Combination of Ultra-Wide Band Characteristic Basis Function Method and Asymptotic Waveform Evaluation Method in MoM Solution

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***Abstract*-A novel technique which combines the ultra-wide band characteristic basis function method (UCBFM) and the adaptive multi-point expansion algorithm for asymptotic waveform evaluation (AWE) method is proposed. The UCBFM and the AWE method are two kinds of model order reduction (MOR) methods, and their proposed combination can be applied for fast evaluation of wide band scattering problems. In the proposed approach, ultra-wide band characteristic basis functions (UCBFs) is solved according to the highest frequency in the range of interest, and the adaptive multi-point expansion algorithm for AWE is based on a simple binary search algorithm. Provided numerical results validate the proposed method, and suggest that it has a high efficiency.**

***Index Term*-model order reduction methods; ultra-wide band characteristic basis function method; asymptotic waveform evaluation; method of moments; scattering problem**

I. Introduction

One of the most popular methods for radar cross sections (RCS) prediction is the frequency domain integral equation solved using method of moments (MoM) [1], but it places a heavy burden on the CPU time as well as memory requirements when electrically large structures are analyzed. Moreover, they require the impedance matrix to be generated and solved for each frequency sample; hence, if the response over a wide frequency band is of interest, the MoM is computationally intensive. This problem is especially serious when the RCS is highly frequency dependant and fine frequency steps are required to get an accurate representation of the frequency response.

Several techniques have been proposed to alleviate this problem. In [2, 3], the characteristic basis function method (CBFM), is able to reduce the size of the MoM matrix. In the CBFM, the object is divided into a number of blocks, and high-level basis functions called characteristic basis functions (CBFs) are derived for these blocks, which are discretized by using the conventional triangular patch segmentation and Rao-Wilton-Glisson (RWG) basis functions [4]. In [5], the asymptotic waveform evaluation (AWE), is proposed for predicting the RCS over a band of frequencies. Since it needs the MoM matrix inversion at central frequency, the AWE technique can hardly deal with wideband electromagnetic scattering problems from electrically large object or multi-objects. So in [6], AWE based on CBFM, is proposed to analyze wideband electromagnetic scattering problems. This method uses the mutual coupling method for generating CBFs, that is time consuming and memory demanding, in CBFM and applies single-point expansion for AWE. In [7], A simple binary search algorithm is described to apply AWE at multiple frequency points to generate an accurate solution over a specified frequency band. Since the CBFs depend upon the frequency, they need to be generated repeatedly for each frequency. Hence, in [8], the ultra-wide band characteristic basis function method (UCBFM) is used on a wide band, without having the generation of CBFs for each frequency repeatedly. The CBFs calculated at the highest, termed UCBFs, entail the electromagnetic behavior at lower frequency range; thus, it follows that they can also be employed at lower frequencies without going through the time consuming step of generating them again. However, in the UCBFM, it is still time consuming for the computation of wideband RCS since it requires repeated solving of the reduced matrix equations at each frequency.

In this paper, The combination of the UCBFM with the adaptive multi-point expansion algorithm for AWE is introduced for fast evaluation of wide band scattering problems. In the following sections, the principles of the UCBFM and the AWE based on a simple binary search algorithm are outlined firstly, and then the combination scheme of the UCBFM/AWE method is developed. Finally, two classical scattering problems are analyzed and the comparisons of the proposed method and traditional methods are provided.

II. Formulation

A. UCBF Method [8]

Let us consider a complex 3-D object illuminated by a plane wave. In a conventional MoM, the whole surface is divided into triangles with size ranging from $\lambda/10$ to $\lambda/20$. Applying this to the electric field integral equation, one can obtain a dense and complex system of the form

$$Z(k)I(k)=V(k) \tag{1}$$

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In (1), Z is the MoM matrix of dimension $N \times N$, I and V are vectors of dimension $N \times 1$, where N is the number of unknown current coefficients and k is the wave number of the free space. For large and complex problem, the matrix filling and matrix equation solving are quite time consuming.

The CBFM begins by dividing the object to be analyzed into blocks. For the best division scheme and the number of blocks M one may refer to [9]. These blocks are characterized through a set of CBFs, constructed by exciting each block with multiple plane waves (MPW), incident from N_{PW} uniformly spaced θ and φ-angles. To calculate the CBFs on the generic ith block , one must solve the following system

$$Z_{ii}\left(k\right)J_i^{CBF}=V_i^{MPW} \tag{2}$$

In (2), Z_{ii} is an $N_i \times N_i$ sub-matrix corresponding to the ith block, J_i^{CBF} is a $N_i \times N_{\mathrm{PW}}$ matrix containing original CBFs, and V_i^{MPW} is a $N_i \times N_{\mathrm{PW}}$ matrix containing excitation vectors, where N_i is the number of unknowns relative to ith block. In order to extract Z_{ii} from the original MoM matrix, a matrix segmentation procedure can be used. Next, a new set of orthogonal basis functions, which are linear combinations of the original CBFs, are constructed via the singular value decomposition (SVD) approach. Thus, the redundant information because of the overestimation is eliminated. For simplicity, one can assume that the average number of CBFs after SVD is K. Consequently, the solution to the entire problem is expressed as a linear combination of the $M \times K$ CBFs, as follows

$$I(k)=\sum_{m=1}^{M}\sum_{k=1}^{K}\alpha_m^k\left(k\right)J_m^{CBF_k}\left(k\right) \tag{3}$$

where $J_m^{CBF_n}$ is the nth CBF of the mth lock. By using the above CBFs, the original large MoM matrix can be reduced, and unknowns are changed to weight coefficient vector α whose order is much smaller than original current coefficient vector I. Finally, after solving the reduced system and substituting solution back to (3), one can obtain the solution of single frequency point. For the multi frequency point, although the above procedures of the CBFM can be repeated, the UCBFM is more efficient.

The ultra-wide band characteristic basis functions (UCBFs) is the CBFs generated at the highest frequency. Since the UCBFs can adequately represent the solution in the entire band of interest, they are used for lower frequencies without going through the time consuming step of generating them again. Fig.1 shows the flowchart of the UCBFM.

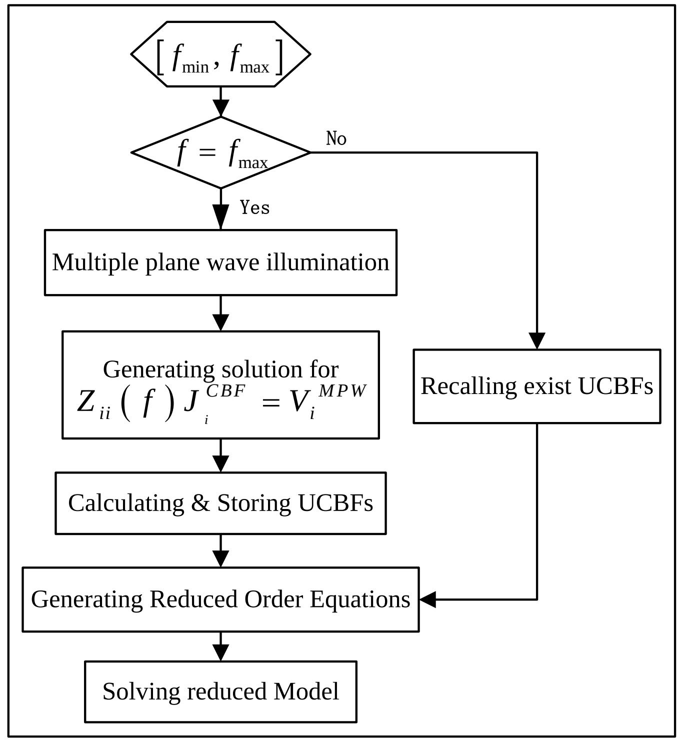

Fig 1. Flowchart of the UCBFM

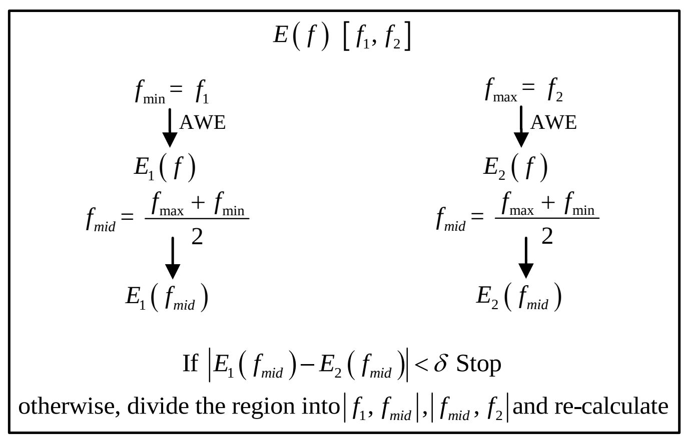

Fig 2. Flowchart of the adaptive multi-point expansion algorithm.

B. AWE Method [7]

For the given frequency f_0 (corresponding to the wave number k_0) in a wide band, current coefficient vector $I(k_0)$ can be obtained one by one by solving (1) by using the above UCBF. It is still time consuming for wide-band problems. For such problems, some interpolation/extrapolation schemes like the AWE method are always powerful. The basic procedure of the AWE is to first expand the $I(k)$ into Taylor series, then to use the Padé approximation to generate the rational function as

$$I\left(k\right)=\sum_{n=0}^{L+M}m_n\left(k-k_0\right)^n=\frac{\sum_{i=0}^{L}a_i\left(k-k_0\right)^i}{1+\sum_{j=1}^{M}b_j\left(k-k_0\right)^j} \tag{4}$$

where the moment vectors m_n can be calculated by the following recursive formula

$$m_0=Z^{-1}\left(k_0\right)V\left(k_0\right) \tag{5}$$

$$m_n=Z^{-1}\left(k_0\right)\left[\frac{V^{(n)}\left(k_0\right)}{n!}-\sum_{i=1}^{n}\frac{Z^{(i)}\left(k_0\right)m_{n-i}}{i!}\right] \tag{6}$$

where $Z^{(i)}(k_0)$ is the ith derivative with respect to k of $Z(k)$ and evaluated at k_0. Similarly, $V^{(n)}(k_0)$ is the nth derivative with respect to k of $V(k)$.The coefficients a_i and b_j are obtained by solving the following linear equation

$$\sum_{j=1}^{M} m_{L+i-j} b_j = -m_{L+i} \quad i = 1, 2, ..., M \tag{7}$$

$$a_i - \sum_{j=0}^{i-1} m_j b_{i-1} = m_i \quad i = 1, 2, ..., M \tag{8}$$

where $b_0 = \{1,1,...,1\}$.

The adaptive algorithm for the AWE is shown in Fig.2, where $E(f)$is the desired quantity. When single-point expansion does not satisfy the requirement over the whole frequency band, the multi-point expansion becomes necessary. The adaptive multi-point expansion algorithm given in Fig.2 is available for this purpose.

C. UCBFM/AWE Method

It can be seen from (5)–(6) that the computation of the derivatives of $Z(k)$ and $V(k)$ is the key step in the AWE technique. In order to avoid solving the inversion of large matrix in (5) and (6), UCBFM is combined with the adaptive multi-point expansion algorithm for AWE technique to calculate the wide band scattering characteristics from electrically large or multiple objects. The flowchart of UCBFM/AWE is shown in Fig.3. The coefficients m_n in (5) and (6) are calculated by UCBF method. In UCBFM/AWE algorithm, the current distribution at each frequency within the band is expressed as a linear combination of UCBFs.

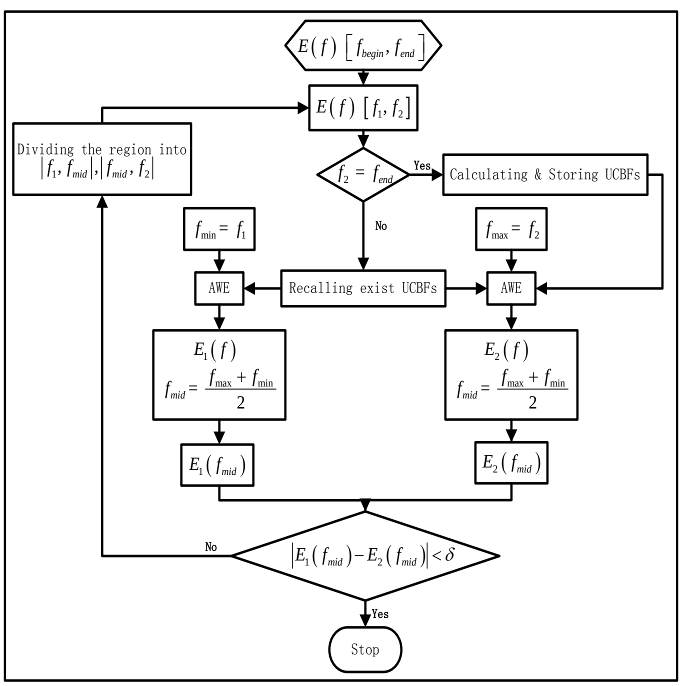

Fig 3. Flowchart of UCBFM/AWE method.

III. Numerical Results

To demonstrate the efficiency and accuracy of the UCBFM/AWE method, two numerical examples are investigated. The objects of the numerical simulations are illuminated by a normally incident theta-polarized plane wave; a conventional triangular patch segmentation and RWG basis function are employed [4]. All the simulations were run on a notebook equipped with 2 Dual Core at 2.3GHz (only one core was used) and 8GB of RAM.

TABLE I
Computational times for the Different methods of a sphere

	Conventional MoM	CBFM/AWE	UCBFM/AWE
Total time	203.227 m	32.908 m	15.453 m
Saving	—	83.81%	92.39%

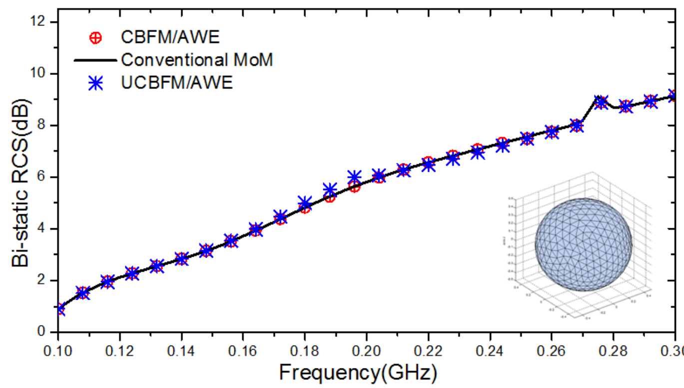

Fig 4. Bi-static RCS of the PEC sphere from 0.1GHz to 0.3GHz using different methods.

TABLE II
Computational times for the two methods of a plate

	Conventional MoM	UCBFM/AWE
Total time	30.127 h	6.814 h
Saving	—	77.38%

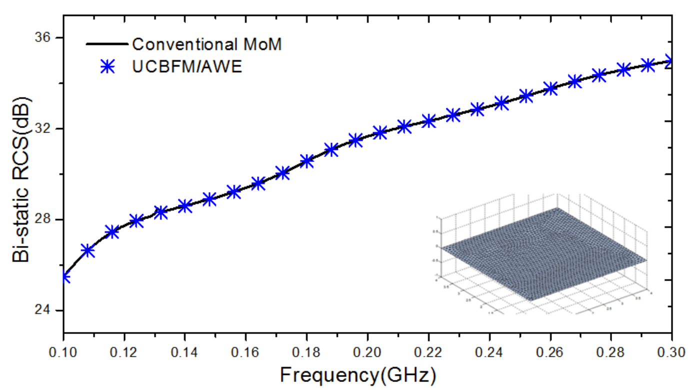

Fig 5. Bi-static RCS response of the PEC plate by UCBFM/AWE and conventional MoM method.

The first example is the RCS by a PEC sphere with diameter of 2.0*m* from 0.1GHz to 0.3GHz. The geometry is automatically divided into two blocks, as shown in Fig.4. The discretisation in triangular patches is carried out at the highest frequency, with a mean edge length of 0.1λ, which involves almost 1293 unknowns. After SVD procedure we totally obtain 228 UCBFs. We set $L = M = 4$, $\delta = 0.1$for AWE expansion and chose $N_{\mathrm{PW}} = 800$ for UCBFM. The results are compared with those derived by using CBFM/AWE and conventional MoM, as shown in Fig.4. The RCS response of the PEC sphere is for the observation angle at $\theta = 180^{\mathrm{o}}$, $\varphi = 0^{\mathrm{o}}$. In UCBFM/AWE method, the expansion points are 0.1GHz and 0.3GHz, with a frequency increment of 1MHz. While in CBFM/AWE technique the expansion points are: 0.1GHz, 0.15GHz, 0.2GHz and 0.3GHz, with a same frequency increment. For conventional MoM method the step of frequency sweep is 5MHz. The bi-static RCS solved by UCBFM/AWE and CBFM/AWE coincide very well with the conventional MoM. Hence, the presented method is accurate in wide band electromagnetic scattering analysis.

To show the efficiency of the UCBFM/AWE, the total simulation time which include matrix filling time and solving time is shown in Table I. With a frequency increment of 1MHz, the conventional MoM method requires about 203.227 minutes to obtain the solution. However, for UCBFM/AWE method, to obtain the same accuracy, only 15.453 minutes are needed, which is 13.15 times faster than the conventional MoM method.

The next example is a 4*m*×4*m* PEC plate. The frequency range starts from 0.1GHz and terminates at 0.3GHz; a length of 4*m* is equal to 4λ at 0.3GHz. The plate is divided into sixteen blocks, as shown in Fig.5. The discretisation in triangular patches is carried out at highest frequency, with a mean edge length of 0.1λ; this leads to total unknowns of 5299. After SVD procedure we obtain 971 UCBFs. These UCBFs calculated at 0.3GHz can be used for any frequency in the range. The bi-static RCS observed at $\theta = 180^{\mathrm{o}}$, $\varphi = 0^{\mathrm{o}}$ is shown in Fig.4, which calculated by UCBFM/AWE and conventional MoM method, respectively.

The bi-static RCS shows an excellent match for the two methods. In this numerical simulation we set $L = 4$, $M = 4$, $\delta = 0.1$ for AWE expansion and chose $N_{\mathrm{PW}} = 800$ for UCBFM. Over the whole frequency band, we obtain six expansion points, with a frequency increment of 1MHz for Padé approximation, while the step of frequency sweep is 5MHz for conventional MoM method. Table II shows the efficiency of the presented method. With a frequency increment of 1MHz, we obtain a total simulation time of about 6.814 hours with UCBFM/AWE , whereas the use of conventional MoM leads to 30.127 hours.

IV. Conclusion

In this paper, a new approach that combines the UCBFM with the adaptive multi-point expansion algorithm for AWE is successfully implemented to fast and efficiently analyze wide band scattering problems. Numerical results given demonstrate the high accuracy and efficiency of the proposed method. The scattering problem of very electrically large objects and multiple objects can be handled, since large matrix inverse computation is not necessary in the hybrid method and UCBFM allows the decrease of time in CBFs compared to that obtained by CBFM. The AWE method in the UCBFM was used in order to further reduce the computational time required for wide band analysis.

References

[1] R.H. Harrington, *Field Computation by Moment Methods*, New York: Macmillan, 1968, pp.49-70.

[2] V.V.S. Prakash and R. Mittra, “Characteristic Basis Function Method: A new technique for efficient solution of method of moments matrix equation,” *Microwave Opt. Technol. Lett.*, vol.36, no.2, pp.94-100, Feb. 2003.

[3] E. Lucente, A. Monorchio and R. Mittra, “An iteration free MoM approach based on excitation independent characteristic basis functions for solving large multiscale electromagnetic scattering problems,” *IEEE Trans. Antennas Propag.*, vol.58, no.7, pp.999-1007, Apr. 2008.

[4] S.M. Rao, D.R. Wilton and A.W. Glisson, “Electromagnetic scattering by surfaces of arbitrary shape,” *IEEE Trans. Antennas Propag.*, vol.30, no.3, pp.409-418, Feb. 1982.

[5] C.J. Reddy, M.D. Deshpande and C.R. Cockrell, “Fast RCS computation over a frequency band using method of moments in conjunction with asymptotic waveform evaluation technique,” *IEEE Trans. Antennas Propag.*, vol.46, no.8, pp.1229-1233, Aug. 1988.

[6] Y.F. Sun, Y. Du and Y. Shao, “Fast computation of wideband RCS using characteristic basis function method and asymptotic waveform evaluation technique,” *Journal of Electronics(CHINA)*, vol.27, no.4, pp.453-457, Apr. 2010.

[7] J.P. Zhang and J.M. Jin, “Preliminary study of AWE method for FEM analysis of scattering problems,” *Microwave Opt. Technol. Lett.*, vol.17, no.1, pp.7-12, Jan. 1998.

[8] M.D. Gregorio, G. Tiberi, A. Monorchio and R. Mittra, “Solution of wide band scattering problems using the characteristic basis function method,” *IET Microw. Antennas Propag.*, vol.6, no.1, pp.60-66, Jan. 2012.

[9] K. Konno, Q. Chen, K. Sawaya and T. Sezai, “Optimization of block size for CBFM in MoM,” *IEEE Trans. Antennas Propag.*, vol.60, no.10, pp.4719-4724, Jan. 2012.