthogonal marching pursuit)[5], CoSaMP (compressive sampling matching pursuit)[6]. This is mainly induced by strong correlation between dictionary atoms. OMP only selects an optimal atom each time, which is more adaptive than other greedy algorithm. However, OMP also calculates the inner product between dictionary atom and observed data, which is severely affected by the correlation of atoms and noise. To address the impact of these negative factors, one sort of dictionary precondition method by alternating projection is proposed by Tropp and Schnass[7]. They construct the sensing dictionary to get the mutual coherence achieve Welch bound, however, this method does not guarantee the existences of equiangular tight frame for any size. Bo L. et al. proposed a similar method[8] which constructs the sensing dictionary and measurement dictionary to improve performance. These two methods above are both aimed at reducing mutual coherence or accumulative mutual coherence of dictionary, and the defection is that the ratio between number of columns and rows of dictionary (redundancy ratio) need relatively to be smaller. Therefore, coherence accumulation time also need longer for these methods.

In practice, optimum sensing dictionary not only depends on redundant dictionary, but also on observed signal. If observed signal can be used effectively, resolution performance will greatly be improved. Based on the assumption, we propose a weighted OMP algorithm, which constructs the weight vector through the cycle iteration to the observed signal, as well as need less accumulation time.

II. DOPPLER RESOLUTION SPARSE MODEL

For coherent radar system, if the adjacent M detection has the same cycle T_r and the carrier frequency of the transmitted signal is a constant f_0 , M echo signals from the same distance unit of the same target constitute M points frequency coherent signal. Assumed the entire process is divided into P

range units, synchronization pulse is transmitted in each radar probe cycle. Then the p^{th} range unit, m^{th} probe cycle signal can be expressed as $x_p(m)$ $(1 \le p \le P, 0 \le m \le M-1)$, as shown in Figure 1. Traditional approach solving the target Doppler frequency is making $L \ge M$ points FFT to the signal $x_p(m)$, which also can be implemented by FIR digital filter.

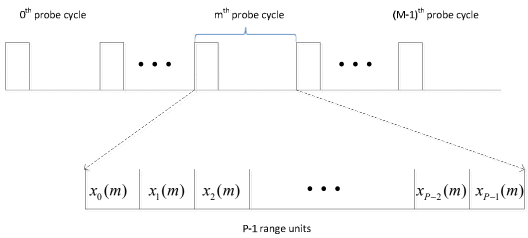

Figure 1 the adjacent M detection cycle

Unambiguous Doppler range is $0 \sim f_s$, where $f_s = 1/T_r$. For the same range, spatial angle units, Doppler resolution of multiple targets can be transformed into the situation of Doppler frequency range is divided into N discrete frequency points f_i, $0 \le i \le N$. Different Doppler frequency vector is expressed as:

$$\varphi(f_i) = \exp\left\{ j\frac{2\pi}{f_s}[0,1,\cdots,M-1]^T f_i \right\}, \quad i = 1,\cdots,N \qquad (1)$$

Which constitute the measurement dictionary

$$\Phi = [\varphi(f_1), \varphi(f_2), \cdots, \varphi(f_N)] \in \mathbb{C}^{M \times N} \qquad (2)$$

Assumed there exists K targets corresponding to different frequency points among $0 \sim f_s$. Based on the discrete grid division, if one target exists at the frequency point, the target amplitude that corresponds to the frequency is nonzero, otherwise the corresponding amplitude is zero. This allows the target amplitudes in the entire unambiguous frequency range constitute a sparse signal by sparsity is K, i.e. x has K non-zero elements. After matched filtering, residual noise of each pulse corresponds to the given range unit is denoted n, then the signal from the unit after matched filter can be expressed as

$$y = \Phi x + n \qquad (3)$$

This issue meets the basic model of the CS theory, greedy algorithm can be used to reconstruct the Doppler frequency.

III. ALGORITHM IMPLEMENTATION

For a given overcomplete dictionary Φ, the possibility of each atom being the optimum atom depends on observed signals. Once the possibility is obtained, OMP can be improved by weighted matrix which is related to optimum atoms.

Theorem 1 Let observed signal y be K-sparse in Φ, i.e. $y = \Phi x = \sum_{i \in \Lambda_0} \varphi_i x_i$, the optimum atoms index $|\Lambda_0| = K$, OMP using the sensing matrix Ψ will always select components of the true index Λ_0 if

$$\mu_1(K, \Phi, \Psi) + \mu_1(K-1, \Phi, \Psi) < \beta \qquad (4)$$

Where $\mu_1(K,\Phi,\Psi) = \max_{|\Lambda_0|=k} \max_{i \notin \Lambda_0} \sum_{j \in \Lambda_0} \left| \langle \varphi_i, \psi_j \rangle \right|$, i.e. accumulative mutual coherence, and $\beta = \min_i \left| \langle \varphi_i, \psi_i \rangle \right|$, i.e. diagonal coherence parameter. The proof refers to [7].

By the theorem above, optimum sensing dictionary should make μ_1 as possible as small and β as possible as big[9]. Considered the dictionary is normalized, the maximum value of β is one. Noted that the correlation between sensing dictionary atoms and other non-optimal atoms do not affect reconstruction performance. Therefore, construct the optimum sensing dictionary as follows.

$$\min \left\| \Psi^H \Phi_{\Lambda_0} \right\|_F^2 \quad s.t. \quad \psi_i^H \varphi_i = 1, i = 1,\cdots,N \qquad (5)$$

The optimization problem in (5) requires the optimum atoms index Λ_0, however, the index is unknown in practice. Instead of Φ_{Λ_0} by ΦW in the optimization process, then

$$\min \left\| \Psi^H \Phi W \right\|_F^2 \quad s.t. \quad \psi_i^H \varphi_i = 1, i = 1,\cdots,N \qquad (6)$$

Where $W = diag\{w_1,\cdots,w_N\}, w_i \in [0,1]$ denotes weighted matrix. Since the absolute inner product between observed signal and redundant dictionary reflects the optimum atoms index information in a certain extent. Therefore, we can initialize $W = diag\{|\Phi^H y|\}$. In extreme case, the weighted value is one corresponding to optimum atoms, otherwise zero.

For (6), by Lagrangian function

$$L(\lambda_i, \psi_i) = \frac{1}{2}\left\| W \Phi^H \psi_i \right\|_2^2 + \lambda_i \left(\psi_i^H \varphi_i - 1 \right) \qquad (7)$$

based on the definition of matrix norm, the first term on the right of (7) is written as

$$\min \left\| \Psi^H \Phi W \right\|_F^2 = \min \sum_{i=1}^{N} \left\| W \Phi^H \psi_i \right\|_2^2 \qquad (8)$$

Differentiating to ψ_i and λ_i respectively.

$$\frac{\partial L(\lambda_i, \psi_i)}{\partial \psi_i} = \Phi W^2 \Phi^H \psi_i + \lambda_i \psi_i = 0 \qquad (9)$$

$$\frac{\partial L(\lambda_i, \psi_i)}{\partial \lambda_i} = \psi_i^H \varphi_i - 1 = 0 \qquad (10)$$

By (9) and (10),

$$\psi_i = \frac{R^{-1}\varphi_i}{\varphi_i^H R^{-1} \varphi_i} \qquad (11)$$

where $R = \Phi W^2 \Phi^H$.

The main steps of the weighted OMP algorithm are summarized below.

Input: *K , Φ , y , iterations: $J = 20$ (select the iterations to meet the accuracy requirements)*

Process 1: calculate the sensing dictionary

Initialization:

$$W = diag\left\{\left|\Phi^H y\right|\right\}$$

Iteration: at the j^{th} iteration ($1 < j \le J$), go through the following steps

(1) calculate $R = \Phi W^2 \Phi^H$

(2) update $\psi_i = \dfrac{R^{-1}\varphi_i}{\varphi_i^H R^{-1}\varphi_i}$

(3) update $W = diag\left\{\left|\Psi^H y\right|\right\}$

End of the iteration, the sensing dictionary is Ψ .

Process 2: Doppler resolution

Using the constructed sensing dictionary Ψ renew OMP algorithm.

Initialization: $a = 0$, $r = y$, $I = \phi$

Sensing : $i = \arg\max_j \left|\langle \psi_j , r \rangle\right|$

Reconstruction: $I = I \cup i$, $a = \Phi_I \Phi_I^\dagger y$, $r = y - a$

Where $\Phi_I^\dagger := \left(\Phi_I^H \Phi_I\right)^{-1} \Phi_I^H$ denotes the pseudo-invserse of the matrix Φ_I , and " H " stands for matrix conjugate transpose.

IV. SIMULATION RESULTS

In this section, we present several experimental results for our Doppler resolution method. First, we compare the resolution performance of our approach to FFT, OMP. Next, we show the bias of OMP and our approach and analyze the reason.

We consider three targets exist in the same range unit and angle unit. Furthermore, coherence accumulation pulses M = 32, pulse repetition frequency f_s = 500Hz, frequency domain grids N = 256, SNR= 10dB, the real Doppler frequencies are f_1 = 309.8039Hz, f_2 = 329.4118Hz, f_3 = 466.6667Hz. The grid numbers in the frequency domain shows that the frequency point number between f_1 and f_2 is five.

The result in Figure 2 shows that FFT method do not distinguish f_1 and f_2 due to their close proximity. Although OMP gives sharp peaks, but f_1 estimated is biased. Our method locates the three frequencies accurately, only the amplitude fluctuates compared to real signals since the effect of Gaussian white noise.

In the second experiment, we investigate bias by considering localization of two signals more closely and varying the frequency separation between them. We plot the bias of each of the two signal location estimates as a function of the frequency separation when one frequency is held fixed at the 55th frequency point, and the other change from 57th to 68th frequency point. After a Monte Carlo experiment, the estimation bias is calculated. The definition of bias is $bias = \dfrac{1}{Mon}\sum_{mon=1}^{Mon}\left|\hat{f}_{num} - f_{num}\right|$, $num = 1,2$. Where $Mon = 50$, $\hat{f}_{num}$, f_{num} represent the estimated and real frequency points respectively. The SNR is 10dB, M = 32, N = 256, f_s = 500Hz. The simulation result in Figure 3 illustrates when our approach is applied in the close proximity of the frequency resolution, the performance is much better than OMP. To facilitate the viewing, the sign of the longitudinal axis in Figure 3 does not represent the frequency deviation direction, only denotes the size of the deviation.

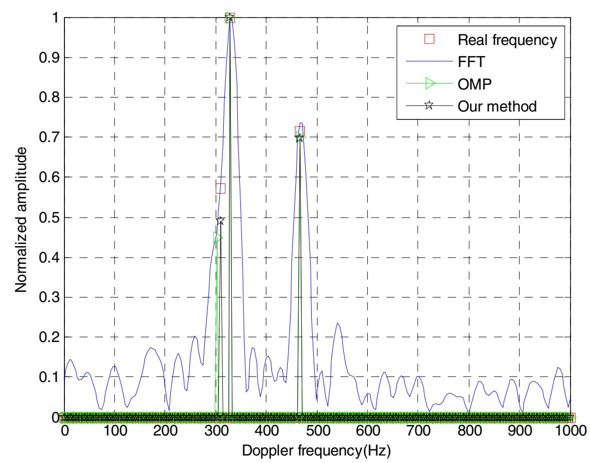

Figure 2 Doppler estimation of three targets

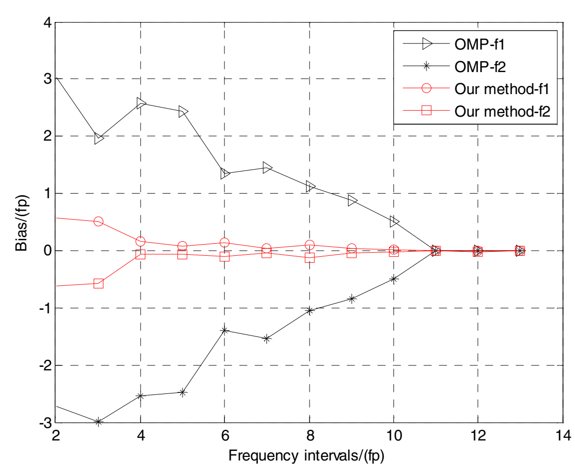

Figure 3 two signals estimate bias
(coordinate units: fp represents frequency point intervals)

In addition, within seven frequency point interval, OMP has obvious fluctuation. This can be explained by the following two reasons. Firstly, the result is partially caused by high cycle correlation character of the measurement dictionary atoms, which is corresponding to cycle fluctuation of Gram matrix[10] constructed by redundant Fourier matrix. Secondly, under the condition of the simulation above, the resolution have not located such close target frequencies accurately. Beyond seven frequency interval, OMP is gradually stable.

V. DISCUSSION AND CONCLUSIONS

In this paper, we construct the optimum sensing dictionary by using of the observed signal information, and analyze the rationality of weighted matrix theoretically. Next, a weighted OMP algorithm is established. The simulation shows our approach is applicable to the case of the target Doppler frequency close. Compared with the existing algorithms, especially for greedy algorithm, the estimate accuracy has been further improved. Since the observed signal is introduced to generate the weight matrix in the process, the complexity of algorithm will increase. Therefore, future work will focus on the computational efficiency.

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