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# Verifying chaotic dynamics from experimental data

Michael Small†, David M. Walker‡, Antoinette Tordesillas‡

†School of Mathematics and Statistics, University of Western Australia, Crawley, WA 6009, Australia
‡Department of Mathematics and Statistics, University of Melbourne, Parkvillle, Victoria 3052, Australia
Email: michael.small@uwa.edu.au

**Abstract**—Often, one is faced with measured time series data from some (presumed to be deterministic) dynamical system. The problem is to correctly infer the true, or at least, likely, underlying dynamical system from data alone. A variety of methods exist to achieve this — under the general umbrella on nonlinear modeling and machine learning. These methods fit a surface (usually smooth) to the data in such a way that that surface can be used as a proxy for the evolution operator of the original system. Unfortunately, different methods produce different results. Worse still, due to the nonlinearity inherent to the problem, even the same method will produce a range of distinct local minima. The aim of this report is to apply an ensemble of dynamical measures of system behaviour to show how one can determine which models *behave* most like the underlying data.

## 1. Introduction

For most experimental nonlinear systems it is not possible to write down a closed-form analytic description of the dynamics. In such situations it is therefore appealing to apply the methods of delay reconstruction [7] and nonlinear modelling [4] to estimate the underlying evolution operator. One can then use this estimate to study properties of the dynamical system which it represents, and then extrapolate this to the experimental system of interest. This second step can be rather problematic. In 2009, Small and Carmeli [5] re-examined an earlier work of Marquet and co-workers [3]. Marquet *et al.* [3] sought to construct a global nonlinear model from time series data of Canadian Lynx populations. Based on their models they demonstrated that the data is potentially consistent with chaos — moreover, they were able to claim that the chaotic dynamics of the models was direct evidence of chaos in a real ecosystem. While it is true that their model — a best fit to the observed data — did indeed exhibit the desired "interesting" chaotic dynamics, it would not be true to infer that this is the only reasonable explanation of this data. In [5] we showed that it is equally likely that the data could be described as a periodic orbit. Moreover, for our models (using a different modeling algorithm from Marquet and co-workers, which we will explain below) when interesting chaotic dynamics did arise this turned out to be transient — with a time scale much longer than the time scale of the original data. In Fig. 1 we plot the original data used by both [3] and [5] together with an ensemble of these model simulations.

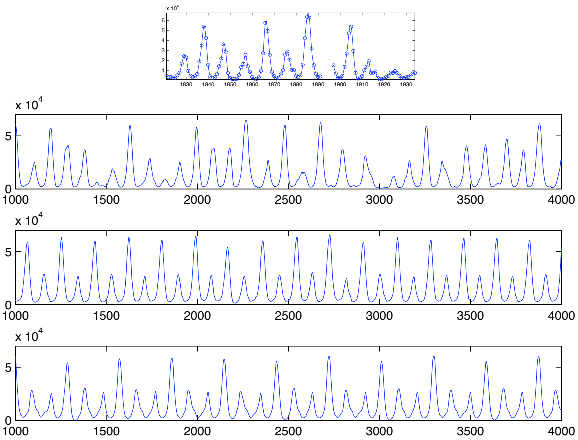


Figure 1: **The Canadian Lynx time series : chaotic or periodic?** The upper panel shows the original data (open circles) from which various global nonlinear models have been built. The lower panel depicts the dynamical behaviour of three such models. The horizontal axis in the upper panel is the observation year: one data point per year. In the lower three panels the horizontal axis are in units of model time steps: 10 time steps per year. The dynamics observed in the lower three panels are: chaotic, almost periodic and exactly periodic.

For the Canadian lynx data depicted in Fig. 1 model simulations — with the addition of a dynamic noise term — all appear qualitatively to behave very like the observed time series (this was extensively demonstrated in an earlier contribution to an earlier iteration of NOLTA [5]).

Given that the single data set can give rise to a range of equally plausible models, the natural question which arises is: which model is best? For short term predictability one can quantify the performance of various models with "honest" prediction error: the root mean square model error when applied to previously unseen data. A useful proxy for this measure, circumventing the need to acquire additional data is minimum description length [2]. Unfortunately, this does not necessarily translate to the best model for long term prediction. Nor does this necessarily lead to a good model description of the underlying system dynamics. In some sense, it is not really appropriate to ask "which model is correct?" Since the data is always finite and contaminated by noise, there will always be a range of plausible explanations — although some may be better than others. What we attempt to do here is find, from a range of alternatives, which model best matches the observed data.

To achieve this aim we do the obvious thing. For each candidate model we compute a range of desirable features from noisy simulations of that model. We compare these statistics to the same quantities computed from the data, and then seek the best match. In a sense, this is a reformulation of the strategy proposed in [6] to extend surrogate data methods to arbitrary model classes. The main differences are that we do not restrict ourselves to comparing a single distribution, nor are we constrained by the need for a well formulated null hypothesis. In practice we find that the existing nonlinear time series measures are useful indicators of which models are good. But that these traditional measures work even better when supplemented by motif rank distribution and other complex network measures which were first introduced in [9]

In the next section we briefly introduce the experimental system of interest and describe the range of computational and statistical tools we employ. Following a brief summary of the results of these calculations we conclude.

## 2. The dynamics of jamming-unjamming in a granular assembly

The system we study here is a particle-based (discrete element) simulation of a granular assembly in the so-called "critical state regime". That is, a system of particles in a 2-D box subject to axial compression at a constant rate (this constant rate is what allows us to treat this as a time series) and laterally confined with constant pressure. This assembly exhibits a fluctuating global shear stress. In the sort of physical system modeled by this simulation, this critical state coincides with a fully developed single persistent shear band. Clearly this is an important physical system for the understanding of a wide range of geophysical phenomena including, for example, the dynamics of mature faults. However, the complex nature of the system means that the best available models are computation models of the large scale complex system. We are interested in determining whether a simpler mathematical description may suffice. Data from this process — which is used extensively throughout this report — is illustrated in Fig. 2. The underlying simulations are described in more detail elsewhere [8].

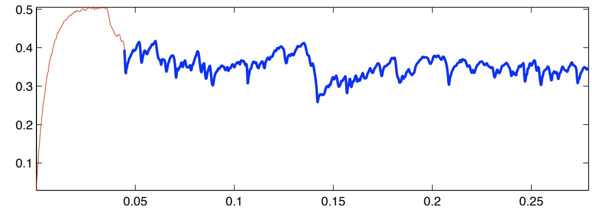


Figure 2: **Biaxial compression test** The observed time series of the bulk measurement of the shear stress ratio with respect to axial strain. The strain interval of interest is indicated by the bolder blue trace and covers the high strain post-peak regime (so-called critical state) where the material has failed and the response is in approximate steady state exhibiting characteristic stick-slip (jamming-unjamming) dynamics.

## 3. What is the best model?

We now turn our attention to answering the central question of this paper: which model will do best for the data described in the previous section? Moreover, what does that model really tell us about the underlying dynamics?

Figure 3, 4, and 5 depict comparisons of the behaviour of the model and the time series for models exhibiting: a stable node (Fig. 3); a stable focus (Fig. 4); and, transient chaos (Fig. 5). In each case we employ the Gaussian Kernel Algorithm [1, 10] to measure correlation dimension, entropy and noise level (three nonlinear measures dependent on a time delay-reconstruction of the data). We also show higher order properties of the data probability distribution: kurtosis and skewness. These computations show that the data and model simulations are most similar when the model exhibits transient chaos.

The point is further confirmed when we employ the complex network based measures introduced in [9]. Fig. 6 illustrates that the simulations most consistent with the data come from models which exhibit a transient form of chaos.

Moreover, in Fig 7 when we examine this system more closely we see two distinct chaotic dynamics: one fast and small and one slow and large. These two dynamics have independent basins of attraction, and in the presence of noise the system dynamics switch between the two states. The large slow state corresponding to the stick-slip phenomena in the original data and the smaller fast system to more

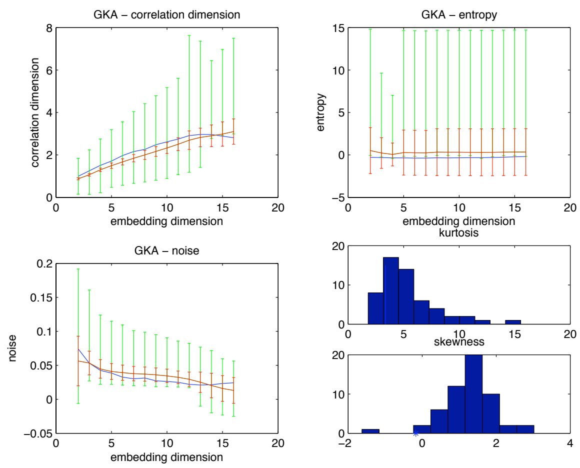


Figure 3: **Stable node** We compare the behaviour of model simulations to the original data for models exhibiting a stable node. For the three error bar plots we show comparison of the model simulations and original data with measures derived from the Gaussian Kernel algorithm : correlation dimension, entropy, and noise level. In each case we plot the mean and standard deviation (the tighter red error bars) and the full range (green). Distribution of both kurtosis and skewness, together with values for the data and illustrated as histograms. We use 100 simulations from a model of the data driven by noise. In the absence of noise this model exhibits a stable node.

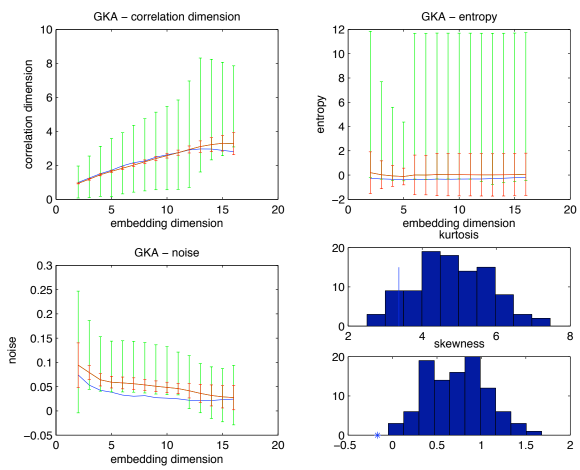


Figure 4: **Stable focus** The data displayed here is in the same format as Fig. 3. In the absence of noise this model exhibits a stable focus.

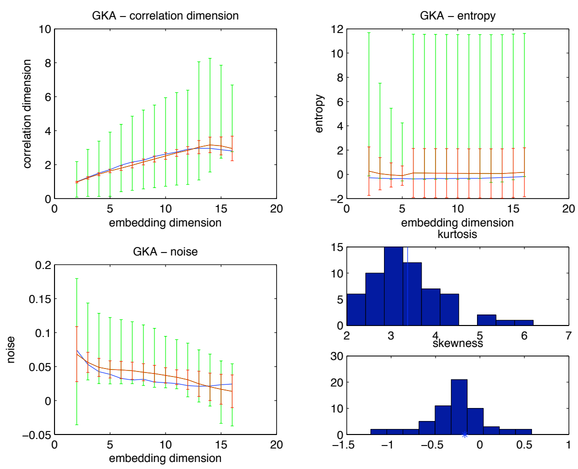


Figure 5: **Transient chaos** The data displayed here is in the same format as Fig. 3. In the absence of noise this model exhibits a transient chaos (typically over a time scale longer than the observed data) and a stable fixed point.

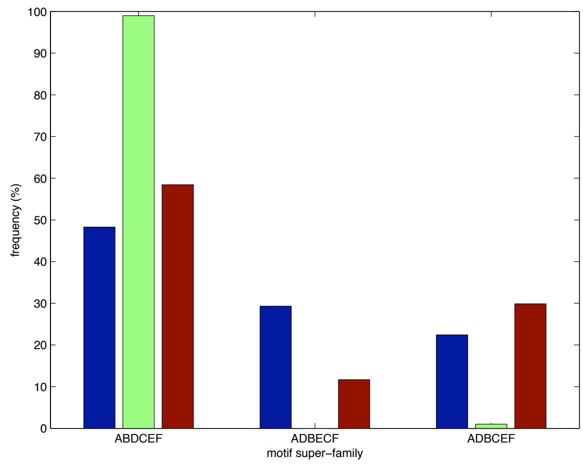


Figure 6: **Motif frequency distribution** Models simulation exhibited one of three distinct motif super-family memberships: ADBCEF (the same as observed for the data), ABDCEF, and ADBECF. For each of these motif-super-families the cluster of three vertical bars indicates the frequency (raw count) with which models exhibiting each of these motif super-families also exhibited: a stable node; a stable focus, or transient chaos. Clearly, the stable focus is not consistent with the data whereas transient chaos models offer the most likely explanation.

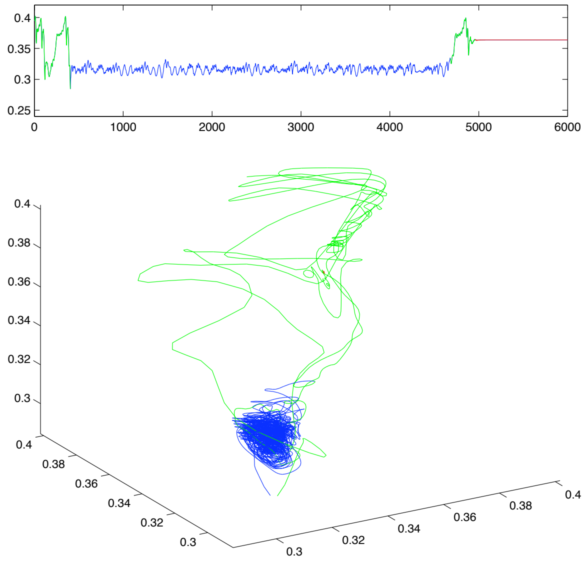


Figure 7: **Sample trajectory** The top panel depicts a single noise free simulation exhibiting transient chaos-type dynamics: note the two distinct chaotic dynamic regimes — in addition to the eventual stable fixed point. The lower panel is an embedding of the same data (colour coded to depict different dynamical regimes).

complex irregular interaction. The existence of a stable node (to which the system eventually converges) is interesting but probably as likely to be a numerical artifact (the time scale required for this feature to emerge exceeds the available data.

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