Microwave Radiation Image Reconstruction Based on Combined TV and Haar Basis

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***Abstract*—Due to the complicated structure of microwave radiometric imaging system and the massive amount of data collection in one snapshot, it is difficult to achieve good performance by interferometry based on the Nyquist sampling and conventional microwave radiation imaging method. In this paper, we use the random observation matrix to sparsely sample microwave radiation image on the basis of digging compressible information of microwave radiation image, reducing the amount of data collection. Considering that sparseness of microwave radiation image on traditional sparse basis (such as: TV and Haar wavelet sparse basis) can't meet the requirements, we combine TV and Haar wavelet sparse basis to sparsely represent microwave radiation image, adopt the method of OMP algorithm finding the optimal atom to reconstruct the original microwave radiation image. We use synthesis data to simulate the three methods of combined TV and Haar wavelet, TV, Haar wavelet. The simulation results show that the proposed method to reconstruct microwave radiation image is better than the reconstruction method of single orthogonal basis.**

***Index Terms*—Aperture Synthesis Radiometers; Combined TV and Haar wavelet; OMP; Compressed Sensing (CS).**

I. Introduction

PASSIVE microwave satellite-borne imaging has become of increasing interest for scientific, military and commercial applications over the last years. If we can get precise soil moisture data by inverting the obtained microwave radiation image, it would enhance the accuracy of weather forecasts by the analysis of soil moisture data, and would effectively monitor drought, flood and other geological disasters. Interferometric synthetic aperture microwave radiometry[1] integrates the small-bore array into large observation bore, doesn't need a mechanical scanning for imaging, and solves the disadvantages of real aperture microwave radiometers. But spaceborne ISAMR of L-Band still need diameter up to 9 m antenna array for 50 km spatial resolution[2]. And with the development of refinement and structuralization of the image, we must increase the diameter of the antenna array to meet the needs of high resolution, ISAMR has evolved into an enormous and complex system, data collected easily reaches tens of millions in one snapshot. So it is difficult to achieve good performance by interferometry based on the Nyquist theory and conventional microwave radiation imaging method. Because of complex structure and low imaging resolution, present microwave radiometric imaging system seriously limits its practical application in regional soil moisture remote sensing.

Compressed Sensing (CS) theory is a great breakthrough in the field of information processing in recent years[3] - [5], it has changed people's traditional concept of information acquisition. The core idea of CS theory is that it applies the sparse prior knowledge of signal representation to the process of signal reconstruction, uses far less than the Nyquist sampling rate to reconstruct original signal, and thus effectively reduces the complexity of the sensor and the sampling system. That is to say, assuming that interferometry on Nyquist sampling collects 100 datum to accomplish the microwave radiation imaging, for example, sparseness is 0.1, using the CS method, it only need about 40 datum to achieve the same imaging resolution. If the signal is sparse enough or compressible, the actual random collection data can be less. With the increasing amount of data acquisition, the advantages of CS method are highlighted increasingly.

In CS method, the sparse representation of the image is a precondition for image reconstruction. In general, natural signal in time domain or the image in space domain isn't sparse, but it can be sparse in Wavelet, Ridgelet, Curvelet and Contourlet transform domain, etc. However, the usage of single orthogonal basis is difficult to sparsely represent the soil microwave radiation image of complex scene. In this paper, we utilize the combined TV and Haar wavelet sparse basis to sparsely represent microwave radiation image, use the observation matrix $[I\ R]$ [6] to sparsely sample microwave radiation image, adopt the method of OMP algorithm finding the optimal atom to reconstruct the data of sparse sampling, acquire actual microwave radiation image.

The rest of this paper is organized as follows. Section II discusses the basic theory. Signal reconstruction method based on combined TV and Haar wavelet is detailed in Section III. Section IV demonstrates the performance of our method, and verifies the performance of three kinds of sparse basis. In Section V, we conclude our work.

II. The Basic Theory

A. Microwave radiation imaging method

In the Aperture Synthesis Microwave Radiation Image (ASMRI), measurement visibility function V(u) and the brightness temperature distribution of the observed scene satisfy the following relation:

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$$V(u)\alpha\iint_{\|\xi\|<1}\left[T(\xi)\Big/\sqrt{1-\|\xi\|^2}\right] \times F_k(\xi)F_l^*(\xi)r(-u\xi/f_0)e^{-j2\pi u\xi}d\xi \quad (1)$$

where $F_{k,l}$ is the normalized antenna voltage pattern, f_0 is the center frequency, r is stripe function, u is the baseline coordinate of normalized wavelength, ξ is direction cosine. Assuming that the number of zero-base line of ASMRI measurement is M, the M equations set as shown in (1) can be expressed as by matrix

$$V = G\cdot T \quad (2)$$

where V is the column vector of $(M+1)\times 1$ visibility function sampling, referred to as visibility sampling, T is the $J\times 1(J>M)$ discrete brightness temperature distribution, model operator G is the shock response of ASMRI. From the view of visibility sampling reconstruction of ASMRI measurement, the brightness temperature distribution of the observed scene is an ill-posed inverse problem [7].

B. The basic theory of CS

The essence of CS theory is a kind of non-adaptive and nonlinear compressible signal reconstruction method, its main content is that on certain basis (called sparse basis) $\Psi=[\psi_1,\psi_2,\cdots,\psi_N]$, the m - sparse description and N sampling signal $x\in R^N$, namely $x=\sum_{n=1}^{N}\alpha(n)\psi_n$, can be precisely reconstructed by the M($m\le M<<N$) linear projection $y(i)=\langle x,\phi_i^T\rangle$, $i\in\{1,2,\cdots,M\}$, on the another incoherent basis $\Phi=\left[\phi_1^T,\phi_2^T,\cdots,\phi_M^T\right]$ (referred to as the measurement basis). Using the matrix form, measurement process can be referred to as $y=\Phi x=\Phi\Psi\alpha$. Where α is the m - sparse transformation coefficient vector, M dimension column vector y is measurement vector, $M\times N$ dimension matrix Φ is measurement matrix. Its goal is accurately reconstructing or approximating signal x by the measurement vector y obtained by M (M<<N) measurements.

The condition $m\le M<<N$ shows that CS theory is mainly to solve signal reconstruction problem under the undersampling condition. Condition M<<N makes the reconstruction of the signal x essentially become a ill inverse problem, but the first reason that it is possible to recover signals from a small amount of measurement is the sparsity of original signal x itself, while the second reason is incoherence between the measurement basis with sparse basis, therefore, in order to ensure the reconstruction, measurement matrix must satisfy certain constraints. Candes, etc[4] gives the following conclusion: in order to reconstruct sparse or compressible signal, measurement matrix must meet Restricted Isometry Property (RIP) of certain parameters. Here we first present the definition of RIP.

Definition 1(RIP): For the matrix $\Phi\in R^{M\times N}$, if all meet the index set of $|I|\le m<M$, $I\subset\{1,2,\cdots,N\}$ and arbitrary vector $v\in R^{|I|}$, there is constant $0<\delta<1$ making that:

$$(1-\delta)\|v\|_{l_2}\le\|\Phi_I v\|_{l_2}\le(1+\delta)\|v\|_{l_2} \quad (3)$$

true, so calling the matrix meets the RIP of parameters (m,δ), where Φ_I is the sub-matrix composed by the column vectors of index set $I\subset\{1,2,\cdots,N\}$ referring to Φ, and the infimum of all parameters δ making the type (3) set up refers to as Restricted Isometry Constant (RIC), refers to as δ_m [8].

Let us denote measurement matrix $\Phi=[I\ R]$, the Φ matrix can be written as:

$$\Phi=\begin{pmatrix} 1 & 0 & \cdots & 0 & \phi_{1\,M+1} & \cdots & \phi_{1\,N} \\ 0 & 1 & \cdots & 0 & \phi_{2\,M+1} & \cdots & \phi_{2\,N} \\ & & \ddots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \phi_{M\,M+1} & \cdots & \phi_{M\,N} \end{pmatrix}$$

By using $[I\ R]$ as the measurement matrix, the first M sensor nodes do not have any computation load, and the rest of nodes have the same computation and communication load as in the basic random measurement matrix.

Theorem 1: Let R be a $M\times(N-M)$ matrix with elements drawn according to $N(0,1/M)$ and let I be an $M\times M$ identity matrix. If $M\ge C_1 K\log\left(\frac{N}{K}\right)$, then $[I\ R]$ satisfies the RIP of order K with probability exceeding $1-3e^{-C_2 M}$, where C_1 and C_2 are constants[6].

III. Signal Reconstruction Method Based on Combined TV and Haar Wavelet

A. Sparse representation

The precondition of CS theory is that the signal is sparse or compressible, in order to make the model simplification, only considering length N and discrete real signal x, named x(n), $n\in[1,2,\cdots,N]$. Known by the signal theory, x can be expressed with linear combination of a set of basis $\Psi=[\psi_1,\psi_2,\cdots,\psi_N]$, $x=\sum_{n=1}^{N}\alpha(n)\psi_n$, where $\alpha(n)=\langle x,\psi_n\rangle$, α and x are $N\times 1$ matrix, Ψ is a $N\times N$ matrix. When the signal x on certain basis Ψ has only K<<N nonzero coefficients $\alpha(n)$, named Ψ as the sparse matrix of signal x[9].

The image is not sparse in time domain, so we use TV, Haar wavelet basis and combined TV and Haar wavelet to transform the image to sparse domain, observe three sparse transformation results and compare the sparsity between them.

Using these three sparse basis to analyze the sparse prior knowledge of the image, using threshold processing method, the sparse ratio of TV transformation is 0.206, the sparse ratio of the third-order Haar wavelet transformation is 0.282, the sparse ratio of TV + third-order Haar wavelet transformation is

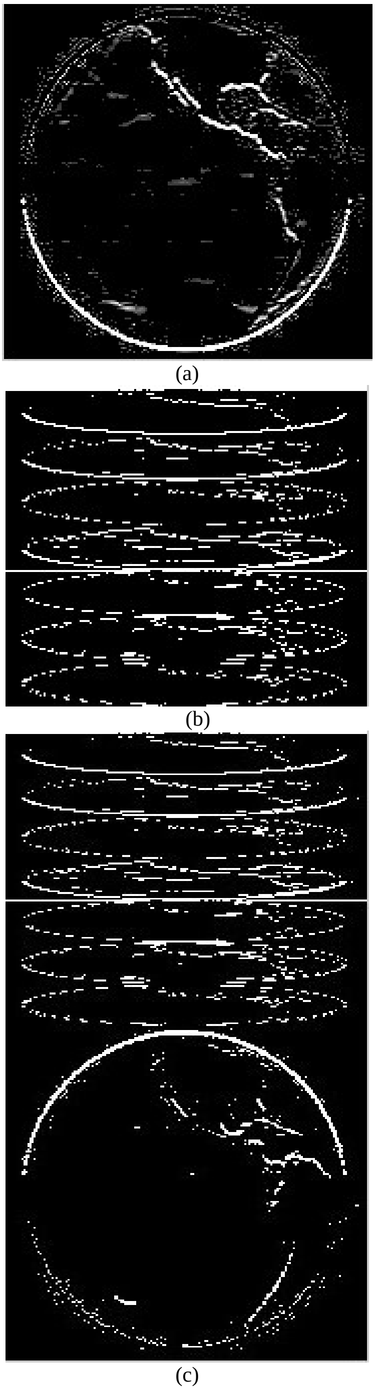


Fig. 1. The transformation of image using sparse basis: (a) The transformation of the image using TV, (b) The transformation of the image using third-order Haar wavelet, (c) The transformation of image using TV + third-order Haar

0.178, we can conclude that sparse property of Combined TV and Haar wavelet sparse basis is the best.

B. Image reconstruction method

For a signal $x \in R^{N\times 1}$, via the M linear measurement, $f = \Phi x$, $\Phi \in R^{M\times N}$, here each row of Φ can be regarded as a sensor, it acquires a part of the signal information by multiplying the signal. Analyzing the sparse prior knowledge of the signal, namely $\theta = \Psi x$, $\Psi \in R^{N\times N}$, we obtain a strict mathematical optimization problem[10]:

$$\min_x \|\theta\|_0 \quad Subject\ to \quad f = \Phi x \tag{4}$$

Extracting the image blocks, regarding each column of the image as a vector, namely an image block x_i, we can get an optimization problem:

$$\min_x \sum_i \|\theta - \Psi x_i\|_2^2 + \lambda \|f - \Phi x_i\|_2^2 \qquad s.t. \quad \|\theta\|_0 \le s \quad \forall i. \tag{5}$$

This problem is a NP hard problem, but it can be solved by greedy algorithm. The current epidemic greedy algorithm is OMP algorithm. We analyze sparsity of the image using TV, Haar wavelet basis and combined TV and Haar wavelet basis, finding the sparse coefficient and its position of the sparse representation of the observation vector, so as to reconstruct the image, and then combine all the blocks together, gaining the reconstructed image.

Algorithm steps are organized as follows:

Input: *measurement matrix* Φ , *sparse matrix* Ψ , *measurement vector* f *, the number of iterations K;*

Output: *reconstructed original signal* x_i *;*

Initialization: *residual* $r_0 = f$ *, Index set* $\Lambda_0 = \varnothing$ *, t = 1, restoration matrix* $\mathrm{T} = \Phi\Psi^{-1}$*,* $\hat{x} = 0$ *;*

Iterative steps 1-6*:*

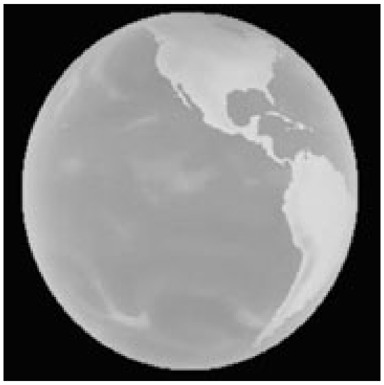
- *Step 1: finding the foot mark of the maximum product between residual r with column* T_q *of restoration matrix, namely* $\lambda_i = \arg\max_{j=1\ldots N} \left|\langle r, T_j \rangle\right|$ *;*
- *Step 2: updating the index set* $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_i\}$*, recording the atom set* $W_t = \left[W_{t-1}, T_{\lambda_i}\right]$ *found in the restoration matrix;*
- *Step 3: getting* $\hat{x}_t = \arg\min \|f - W_t \hat{x}\|_2^2$ *through the least-square method;*
- *Step 4: updating the residual* $r_t = f - W_t \hat{x}_t$*,* $t = t+1$ *;*
- *Step 5: judging whether t>K, if met, it will stop, if not satisfy, then perform step 1;*
- *Step 6: the reconstruction of the original signal* $x_i = \Psi^{-1}\hat{x}_t$ *.*

Finally we gain the original image x via the integration of the reconstructed image block x_i.

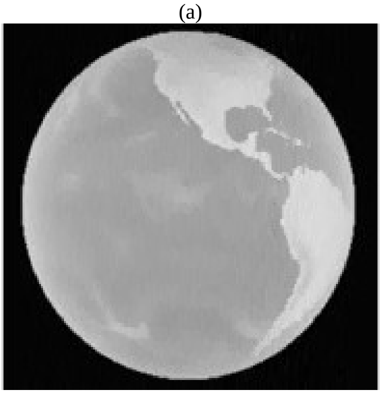
IV. Experiment

In this section, we conduct simulating experiments of three kinds of sparse basis, comparing the performance of three kinds of sparse basis, finally concluding that what kind of sparse basis is better to reconstruct the signal. We use the microwave radiation image, its size is 180×180, firstly observe signal by observation matrix $[I\ R]$, and the size of the observation matrix is 140×180, then analyze the sparse prior knowledge of the image, based on the optimization problem. We use the OMP reconstruct the sparse image by combined TV and Haar wavelet. In simulation, the directed OMP algorithm can't find the atoms of TV basis in the process of finding the atoms, so we use the alternate iterative method to find atoms. Finally, we can find the atoms to minimize residual, accelerating the convergence speed and reducing the number of iterations.

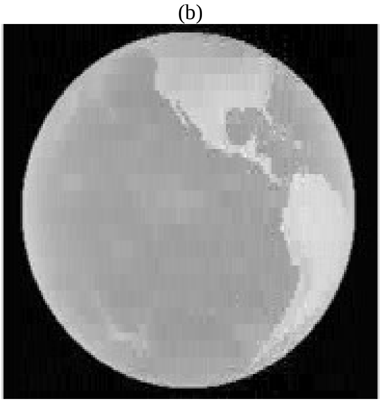
By comparing the performance of the three kinds of sparse basis, under the same sampling points, the PSNR of sparse basis for combined TV and Haar wavelet is highest, but also its running time is longest; sparse basis for TV is the center, the PSNR is high, it also has a shorter running time; the PSNR of sparse basis for Haar is minimum, and running time is slightly longer than TV. So we pay attention to both quality and speed, and improve reconstruction algorithm to achieve better results.



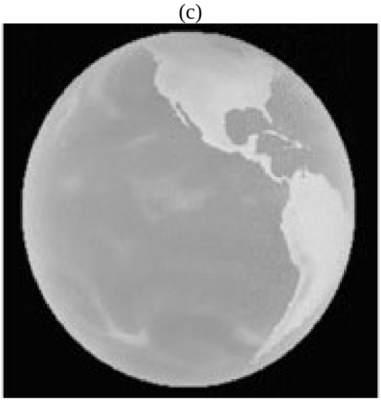
(a)



(b)



(c)



(d)

Fig. 2. Performance of method: (a) The original image, (b) The reconstructed image using TV, (c) The reconstructed image using third-order Haar wavelet, (d) The reconstructed image using TV + third-order Haar wavelet

TABLE 1 The comparison of three sparse basis performance

Sparse basis	MSE	PSNR	M	Running time
TV	5.3473	40.8494	140	9.8594
HAAR	22.4322	34.6221	140	12.4219
TV+HAAR	1.7334	45.7419	140	118.9688

V. CONCLUSION

In this paper, we propose and analyze the microwave radiation image reconstruction based on combined TV and Haar Basis, when the sampling is below Nyquist theory. Via simulating experiments, we can know that the the proposed method better than signal TV, Haar wavelet sparse basis. We will pay attention to both quality and speed, and improve reconstruction algorithm to achieve better results.

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