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Abstract—Synchronization is common phenomena in various systems, such as mechanical systems, biological systems, electrical systems and so on. These synchronizations are typically caused by mutual couplings between oscillators. In contrast, synchronization by common white noise, which is different from the synchronization by mutual couplings, has been observed in neurons and electrical circuits. In this paper, we investigated synchronization by common colored noise using electric circuits. We used square-wave oscillators by operational amplifiers and $1/f$ noise as colored noise. In the experiments, we applied the $1/f$ noise to two square-wave oscillators, measured outputs voltage from the oscillators, and calculated a phase difference between the oscillators. As a result, we confirmed that synchronizations by common colored noise of $1/f$ noise are observed from the electric circuits. The results indicate that the theory of synchronization by common colored noise can also be applied to real systems, such as electric circuits.

1. Introduction

Synchronization is common phenomena that have often been observed in various systems[1]. For instance, in mechanical systems, metronomes synchronize[2]. In Ref.[3], we can see ten metronomes easily synchronize. In biological systems, calling behaviors frogs synchronize [4]. In Ref.[5], synchronization on a frastrated system of three frogs is observed and its mathematical model of the calling behavior has been proposed. Other typical examples from biological systems is synchronized flashes of fireflies[6]. In Ref.[6], for example, synchronization of firefly flash is reported.

Although these examples come from different fields, they are common in the sense that basic elements–metronomes, frogs and fireflies–have a constant rhythm. They are often called oscillators. Then, these synchronizations are typically caused by mutual couplings between the oscillators.

In contrast, synchronization by common noise, which is different from the synchronization caused by mutual couplings, has been observed in neurons and electrical circuits. Mainen and Sejnowski showed that reliability of spike generation improves when a neuron of rat neocortical slices receives the noise current compared with the case of the constant current[7]. In addition, Neimann and Russell experimentally showed that two uncoupled neurons of paddlefish show stochastically synchronized firing patterns by common noise current[8]. These results imply that common noise induces stochastic synchronization without mutual couplings.

In 2004, Teramae and Tanaka analytically investigated the synchronization of limit-cycle oscillators induced by common white noise[9]. They showed that the phase difference between oscillators exponentially attenuated by the common white noise. In addition, they also showed that analytically obtained Lyapunov exponents predict those obtained by numerical experiments very well. Using electrical circuit implementation, common noise-induced synchronization is also observed. Yoshida, Sato and Sugamata observed common noise-induced synchronization by white noise using multivibrators[10]. Arai and Nakao also observed common noise-induced synchronization by random pulses using LED flashing circuits[11].

On the other hand, Teramae and Tanaka analytically investigated synchronization of limit-cycle oscillators induced by common colored noise with various spectrum structures[12]. Kurebayashi, Fujiwara and Ikeguchi an-
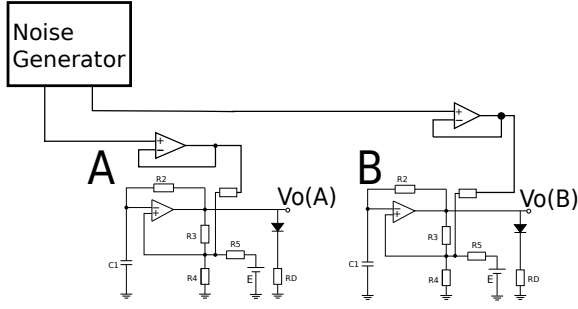
Figure 1: Circuit diagram of the whole system.

Figure 2: A square wave oscillator used in the experiment.

alytically investigated common colored noise induced synchronization[13]. They derived frequency distribution of the phase difference between oscillators[13]. As an example of research by electric circuits that implements common colored noise induced synchronization, Nagai and Nakao observed common colored noise induced synchronization using electric circuits[14]. They used random telegraph noise as an input to the electric circuits.

In this paper, we investigated the common colored noise induced synchronization using electrical circuits. To analyze the common colored noise induced synchronization in a more universal way, we analyzed common colored noise induced synchronization in electric circuits. In this paper, we used $1/f$ noise as a typical example of colored noise. We used two square-wave oscillators by operational amplifier and one $1/f$ noise generator. Then, we applied the colored noise of $1/f$ spectral structure to two uncoupled square-wave oscillators and investigated the distribution of the phase difference between these two oscillators.

2. Experiments

2.1. Setup

The experiments were performed using the electric circuit shown in Fig.1. We used two oscillators and one $1/f$ noise generator.

2.1.1. Oscillator

In the experiments, we used two square-wave oscillators by operational amplifier. In Fig.2, LM358N was used as the operational amplifier that is capable of operating in a single power supply. The circuit parameters were set as follows: $C_1 = 1[\mu F]$, $R_2 = 1[M\Omega]$, $R_3 = R_4 = R_5 = 100[k\Omega]$, R_D = 330$[\Omega]$, and $E = 9.0[V]$. We used LED to confirm synchronization (Fig.3). Voltages of positive power supplies to the operational amplifier was fixed at +9.0[V]. This oscillator generates square-waves by comparing the difference between the positive input voltage V_+ and the negative input voltage V_- of the operational amplifier. Then, the out-
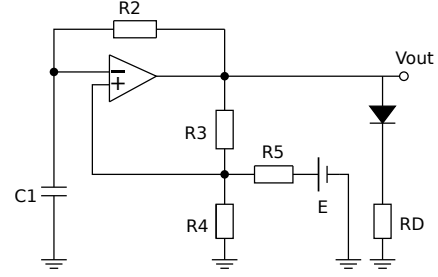
put voltage V_{out} is described as follows:

$$V_{out} = \begin{cases} V_{max} & (V_+ > V_-), \\ 0 & (V_+ \leq V_-). \end{cases} \tag{1}$$

Also the negative voltage V_- is described by the following differential equations:

$$\begin{cases} R_2 C_1 \frac{dV_-}{dt} = -V_- + E & (V_+ > V_-), \\ R_2 C_1 \frac{dV_-}{dt} = -V_- & (V_+ \leq V_-). \end{cases} \tag{2}$$

Figure 3 shows the real electric circuit that produces the square-wave.

2.1.2. $1/f$ noise

To realize common colored noise, we used $1/f$ noise. Then, we designed a circuit that generates $1/f$ noise shown in Fig.4. In the $1/f$ noise generator, we used MOSFET 2SK241GR. This MOSFET generates weak $1/f$ noise. Thus, we amplified it by the operational amplifier LF356. The circuit parameters are set as follows: $R_1 = R_3 = 1[k\Omega]$, $R_2 = 10[k\Omega]$, $R_4 = 100[k\Omega]$, $C_1 = 2.2[\mu F]$, $C_2 = C_3 = 0.1[\mu F]$.

To check whether the circuit shown in Fig.5 really produces the $1/f$ noise, we measured the output voltage V_o
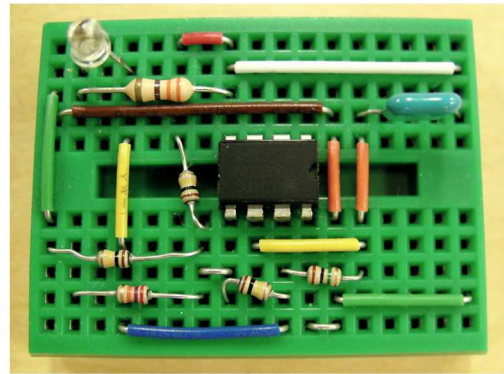

Figure 3: An electric circuit of the square-wave oscillator used in the experiments.
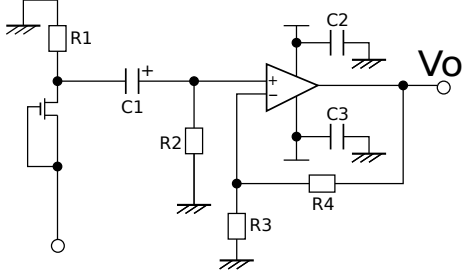
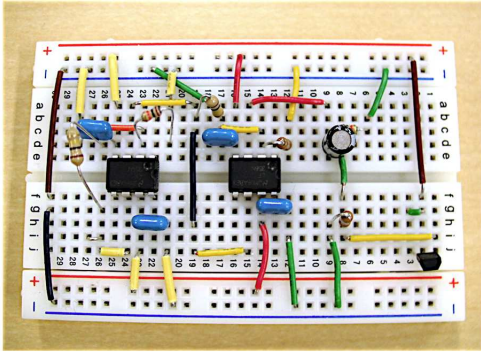Figure 4: Circuit diagram of a $1/f$ noise generator.

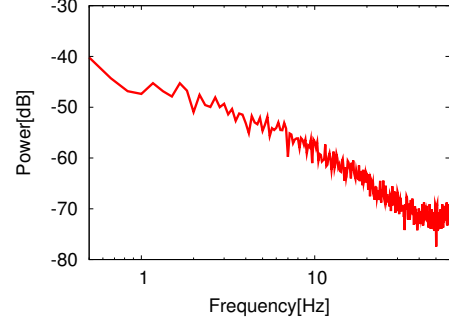Figure 5: The $1/f$ noise generator used in the experiments.

Figure 6: The power spectrum of outputs from the colored noise generator of $1/f$ type.
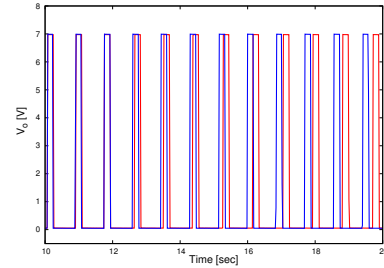

Figure 7: Superimposed outputs of the two square wave oscillators A and B. Red line shows an output square wave of the oscillator A, and blue line is that of the oscillator B.

and evaluated its power spectrum. Its power spectrum has a negative logarithmic slope. We measured V_o by the data logger with sampling frequency 100[Hz] and 16-bit quantization. Figure 6 shows the power spectrum of the output voltage V_o from the $1/f$ noise generator. We confirmed that its outputs surely have a $1/f$ characteristic by Fig.6.

2.2. Method

Figure 1 shows the circuit diagram of the whole system. It consists of one noise generator and two oscillators. The left oscillator in Fig.1 is referred to as the oscillator A, and the right one as the oscillator B. In order to disconnect these two oscillators, the voltage follower was inserted between the oscillators and the noise generator. After turning on the oscillators, we applied the $1/f$ noise from the noise generator, and measured voltages V_o(A) and V_o(B). We measured these voltage values by the data logger with the sampling frequency 50[Hz] and 16-bit quantization.

In the experiment, we applied the $1/f$ noise to the positive input voltage V_+ of operational amplifier. Usually V_+ is constant if no inputs are applied. Then, the period of the output voltage from the oscillator is constant. However, the period of the square-wave oscillator is changed by applying the $1/f$ noise to V_+.

We also defined the phase difference between the output voltages of the two oscillators to calculate synchronization quantitatively. We defined t_{A_n} and t_{B_n} as the time when the

outputs voltage of A and B exceed their average the outputs voltage of the nth time. Then we defined the phase difference θ_n as follows:

$$\theta_n = 2\pi\left(\frac{t_{B_n} - t_{A_{n-1}}}{t_{A_n} - t_{A_{n-1}}}\right). \tag{3}$$

To normalize the range of the phase difference to $(-\pi, \pi)$, we converted Eq.(3) by $\phi_n = \theta_n \pmod{2\pi}$. Then, we calculated the frequency distribution of the phase difference ϕ_n, and examined if the two oscillators synchronize by common colored noise whose spectral structure is $1/f$ type.

3. Results

Figure 7 shows the superimposed outputs of square-wave oscillators A and B. Figure 8 shows the phase difference ϕ_n between two oscillators. The highest frequency of the phase difference ϕ_n is 0 in Fig.8. However, the peak phase difference ϕ_n is not perfectly 0, and non-zero phase difference exists. Focusing on the distribution of the non-zero phase difference, it has a uniform distribution because no particular peak exists. If the phase differences approaches 0, two oscillators are phase-locked by $1/f$ noise.

In Ref.[13], when applying common colored noise to the oscillators, a kind of clustering possibly appears. However, in our experiments, we could not observe the clustering.

This reason is that the clustering does not occur if the phase reduction cannot be applied to the oscillators[13]. In the experiment, the shape of limit cycles constantly changes if the 1/f noise is applied to square-wave oscillators. Then, in our case the phase reduction cannot be applied.

We considered that the common $1/f$ noise induced period modulation of oscillators. Then the probability of the phase difference ϕ_n has a highest peak on 0. Thus, we observed the two oscillators synchronized stochastically by the common colored noise of $1/f$ type.
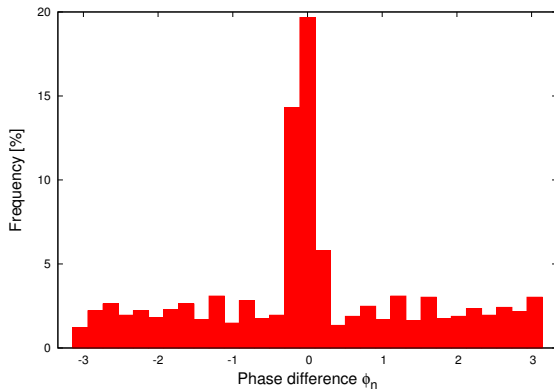

Figure 8: Frequency distribution of phase difference ϕ_n between two oscillators when common $1/f$ noise is applied.

4. Conclusions

In this paper, we investigated common colored noise-induced synchronization in electric circuits. We used square-wave oscillators and $1/f$ noise generator to observe noise-induced synchronization in electric circuits.

We implemented the electric circuits that have two square-wave oscillators as shown in Fig.2. Then we experimented common colored noise induced synchronization using the square-wave oscillator and the common colored noise generator. From the experiment, we observed that $1/f$ noise induced synchronization. We found that the probability of phase difference ϕ_n between two oscillators has a highest peak on 0. However the phase difference ϕ_n is not perfectly zero.

From these results, we concluded that $1/f$ noise induced stochastic synchronization. Therefore, common colored noise induces stochastic synchronization, and the theoretical analysis of Ref.[13] can be widely applied.

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