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Intrinsic Localized Oscillations and Their Interactions in an Articulated Structure with Softening Elastic Couplers

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Abstract– This paper demonstrates numerically excitation of intrinsic localized modes (ILMs) and their interactions in transverse oscillations of an articulated structure with free ends. The structure consists of many, identical rigid members, which are connected to adjoining ones by elastic couplers yielding nonlinear restoring moments in angular displacements, and are supported by linear springs at center of mass of each member. The structure extends straight in equilibrium and the displacements are restricted in a plane. Eigen-frequencies of the linearized system are confined in a band with a lower limit determined by the elastic supports. Here concerned is a case that the coupler is of soft-spring type in angular displacements between adjacent members, where the restoring moment has the maximum or minimum. For a certain range of parameters involved in the system and initial conditions, a movable ILM can be excited at a frequency lower than the smallest eigen-frequency. If two ILMs of equal amplitude are excited simultaneously, it is revealed that (1) they are coalesced into a single ILM, (2) they are collapsed into ripples or (3) they are separated to be trapped at both ends.

1. Introduction

Intrinsic localized modes or discrete breathers (DBs) are discovered by the pioneering studies in the 1980s, for example, Sievers and Takeno [1]. They are temporally periodic and spatially localized oscillations that occur stably in spatially discrete and perfectly periodic nonlinear systems without any defects, impurities or dissipative forces. Many investigations are now still being made theoretically and experimentally. (For example, see Flach and Gorbach [2].)

While those models are concerned with longitudinal oscillations in lattice dynamics, a periodic and articulated structure has been proposed to model transverse oscillations in large-scaled structures [3]. Although the structure consists of many identical units, it is emphasized that the number of units is large but finite. Thus the structure may be regarded as a periodic one locally but it is not so in a global and exact sense.

The simplest structure consists of combination of a number of identical units, such as beams or panels connected to adjoining ones by couplers, and, therefore, possess locally spatial periodicity. Supposing that each

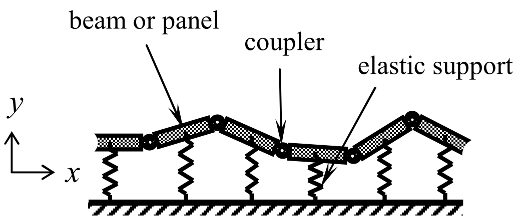

Fig. 1 Model of a spatially periodic and articulated structure supported elastically by linear springs at each center of unit.

coupler gives nonlinear restoring torque M in the form of $M = K\,\Delta\phi + K_C\,(\Delta\phi)^3$ to the angle $\Delta\phi$ between two adjacent members, where K and K_C are constants, we have revealed the existence of ILMs numerically and examined their characteristics and behaviors of propagation [3, 4].

In this paper we consider the articulated structure supported by linear springs at each center of the member (see Fig. 1). While the linear dispersion relation in the case of no supports permits wave propagation at a frequency lower than the cutoff frequency ω_0, the existence of the supports pushes up the bottom of the dispersion curve and yields a stopping band in low frequency. There we have found that the low-frequency ILMs can be excited indeed but the domain of existence of the ILMs in the plane of parameters characterizing the structure is considerably restricted in comparison with the case of usual high-frequency ILMs [5]. Our calculations show that, in the case that $K_C / K \equiv \kappa > 0$ (hard-spring type), the low-frequency ILMs are likely to stay around the center or at the end of structure where they were excited, whereas, in the case that $\kappa < 0$ (soft-spring type), the ILMs are likely to be movable.

In this paper we consider the interactions between two movable ILMs excited in the case that $\kappa < 0$.

2. Model and Formulation

The structure consists of N ($N \gg 2$) identical and rigid members, which are of finite length and of uniform density along the axis, connected to adjoining ones at both ends by identical couplers giving nonlinear restoring moment and supported at each center of mass by linear springs (Fig. 1). Each member may be a beam or a panel, and the mass of the coupler and the length of the interval of the junction are assumed to be negligible. A member with couplers at both ends forms one unit of the structure. Supposing that the number of the units N is large but finite, they are numbered consecutively by integer j ($1 \le j \le N$) and physical variables pertaining to the unit j are denoted by attaching subscript j. At the left end of the 1st unit and the right end of the N th unit, the structure is assumed to be free.

Motions of the structure are restricted in the x-y plane where the x-axis is taken along the structure in equilibrium. It is assumed that restoring force by linear spring acts the unit at its center only in the y direction. In the j th unit, the position of the center of mass of the unit is denoted by ($x_j(t)$, $y_j(t)$) and the angle of the centerline to the x-axis is denoted by $\phi_j(t)$, t being the time. Suppose that the restoring moment (torque) M_j, which is affected by the coupler at the left end of the j th unit, is given by a linear plus cubic function of difference in angle between two centerlines of the adjacent units in the following form:

$$M_j = K(\phi_j - \phi_{j-1}) + K_C(\phi_j - \phi_{j-1})^3,$$

for $2 \le j \le N$, K (> 0) and K_C being constant. We consider the case that $K_C < 0$ in this paper.

Equations of motions are easily derived by applying the Newton's law of motions to each unit: the equations for translation and rotation about the center of mass. The conditions for continuity of displacement at each junction are required for geometrical constraints. The variables in these equations and conditions are normalized by adequate quantities with two parameters: κ for the nonlinearity in the response of coupler to rotation and η for the stiffness of linear spring to deflection. The total energy of the system is conserved of course.

The linear dispersion relation to the equations is derived by assuming a sinusoidal wave for y_j (or ϕ_j) in the form of $\exp\left[i\left(kj - \omega t\right)\right]$, k and ω being a dimensionless wave number and an angular frequency, respectively, as follows:

$$\omega = \omega_0 \left[\frac{\sin^4(k/2) + (\eta/16)\cos^2(k/2)}{3 - 2\sin^2(k/2)} \right]^{1/2},$$

where $\omega_0 = \sqrt{48}$. The function on the right-hand side is periodic in k with periodicity 2π, but the shortest wavelength occurs physically at $k = \pi$ (π-mode or zigzagged configuration). The width of the passing band is dependent on the value of η and is the smallest at η = 48. It is seen that π-mode takes $\omega = \omega_0$ irrespective of the value of η and the passing band lies under ω_0 for $\eta \le 48$. In the following calculations, we take η = 48 for the narrowest passing band.

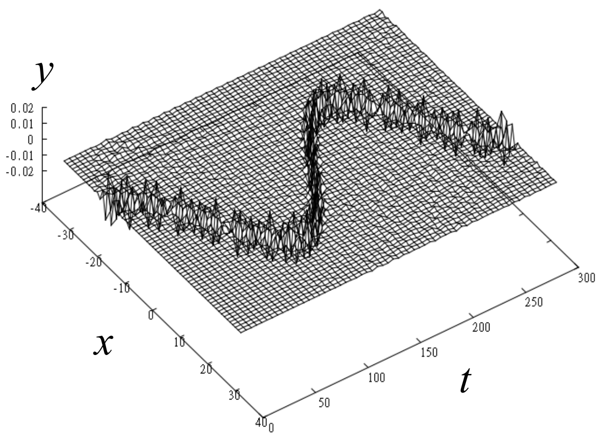

Fig. 2 Typical mobile ILM in the structure for σ = 16 (κ = -340, $A = \pi/90$, α = 0.9).

3. Numerical Results

3.1. Mobile ILMs

The simultaneous equations derived above are solved numerically with the free boundary condition at both ends of the structure and appropriate initial conditions for the combinations of values of κ and η. We have taken N = 64, κ < 0 and the initial conditions which are given by the values for ϕ_j in the form of the π mode whose envelope is modulated in the form of a sech-type pulse as

$$\phi_j = (-1)^j A \,\mathrm{sech}\left[\alpha(j - c - \sigma)\right]$$

for $1 \le j \le N$, where A, α and σ are constants, c (= $N/2$ + 1/2) is the center of structure and σ ($0 \le |\sigma| \le N/2$) indicates the offset of the center position of modulation from c. The initial positions, $x_j(0)$ and $y_j(0)$, are determined by the conditions of constraints if one set of the values of $\phi_j(0)$ for all j is prescribed. All initial velocities are taken to vanish. For the adequate combinations of values of κ and η and initial conditions, single localized oscillations can be excited and they generally move in the structure (i.e. single mobile ILMs) except for some special cases when the structure is disturbed initially just at its center or near one end of it. In almost cases the ILMs become undulating around the center depending on the value of $|\sigma|$. A typical solution is displayed in Fig. 2 for σ = 16, κ = -340, $A = \pi/90$ and α = 0.9.

In the following, we will study the interactions between two localized oscillations of the same amplitude positioned symmetrically with respect to the center of structure.

3.2. Interactions between Two Localized Oscillations

To excite two symmetric ILMs of the same amplitude, we adopt an initial condition in the following form:

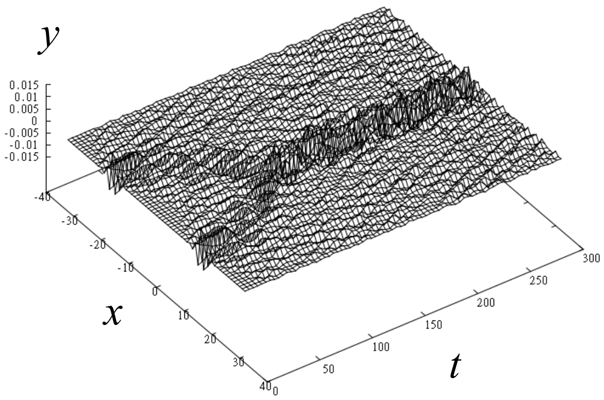

Fig. 3 Typical interactions between two identical localized oscillations ($\sigma = 16$, $A=\pi/180$).

$$\phi_{\mathrm{j}} = (-1)^{j} A\{\mathrm{sech}[\alpha(j-c+\sigma)] + \mathrm{sech}[\alpha(j-c-\sigma)]\},$$

with zero velocities for all j. For the same values of κ and α as the ones in previous section, the equations are solved by the standard 4th-order Runge-Kutta method, monitoring the total energy to be conserved.

Typical solution is displayed in Fig. 3. It is seen that two identical localized oscillations are excited and they immediately come close to each other, coalesce at the center of structure and behave as a single stationary mode of oscillations. The configuration maintains the symmetry with respect to the center and the localized modes oscillate in phase with each other. In most cases of $|\sigma|$ ($0 \le |\sigma| \le 29$), the behaviors become qualitatively similar. For the larger values of $|\sigma|$, the two localized oscillations separate from each other to behave independently.

4. Conclusions

The interactions between two localized oscillations whose envelopes have the same amplitude and are symmetrical with respect to the center of the structure have been numerically studied. The domain of existence of the low-frequency ILMs in the κ-η parameter plane for $\kappa < 0$ is narrow in comparison with that of high-frequency ILMs, therefore, the variety of the interactions is poor [5, 6]. It is revealed that the localized oscillations, when excited in close vicinity of each other, are attractive and, therefore, become to coalesce into single one or collapse into ripples at the center of structure.

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