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Manipulation of Fluxoid by Electromagnetic Perturbation

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Abstract—The Time-Dependent Ginzburg-Landau (TDGL) equation is known as a nonlinear equation that enables us to reproduce variously-scaled physical phenomena represented by the Ginzburg-Landau equation. In this paper, we numerically discuss the dynamics of fluxoids in type-II superconductors based on the TDGL equation. In particular, we discuss the dynamics of trapping of a fluxoid at a pinning point and the release by an external perturbation by electromagnetic wave. The manipulation of fluxoid by the perturbation gives us an approach to quantum systems by classical dynamics.

1. Introduction

Type-II superconductors show the mixing state of flux penetrated into bulk/film. The flux is spatially quantized in the superconductor [1]. The quantized flux is called fluxoid. Fluxoids are pinned at low potential wells caused by impurities in the superconductor. The normal conduction particles, clacks, and so on become the impurities of crystal. The pin is surrounded by superconducting current, which traps a fluxoid in it. The gradient of potential around the pin becomes negative. That is, the fluxoid is attracted. In addition, the force strongly depends on the magnetic characteristics of superconductor, so that the hysteric characteristics appear in the pinning force and magnetic field relationship. The hysteric characteristics produce the possibility of high potential magnets which can be applied in magnetic levitation systems [2]. The application of bulk superconductor is clear but the physics of the fluxoid is not the level of analysis in nonlinear dynamics.

The distribution of magnetic flux for type II superconductor can be described by Time-Dependent Ginzburg-Landau (TDGL) equation [3]. There have been amount of simulations based on the equation for estimating fluids and physics of flux pinning [4, 5]. This paper focuses on the dynamics of fluxoid in type II superconductor by numerical simulations of TDGL equation. In particular, the manipulation of fluxoid is considered through a perturbation by electromagnetic wave.

2. TDGL Equation and Model of Type II Superconductor

For the analysis of fluxoid in a thin film consisting of type II superconductor, a mathematical model based on TDGL equation is introduced. Figure 1 approximates a 2D model of superconducting thin film. The thin film is a square with $L_x = L_y$ and negligible thickness of z direction. TDGL equation is represented by Eqs. (1) and (2)[3].

$$\frac{\hbar^2}{2m_s D}\left(\frac{\partial}{\partial t} + \frac{\mathrm{i}e_s}{\hbar}\phi\right)\psi + \alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m_s}\left(-\frac{\hbar}{\mathrm{i}}\nabla + e_s A\right)^2\psi = 0, \tag{1}$$

$$J = \frac{1}{\mu_0}\mathrm{rot}^2 A = -\sigma\left(\frac{\partial A}{\partial t} + \nabla\phi\right) + \left\{\frac{e_s\hbar}{2m_s\mathrm{i}}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{e_s^2}{m_s}|\psi|^2 A\right\} \tag{2}$$

where, A denotes vector potential, ϕ scalar potential, ψ complex order parameter, and J current density. e_s and m_s are defined as superconducting electron and the mass. α and β are the constants depending on temperature, μ_0 is the permeability of vacuum. σ and D correspond to the constants of conductivity and diffusion coefficient at normal conduction. c is the light speed. The boundary condition is given for the normal vector:

$$\frac{1}{\mu_0}\mathrm{rot}A \times n = H \times n, \quad J \cdot n = 0 \tag{3}$$

Normalization and standardization of the above equation lead the next TDGL equation with no dimension [5]. The standard values are set at

$$|\psi_0|^2 = -\frac{\alpha}{\beta}, \quad H_c = \sqrt{\frac{\alpha^2}{\mu_0\beta}}, \quad \lambda = \sqrt{-\frac{m_s\beta}{\mu_0 e_s^2\alpha}},$$
$$\xi = \frac{\hbar}{\sqrt{-2m_s\alpha}}, \quad \kappa = \frac{\lambda}{\xi}, \quad \eta = \mu_0\sigma D, \quad \tau = \frac{\xi^2}{D}. \tag{4}$$

Variables in Eqs. (1), (2) are

$$x' = x/\xi, \quad y' = y/\xi, \quad z' = z/\xi, \quad t' = t/\tau,$$
$$\psi' = \psi/\psi_0, \quad H' = H/(\sqrt{2}H_c\kappa),$$
$$A' = A/(\sqrt{2}\mu_0 H_c\kappa\xi), \quad \phi' = \phi/(\sqrt{2}\mu_0 H_c\kappa D). \tag{5}$$

The non-dimensional TDGL equation is given by

$$\frac{\partial\psi}{\partial t} = -\left\{(|\psi|^2 - 1)\psi + (\mathrm{i}\nabla + A)^2\psi\right\}, \tag{6}$$

$$\frac{\partial A}{\partial t} = -\frac{1}{\eta}\left\{\frac{\mathrm{i}}{2\kappa^2}(\psi^*\nabla\psi - \psi\nabla\psi^*) + \frac{1}{\kappa^2}|\psi|^2 A + \mathrm{rot}^2 A\right\} \tag{7}$$

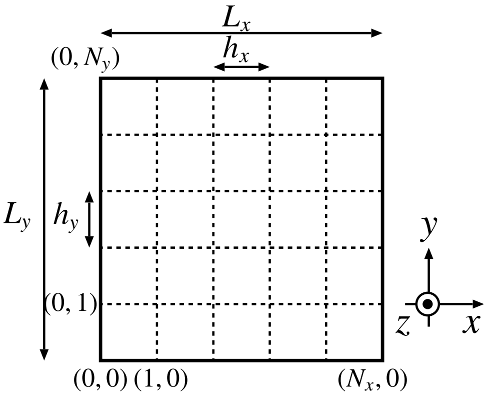


Figure 1: A 2-dimensional model of type-II superconducting thin film.

Table 1: Parameter settings

κ	1.4355
η	4.0440×10^{-2}
ξ	28.5 nm
λ	41.0 nm
L_x, L_y	410 nm
N_x, N_y	20

The boundary conditions are also given by

$$\nabla\psi \cdot n = 0, \quad \mathrm{rot}A \times n = H \times n, \quad A \cdot n = 0 \tag{8}$$

The following discussions are based on the estimation by Eqs.(6)–(8).

3. Simulations

3.1. Setting of quasi-static simulation

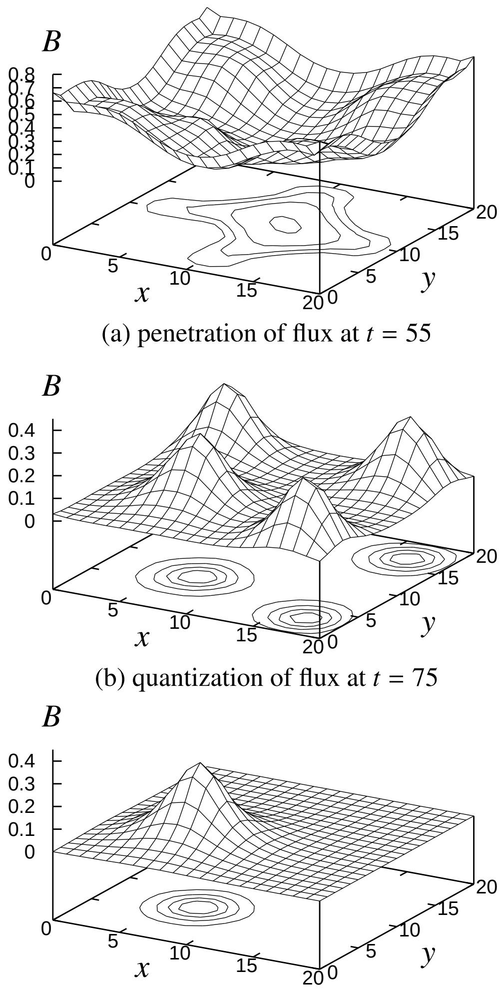
In the simulations, a model of thin film of type II superconductor is considered. The thin film is analyzed at the mesh by $N_x \times N_y$ on x, y plane. h_x and h_y is the unit of mesh in each direction. The TDGL equation for each node gives a set of ordinary differential equations depending on time t and space. The equations can be analyzed by 4th-order Runge-Kutta method. The numerical simulation of flux in the thin film is obtained by the time development of ψ and A. Flux density B is descried by rot$A$. The parameters in the simulation are shown in Tab.1.

We use an assumption: *the complex order parameter* ψ *at pinning point keeps constant without depending on time. Superconducting electron density* $|\psi|^2$ *is set at zero.*

Figure 2 displays the result of simulation for the external magnetic flux toward z-direction. After the penetration of flux, fluxoids appear by the analysis of TDGL equation,

$$H_z = \begin{cases} H_\mathrm{d} - H_\mathrm{d} \cos\left(\dfrac{2\pi t}{80}\right) & (0 \leq t < 80) \\ 0 & (t \geq 80) \end{cases}, \quad H_x = H_y = 0, \tag{9}$$

where $H_\mathrm{d} = 0.26$.



(a) penetration of flux at t = 55

(b) quantization of flux at t = 75

(c) pinned fluxoid at t = 100

Figure 2: Incursion of magnetic flux and pinning of fluxoid.

3.2. Behavior of fluxoid between two pinning points

Here, we examine the dynamical behavior of free fluxoid between two pinning points. Fig. 3 shows the initial condition for simulation. A fluxoid is set at center and pinning centers are set at $w = 4, -4$ along the w axis. We can guess that the fluxoid will move to one of pinning centers. At the initial position, the potential is equivalent to both directions to pins. Therefore, we can expect the dynamics around saddles of potential wells for fluxoid. Fluxoid is not a particle but behaves as a packet of energy.

At the initial condition, the fluxoid has no velocity of displacement: $\dot{w} = 0$. Depending on the initial position on the w-axis, the fluxoid is attracted to one of the two pinning centers. At the same time, when the fluxoid has a velocity of displacement initially, it goes over the potential saddle. We can see the dynamics in simulations. The details are presented in the final paper.

In this analysis, we can see the phase space of dynamics for superconducting fluxoid with considering the distribution of impurities in the thin film. The domain of attraction

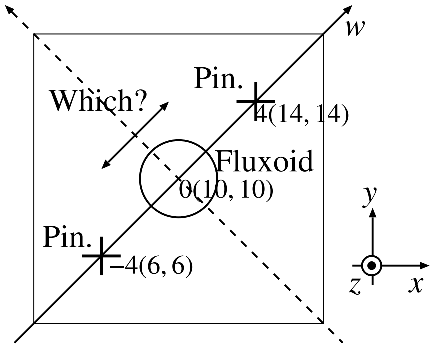


Figure 3: Positional relation between a fluxoid and pinning points in a 2-dimensional model.

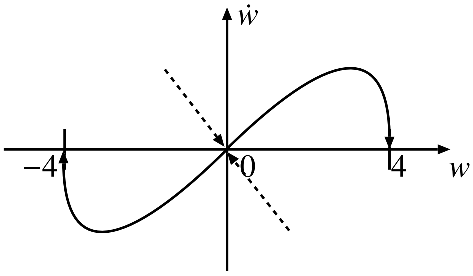


Figure 4: A picture about the behavior of fluxoid near the saddle point formed by 2 pinning points.

for the fluxoid is the dynamical structure of the superconductor, which is related to the pinning and flux flow states under the perturbation of external magnetic field.

3.3. Manipulation of fluxoid by electromagnetic wave

Fluxoid is a distributed magnetic flux, which does not concentrate on a point and is quantized by super conduction electron. Therefore, the position and velocity cannot be clearly defined. Here the velocity of displacement implies a spatial transfer of magnetic field packet. We need to mention that the fluxoid with velocity shows that there appears the disturbance of the vector potential A depending on the velocity.

3.4. Setting of external perturbation by electromagnetic wave

The electromagnetic wave is irradiated to the surface of superconducting thin film. The direction is limited to $+z$ locally. The polarization of the wave is TEM wave. $H = [H_x, H_y, 0]^T$ is kept. H_x and H_y are given by

$$H_x = H_{dx}\cos(2\pi f t + \chi_x), \quad H_y = H_{dy}\cos\left(2\pi f t + \chi_y\right), \quad (10)$$

where H_{dx} and H_{dy} denote the amplitudes of the magnetic field, f the frequency of electromagnetic wave, and both χ_x and χ_y the phase shifts. Then, the boundary conditions (8)

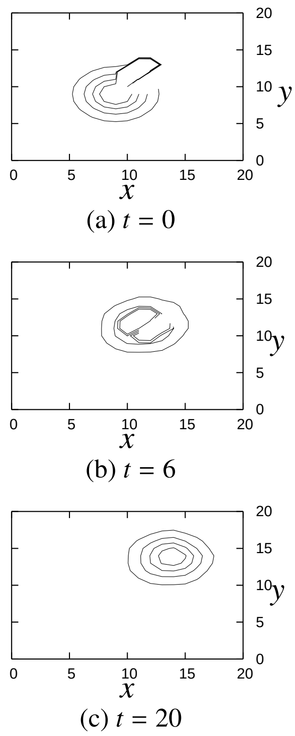


Figure 5: Behavior of fluxoid irradiated by TEM wave.

must satisfy the following conditions:

$$\frac{\partial A_x}{\partial z} = H_y, \quad \frac{\partial A_y}{\partial z} = -H_x. \quad (11)$$

In the 2D model, the difference in z is assumed as the approximation in a slab with $h_z(= h_x)$.

3.5. Manipulation of fluxoid

Here, let us consider how the fluxoid is trapped in a pinning center by the external electromagnetic wave. In the Fig. 3, the fluxoid at $(x, y) = (9, 9)$, which has no initial velocity, moved to a pinning center at $(6, 6)$. The saddle at $(10, 10)$ separates the direction of the behavior of fluxoid. Then, the fluxoid cannot reach the pinning center at $(10, 10)$.

We set the parameters of electromagnetic wave for perturbation as $H_{dx} = H_{dy} = 1$, $f = 5.0 \times 10^5$, $\chi_x = 0$, $\chi_y = \pi$. The TEM wave is irradiated to the points at $(11, 11)$, $(11, 12)$, $(12, 11)$, and $(12, 12)$ during $0 \le t < 10$. Fig. 5 shows the behavior of fluxoid affected by external perturbation. Because of the perturbation, the density distribution of fluxoid was collapsed. Finally, the fluxoid could reach the pinning center $(14, 14)$ over the saddle, which could not be overcome.

Figure 6 shows the superconducting electron distribution $|\psi|^2$ at $t = 1$. In Fig. 6, there exists the superconducting state, which keeps $|\psi|^2 = 1$, and normal conducting state at

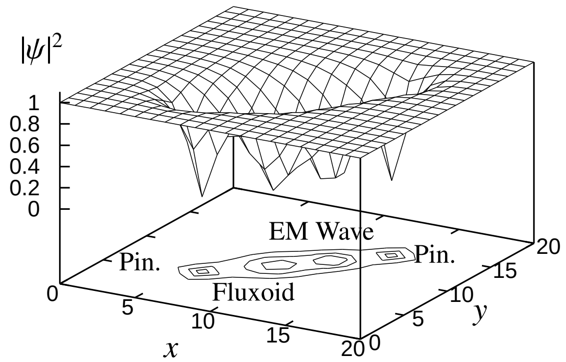


Figure 6: Density distribution of superconducting electrons ($t = 0$) under the case of existing external perturbation.

$|\psi|^2 = 0$. The area irradiated by TEM wave loses the superconductivity, like $|\psi|^2 \sim 0$. It implies that the potential structure by two pinning centers loses the two well distribution. This induces the movement of fluxoid from $(10, 10)$ to $(14, 14)$.

3.6. Release of trapped fluxoid

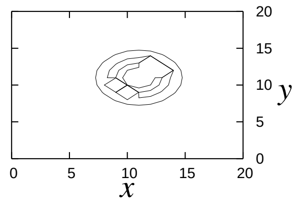
Here let us consider the release of trapped fluxoid from the pinning center by using electromagnetic wave.

Initially, the fluxoid is set at $(11, 11)$ without initial velocity of displacement. That is, the fluxoid is trapped at the pinning center. It is easily imagined that the fluxoid needs much perturbation in order to be kicked out from the well.

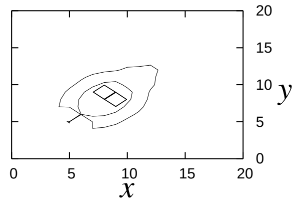
Set the perturbation as $H_{dx} = H_{dy} = 5$, $f = 5.0 \times 10^2$. The perturbation was irradiated, starting from $\chi_x = \chi_y = 0$ at $(10, 10)$, $(11, 11)$, and $(12, 12)$, ended at $(6, 6)$, $(7, 7)$, and $(8, 8)$. Fig. 7 shows the release behavior of fluxoid from the pinning center. This is caused by the temporal exchange of superconducting region to normal conducting region. The sufficient perturbation can overcome the trap of fluxoid.

4. Concluding Remarks

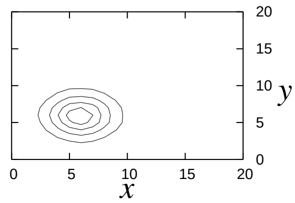
TDGL equation can represent the behavior of fluxoid around pinning centers. The external electromagnetic wave can manipulate the fluxoid by exchanging the superconductor into normal conductor as pinning center. Instead of manipulation, the electromagnetic wave can also release the trapped fluxoid from a pinning center. By the discussion in this paper, we can obtain a clue to to catch and release of quantized magnetic flux in superconductor in thin film. The phenomena is a kind of ratchet mechanisms of magnetic fluxoid



(a) $t = 0$



(b) $t = 13$



(c) $t = 20$

Figure 7: Release of fluxoid irradiated by TEM wave.

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