The Imaging Approach of Sparse Interferometry to Microwave Radiation

Yuanyuan Liu Suhua Chen Lu Zhu
School of Information Engineering East China Jiaotong University, NanChang, 330013 China

***Abstract*—High efficiency image reconstruction and inversion algorithm is one of the key technologies for interference synthetic aperture microwave radiometer. Due to the fact that the brightness temperature of the Earth has a local smoothness characteristic, it could be random sparse interferometry. Based on compressive sensing, this paper proposes a novel imaging approach of sparse interferometry to microwave radiation. According to the sparity of the image and the characteristic of the interferometry, we set up the microwave radiation sparse interferometric imaging model using Total Variation constraints on the basis of the traditional microwave radiation imaging. In the model, we use a new sparse interferometry to sample frequency information on the basis of the sparse antenna array. During the process of microwave radiation inversion imaging, we use the steepest descent method and the alternate iteration method reconstruct. Experimental results show that the proposed approach is able to rapid, accurate and efficient inverse microwave radiation image.**

Index Terms*—interference synthetic aperture microwave radiometer, compressive sensing, random sparse interference method, TV reconstruction algorithm, steepest descent method, Alternating Direction Algorithm, Total Variation, image inversion

I. Introduction

Interference synthetic aperture microwave radiometer (ISAMR) was suggested in the 1980s as an alternative to real aperture radiometry for Earth observation at low microwave frequencies with high spatial resolution [6]. ISAMR integrated small antenna array into a large observation aperture, and imaging without mechanical scan, so it can solves the disadvantages of real aperture microwave radiometer [12]. How to efficient and accurate inverse the synthetic aperture microwave radiation image is the key. According to the Fourier transform relationship between inversion bright temperature of ISAMR and visibility function, He Yuntao [2] and other scholars put forward various inversion algorithm based on Fourier transform, but this algorithm is very high requirement for the hardware system and the imaging error is larger. The present synthetic aperture image inversion methods are mostly deterministic inversion method [7], so it is rarely make full use of the local smoothness characteristics of brightness temperature image. For then now L-Band satellite-based ISAMR, it still needs a diameter of 9m antenna array to achieve 50 km spatial resolution [3], with the microwave radiation imaging to refinement and structured development, we must increase the diameter of the antenna array to satisfy the need of high resolution. ISAMR collect tens of millions data during one observation [8]. So the Nyquist spatial interferometry and conventional microwave radiation imaging method are difficult to achieve.

Compressed Sensing (CS) has become the research hot spots focus in various fields [13], which use the adaptive linear projection to preserve the original structure of the signal, and accurately reconstruct the original signal through numerical optimization [10]. In this method, it is also an increasing dimension problem from measured values to the original signal, which is similar to super-resolution image reconstruction.

Therefore, to against the problem of huge data and less resolution of the traditional microwave radiation interference measuring method, by fully exploiting the spatial structure information of the microwave radiation imaging, from the aspects of CS, using the sparse random interference measuring method of microwave radiation imaging to study [9], this paper put forward a imaging approach of sparse interferometry to microwave radiation, reduce the complexity of the imaging system structure and hardware cost, and break through the inherent limits of the imaging system's spatial resolution to get the high spatial resolution image, which is close to the expensive big aperture imaging system. After several simulation experiments, the results show that the approach can get good image inversion result.

II. Principle theory

A. Principles of CS

The area of CS was initiated in 2006 by two ground breaking papers by Candès, Romberg, and Tao [5], and by Donoho [4]. CS theory mainly includes signal sparse representation, encoding measurement and reconstruction algorithm. If the signal has only a few nonzero elements, then the signal is sparse, so it can project onto an orthogonal transformation matrix, use far lower sampling rate than the traditional Nyquist sampling to sample and compress, and apply the prior information of the original

The work was supported by the Natural Science Foundation of Jiangxi Province (No. 2010GQS0033) and National Nature Science Foundation of china (No. 61162015, No.31101081).
Corresponding author: Lu Zhu (e-mail: luyuanwanwan@163.com)

signal to accurately reconstruct it. The mathematical model of compressed sensing is shown as follows:

The N dimensional real signal $X \in R^{N\times 1}$ is being unfold under a set of orthogonal basis $\{\Psi_i\}_{i=1}^{N}$ Ψ_i (is N dimensional Column vector):

$$\mathrm{x} = \sum_{i=1}^{N} \theta_i \Psi_i \tag{1}$$

The matrix form is:

$$x = \Psi\theta \tag{2}$$

$\psi = [\Psi_1, \Psi_2, ..., \Psi_N] \in R^{N\times N}$ is the orthogonal basis, and satisfies $\Psi\Psi^T = \Psi^T\Psi = I$. $\theta = [\theta_1, \theta_2, ...,\theta_N]^T$ is expansion coefficient vector. Assuming that the coefficient is K-sparse vector, and the number of nonzero coefficient is K, K<<N， so we can use another observation matrix $\Phi \in R^{M\times N}$ (M<<N) which is not relevant with the orthogonal basis dictionary to observe

$$y = \Phi x \tag{3}$$

M linear observations (or projections) $y \in R^M$ can be get. These small amount of linear projection contains enough information to reconstruct signal x. The observations y can be reconstructed from signal x When $\Theta = \Phi\Psi$ satisfies the constraint of restricted isometry property (RIP) [1]. The reconstruction can be expressed by solving the following optimization problem

$$\min_x \left\|\Psi^T x\right\|_0 \quad s.t \quad y = \Phi x \tag{4}$$

But the optimization problem (4) is l_0 norm, and it’s an NP - Hard problem. In order to solve the problem, we usually use the l_1 norm instead of l_0 norm, namely:

$$\min_x \left\|\Psi^T x\right\|_1 \quad s.t \quad y = \Phi x \tag{5}$$

B. ISAMR sparse interferometry

ISAMR has the high perceptive ability of the earth observation at low microwave frequencies with high resolution [11]. It measures the complex correlation between the signals collected by pairs of spatially separated antennas which have overlapping fields of view, yielding samples of the visibility function V (also termed complex visibilities) of the brightness temperature distribution T of the observed scene. The relationship between $V(u)$ and $T(\xi)$ is given by [8]:

$$V(u_{kl}) \propto \frac{1}{\sqrt{\Omega_k\Omega_l}} \iint F_k(\xi)\overline{F_l}(\xi)T(\xi) \times \tilde{r}_{kl}\left(\frac{-u_{kl}\xi}{f_0}\right) e^{-2j\pi u_{kl}\xi} \frac{d\xi}{\sqrt{1-\|\xi\|^2}} \tag{6}$$

where u_{kl} is the spatial frequency associated with the two antennas A_k and A_l (namely, the spacing between the antennas normalized to the central wavelength of observation), the angular position variable ξ is direction cosine(θ and ϕ are the traditional spherical coordinates, $F_k(\xi)$ and $F_l(\xi)$ are the normalized voltage patterns of the two antennas and with equivalent solid angles Ω_k and Ω_l , $\tilde{r}_{kl}$ is the so-called fringe-wash function, which accounts for spatial decorrelation effects, $u_{kl}\xi / f_0$ is the spatial delay, and f_0 is the central frequency of observation. When the nonzero base line number of ISAMR is M, M equations as shown in (6) can be expressed by matrix:

$$V = GT \tag{7}$$

Where V is the column vector of the visibility function sampling, T is the discrete brightness distribution，and G is model operator matrix. We know that the relationship between V and T is actually Fourier transform, so we begin to disperse for the space frequency on the basis of (7), and then randomly selected spatial Fourier frequency component, the microwave radiation sparse random interference measurement model is as follows：

$$V = F_{\wedge} T \tag{8}$$

$F_{\wedge}$ is the sparse observation matrix. When T is sparse, we use $F_{\wedge}$ to interferometry for T on the basis of sparse antenna. By doing this, we can get the useful information which is far less than the amount of microwave radiation image data. While the traditional interferometry imaging method based on Nyquist sampling theory, so the amount of sampling information is relatively big. Fig.1 is the principle of traditional interferometry imaging method, in this method, the antenna array are arranged to get the visibility function V, which must be consistent in Nyquist theory. We, then get the brightness temperature V through the linear reconstruction to the visibility function. Fig. 2 is the principle of new random sparse interferometry. Firstly, we get the microwave radiation V from very sparse antenna arrays just as Fig. 1, similarly using the random sparse observation matrix $F_{\wedge}$ to sparsely sample the spatial frequency. Finally we can use the nonlinear reconstruction algorithm to inversion the brightness temperature.

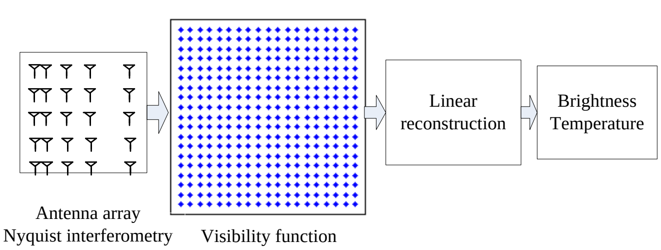


Fig.1.Traditional interferometry imaging method

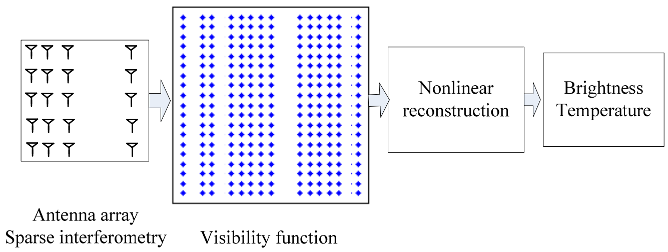


Fig.2. Random sparse interferometry image method

C. The reconstruction algorithm

According to the area smoothness of microwave radiation image, with the thought of CS, we use Total variation (TV) to sparse representation the image, so as to save all the information. The definition of TV is shown as follows:

$$TV(T) = \sum_{i,j} \sqrt{\left(T_{i,j} - T_{i-1,j}\right)^2 + \left(T_{i,j} - T_{i,j-1}\right)^2} \qquad (9)$$

So in the foundation of (2), the microwave radiation image's sparse representation model can be presented as follows:

$$V = F_{\wedge} \Psi \alpha \qquad (10)$$

While α is the useful information which is far less T, $A = F_{\wedge} \Psi$, $A \in R^{M \times N}$ is the random observation matrix, in fact, it is the part of random Fourier observation matrix. The part of random Fourier observation matrix is that to generate a random N×N Fourier matrix firstly, then to sample of the M line randomly to form a new matrix, and unitized for each column lastly. Equation (11) is a morbid linear equation from the mathematical perspective. In order to solve the problem, we build the following TV model based on the principle of CS and the characteristic of temperature image

$$\begin{cases} \min\limits_{w_i \in R^2, T_b \in R^n} \sum\limits_i \|w_i\| \\ \text{s.t. } V = AT \textit{ and } D_i T = w_i \quad \textit{for all } i \end{cases} \qquad (11)$$

While W_i is a secondary variable, $\|\cdot\|$ can be either 1-norm or 2-norm. D_i is the differential operator. How to accurately resolve the formula (11) is the key. The (11) is a convex optimization problem, so we use the Lagrange principle to transfer the constrained problem into unconstrained problem as follows:

$$\min_T \ell_k(T) = \sum_i \left(-v_i^T \left(D_i T - w_i\right) + \frac{\beta}{2} \left\|D_i T - w_i\right\|^2 \right) - \lambda^T (GT - V) + \frac{\mu}{2} \|GT - V\|^2. \qquad (12)$$

While, v_i, λ are weight coefficient, and μ, β are penalty factor. In order to solve the problem, it is necessary to set appropriate value for weight coefficient v_i, λ. However the above problem is still difficult to solve, because it is non-differentiable and non-linear, therefore we utilize the separable structure of the variables and alternating direction algorithm (ADM) [14] to optimize. So the sub-problem of T and W are as follows:

$$\min_T \ell_k(T) = \sum_i \left(-v_i^T \left(D_i T - w_i\right) + \frac{\beta}{2} \left\|D_i T - w_i\right\|^2 \right) - \lambda^T (GT - V) + \frac{\mu}{2} \|GT - V\|^2. \qquad (13)$$

$$w^{t+1} = \arg\min_w \left(\sum_i w_i - v_i^T \left(D_i T - w_i\right) + \frac{\beta}{2} \left\|D_i T - w_i\right\|^2 \right) \qquad (14)$$

In order to resolve W, we firstly fix T, and we use the following 2D shrinkage-like formula [15]

$$w_i = \max\left\{ \left\| D_i T - \frac{v_i}{\beta} \right\| - \frac{1}{\beta}, 0 \right\} \frac{D_i T - v_i/\beta}{\left\| D_i T - v_i/\beta \right\|} \qquad (15)$$

For fixed w_i, the minimization ℓ_k of with respect T_b to becomes a least squares problem, its gradient is

$$d_k(T) = \left(\sum_i \beta_i D_i^T \left(D_i T - w_{i,k+1}\right) - D_i^T v_i \right) + \mu A^T (AT - V) + \frac{\mu}{2} \|AT - V\| \qquad (16)$$

Forcing $d_k(T)$ =0. Theoretically, it is ideal to accept the exact minimizer as the solution of the u-subproblem by directly resolving the (14). However, it is too costly to implement numerically. Therefore, the steepest descent method is highly desirable. The method is able to solve iteratively by applying the recurrence formula

$$T^{k+1} = T^k - \alpha_k d_k \qquad (17)$$

The initial point T_0 is $A^T y$, d_k is the gradient direction of the objective function, α_k is determined by the inexact search.

III. Simulation and Experiment Results

In this section, we present several simulations to proposed method. All simulations were performed under Windows XP and MATLAB R2009a running on a Acer laptop with an Intel Core i3 CPU at 2.4GHz and 2GB of memory. We generated our test from the image moon. The dimension of the moon is 64×64. We introduce the concept of compressed radio, which is the radio between the number of nonzero elements and the total number of elements. In our experiments, we simply set μ =2^8 and β =2^5, the radio are set 0.4, 0.5, 0.6, 0.7, 0.8. The reconstruction result of different radio is given in TABLE 1, and the following quantities are list the number of iteration (Iter), the CPU time (T) in seconds, the peak signal to noise ratio (Psnr), root mean square error (Rmse) and Bias. The quality parameters of the image have been shown in the following TABLE 1.

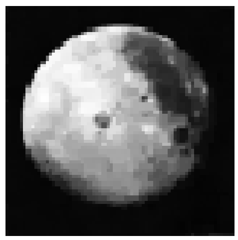
TABLE 1

PARAMETER FOR TV RECONSTRUCTION ALGORITHM

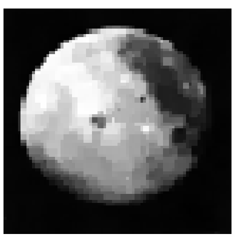
Radio	0.4	0.5	0.6	0.7	0.8
Iter	123	99	109	107	94
T	8.753259	9.759920	11.864406	13.549631	14.832024
Bias	1.0071	0.8687	0.7744	1.0471	0.8821
Rmse	2.1626	1.9728	1.8009	2.2986	1.9267
Psnr	41.4311	42.2290	43.0211	40.9016	42.4345

The inversion images of different radio have been shown as follows:

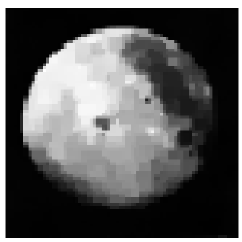
(a)Original image



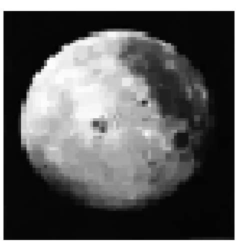
(b) Inverse image, radio=0.4



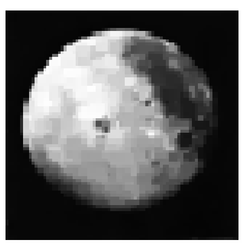
(c) Inverse image, radio=0.5



(d) Inverse image, radio=0.6



(e) Inverse image, radio=0.7



(f) Inverse image, radio=0.8

From the above form and the inversion images, we can be seen that when the ratio is greater than or equal to 0.4, the effect of the inversion image is better, and the Psnr value is bigger. With the increase of the ratio, the time that used to reconstruction becomes longer, and the value of Psnr becomes larger, so the quality of the inversion image becomes better. When the radio is 0.6, we can get the best inversion image. From the table1, we can see that the CPU time is between 8s and 15s, it is very small. So the algorithm is very fast for the image reconstruction.

IV. CONCLUSION

In this paper, we propose the imaging approach of sparse interferometry to microwave radiation, and apply the TV reconstruction algorithm to reconstruct the image. The simulation results show that the proposed method is effective. This approach is better than the traditional algorithm of using G matrix for image reconstruction and can break through the problem by traditional microwave radiation imaging method.

REFERENCES

[1] Shutao Li, Dan Wei. “A Survey on Compressive Sensing,” Acta Automatic Sinca., vol.35, no. 11, pp. 1369-1377, 2009.

[2] Qiusheng Lian, Shuzhen Chen. “Image Reconstrcution for Compressed Sensing Based on the Combined Sparse Image Representation,” Acta Automatic Sinca., vol.36, no. 3, pp. 385-391, 2010.

[3] Ulaby, Moore and Fung. “Microwave Remote Sensing,” Active and Passive., vo.1, Chapter 6. Addison-Wesley: Reading MA, 1981.

[4] D.L.Donoho. “Compressed Sensing,” IEEE Trans.Inf.Theory., vol.52, no. 4, pp. 1289-1306, 2006.

[5] E. Candès, J. Romberg, T. Tao. “Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information,” IEEE Trans. Inf. Theory., vol.52, no. 2, pp. 489-509, 2006.

[6] Yuntao He, Yuesong Jiang,Haiting Chen. “Studies of Optimation and Imaging Properties of Two-dimensional Circle Array for mm-wave Synthetic Aperture System,” JOURNAL OF REMOTE SENSING., vol.11, no. 1, pp. 33-38, 2007.

[7] Kun Chao, Houcai Chen, Zhenwei Zhao. “Image Reconstruction and Inversion Algorithm of Synthetic Aperture Radiometer,” CHINESE JOURNAL OF RADIO SCIENCE., vol.26, no. 5, pp. 881-886, 2011.

[8] Camps A, V Mercè, C Ignasi, et al. “Improved Image Reconstruction Algorithms for Aperture Synthesis Radiometers,” IEEE Trans. Geoscience and Rem. Sen., vol.46, no.1, pp. 146-158, 2008.

[9] Fangmin He, Qingxia Li, Zhihua Zhao. “A Sparse Prior Based Statistical Inversion Approach for Aperture Synthesis Radiomtric Imaging of Extened Source,” ACTA ELECTRONICA SINCA., vol.41, no.3, pp. 417-422, 2013.

[10] Jérôme Bobin, Jean-Luc Starck. “Compressed Sensing in Astronomy,” IEEE Signal Process. Mag., vol.32, no. 5, pp. 718-726, 2008.

[11] Shiyong Li, Xi Zhou. “A Compressive Sensing Approach For Synthetic Aperture Imaging Radiometers,” Progress In Electromagnetics Research., vol.135, pp. 583-599, 2013.

[12] Xi Zhou, Houjun Sun. “NUFFT-Based Iterative Reconstruction Algorithm for Synthetic Aperture Imaging Radiometers,” IEEE Remote Sensing Leters., vol.6, no.2, pp. 273-276, 2009.

[13] http://dsp.rice.edu/cs

[14] J.F. Yang, Y Zhang, and W.T. Yin. “A Fast Alternating Direction Method for TVL1-TVL2 Signal Reconstruction From Partial Fourier Data,” IEEE J. of Selected Topics in Signal Process., vol.4, no.2, pp. 288-297, 2010.

[15] http://www.caam.rice.edu/~optimization/L1/TVAL3/.