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Nonlinear Dynamics and Orbital Instabilities of a Magnetic Resonance Force Microscope Operating in Ultra-High Vacuum

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Abstract– The focus of this paper is on the nonlinear dynamics of a micro-cantilever resonator model proposed for measurement of electron spin via magnetic resonance force microscopy. The resonator model, augmented by the Bloch equations for the magnetization, is analyzed numerically and compared to asymptotic results derived for a low order asymmetric adiabatic limit. Orbital instabilities include coexisting solutions and lengthy chaotic transients that occur below a homoclinic jump-to-contact threshold. A multiple-scales analysis of the limiting adiabatic model enables estimation of the threshold for bi-stable solutions, and prediction of the frequency shift that enables spin detection. A numerical investigation of the dynamical system reveals a global stability threshold beyond which solutions jump-to-contact with the sample. Below the threshold system response is primarily periodic with the exception of distinct solutions that exhibit lengthy nonstationary transients.

1. Introduction

Magnetic resonance force microscopy (MRFM) is an imaging technique that enables acquisition of magnetic resonance images at nanometer scales [1], and has enabled detection of the magnetic spin of a single electron [2]. MRFM employs a vibrating micro-cantilever to directly detect a modulated spin gradient force between the sample spins and a ferromagnetic particle attached to the tip of the cantilever [3]. The motion of the cantilever is detected by an interferometer via a detuning or change in its natural frequency. One of the methods used to measure the frequency shift of the cantilever is entitled 'oscillating cantilever driven adiabatic reversals' (OSCAR), which is implemented by applying an external force to the cantilever, which itself causes cyclic adiabatic inversions to the paramagnetic cluster.

The nonlinear dynamics of the paramagnetic moment in MRFM has been studied using the Bloch equations [4,5] which have revealed both periodic and chaotic like solutions in the presence of a constant radio frequency and time dependent relaxations processes [6]. An estimate of the resonant frequency shift using the OSCAR technique was derived by Berman et al. [7,8], who employed the method of averaging to a symmetric low order resonator equation. In their analysis Berman et al. ignored an imposed bias in the complete equations of motion that is typical of similar dynamical systems derived for Atomic Force Microscopy. Thus, the purpose of this paper is to determine an alternative estimate for the frequency shift incorporating the complete asymmetric configuration via the asymptotic multiple scales method, and to determine the existence of orbital instabilities.

2. Problem Formulation

We consider an initial-boundary value problem (IBVP) for a cantilever with a magnetic tip that is vibrating about a magnetic sample (Figure 1). We employ a Galerkin ansatz and reduce the IBVP to a modal dynamical system near its primary resonance.

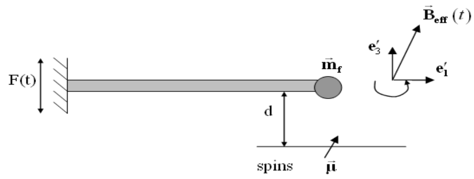

Figure 1 Definition sketch

The nondimensional equations of motion for the reduced dynamical system are:

$$Z'' + \beta Z' + Z + \frac{\Gamma M_z}{(1+Z)^4} = \alpha \Omega^2 \cos(\Omega \tau + \psi_0)$$

$$M'_x = \left[\delta - \frac{\chi}{3}\left(1 - \frac{1}{(1+Z)^3}\right)\right] M_y$$

$$M'_y = \omega_M M_z - \left[\delta - \frac{\chi}{3}\left(1 - \frac{1}{(1+Z)^3}\right)\right] M_x$$

$$M'_z = -\omega_M M_y$$

Where system parameters are:

$$\beta = \frac{1}{Q}, \Gamma = \frac{3\mu_0 m_f \mu}{2\pi m^* \omega_c^2 d^5}, \alpha = \frac{f_0}{d\omega_c^2}, \Omega = \frac{\nu}{\omega_c}, \delta = \frac{\gamma \Delta B}{\omega_c}, \omega_M = \frac{\gamma B_1}{\omega_c}, \chi = \frac{3\gamma \mu_0 m_f}{2\pi d^3 \omega_c}$$

We also consider the adiabatic limit of the magnetic resonance dynamical system to yield:

$$Z'' + \beta Z' + Z \pm \frac{\Gamma}{(1+Z)^4 \left(1 + \left[\omega_M / f(Z)\right]^2\right)^{0.5}} = \alpha \cos(\Omega \tau + \psi_0)$$

where

$$f(Z) \equiv \delta - \frac{\chi}{3}\left(1 - \frac{1}{(1+Z)^3}\right)$$

3. Equilibrium Analysis

Analysis of the equilibrium of the dynamical system reveals three distinct regions of bifurcation (Figure 2). The first region includes a stable trivial equilibrium and an unstable saddle. The transition between the first and the second regions consists of a pitchfork bifurcation that is defined by an unstable trivial solution confined by two adjacent stable solutions. The lower of the stable solutions ends with a saddle-node bifurcation at the transition between the second and third regions. The third region includes an unstable trivial solution and an upper stable solution.

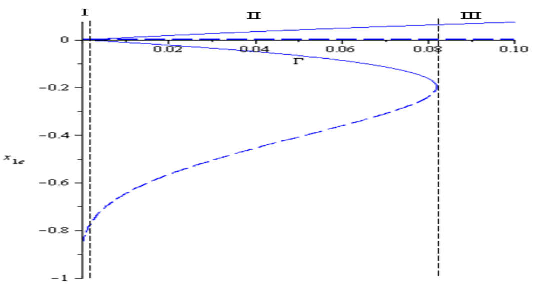

Figure 2 Bifurcation diagram

4. Asymptotic Analysis

We expand the nonlinear restoring force of the adiabatic limit to cubic order

$$Z'' + \beta Z' + \bar{\omega}_1^2 Z + \alpha_2 Z^2 + \alpha_3 Z^3 = \alpha \cos\left(\Omega\tau + \psi_0\right)$$

where

$$\bar{\omega}_1^2 = 1 \mp \frac{\Gamma\chi}{\omega_M},\ \alpha_2 = \pm\frac{6\Gamma\chi}{\omega_M},\ \alpha_3 = \pm\frac{\Gamma\chi}{\omega_M}\left(\frac{64}{3} - \frac{1}{2}\left(\frac{\chi}{\omega_M}\right)^2\right)$$

And employ an asymptotic multiple scales analysis [9]. The analysis yields the following evolution equation for slow time:

$$-2i\bar{\omega}_1\left(D_2 A\right) - i\bar{\omega}_1\bar{\beta}A + \frac{10\alpha_2^2}{3\bar{\omega}_1^2}A^2\bar{A} - 3\alpha_3 A^2\bar{A} + \frac{\bar{\alpha}}{2}e^{i(\sigma T_2 + \psi_0)} = 0$$

Investigation of the steady response of the slowly-varying evolution equations yields a typical softening frequency response (Figure 3) which depicts a region of bi-stable solutions.

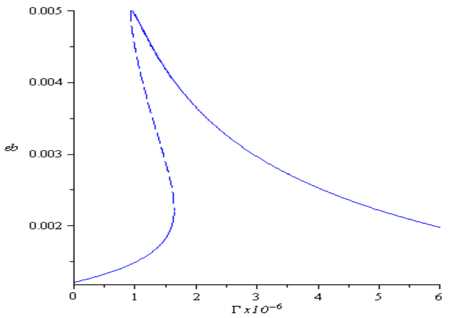

Figure 3 Frequency response

5. Numerical Analysis

Numerical integration of the dynamical system yields a stability map for an allowable parameter space above which solutions 'jump-to-contact' with the magnetic sample (located at z=-1).

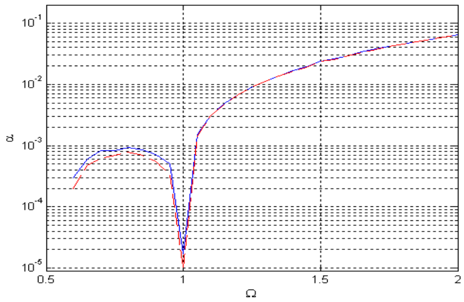

Figure 4 Explosion stability map

Small amplitude solutions below the jump-to-contact threshold are periodic (Figure 5) where the sum of the squares of the magnetic moments is unity [3].

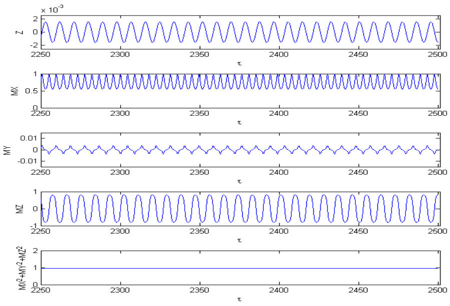
Figure 5 Periodic solution: time-series

The power spectra of the small amplitude dynamics (Figure 6) reveals multiple even and odd harmonics of the magnetic moments typical of dynamical systems with combined even and odd nonlinearities.

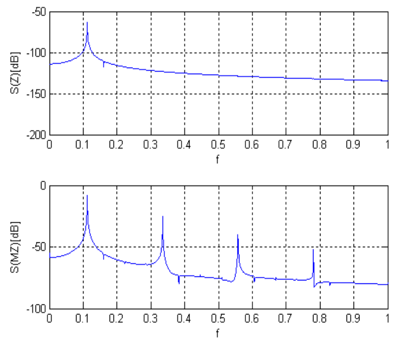

Figure 6 Periodic solution: power spectra

In specific regions of parameter space below the jump-to-contact threshold, finite-amplitude solutions can exhibit lengthy non-stationary transients (Figure 7).

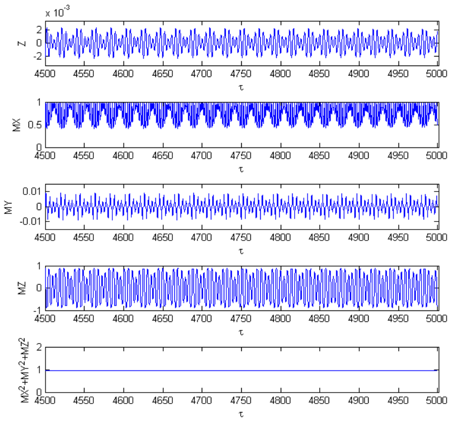

Figure 7 Non-stationary transients: time-series

The power spectra of the non-stationary transients consist of multiple peaks (Figure 8) which define the frequency content of both the mechanical resonator and the magnetic moment. Note that the latter is dense and includes very small frequencies corresponding to the slowly varying envelope of modulated time-series (Figure 7).

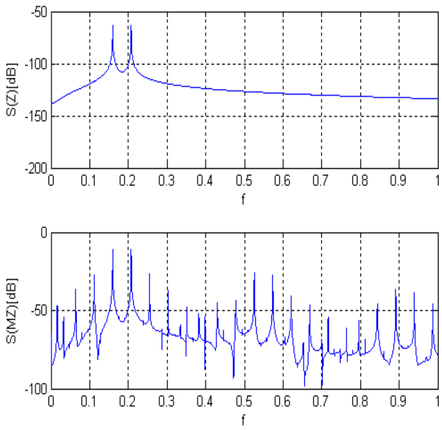

Figure 8 Non-stationary transients: power spectra

Acknowledgments

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