A Graph-Theoretic Approach to Building Layout Reconstruction from Radar Measurements

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***Abstract*-Motivated by the desire to obtain the interior layout of a building from through-the-wall radar measurements, we have developed a building layout reconstruction algorithm based on the minimum spanning tree (MST), which is a graph-theoretical method. Based on the extraction of all the wall-wall-floor trihedrals from radar measurements, we have defined the vertex and edge set of the building layout graph. Then the edge weight is determined according to actual conditions. Finally, the MST method is applied to reconstruct the building layout. Simulation results have shown effectiveness of the method. The techniques in this paper are intended to serve as an exploration into the graph theoretical solution on the building interior layout reconstruction problem.**

I. INTRODUCTION

In recent years, the technology of sensing through walls has received considerable attention. Through-the-wall radar imaging (TWRI) is considered very effective to achieve the objectives of "seeing" through walls. TWRI is highly desirable for a range of organizations, including police, fire and rescue personnel missions, surveillance, first responders, and defense forces [1, 2].

At present TWRI mainly focus on behind-the-wall targets. These through-the-wall radars often require a close position to the wall or they have to be pressed against the wall. However, they are only available for single-wall penetration, providing range, direction, and motion information of moving objects [3-5].

Nowadays there has been growing interest in techniques for determining the layout of a building's interior from radar measurements made from that building's exterior. There are many applications for such an ability, including firefighting and lawenforcement operations.

The newly works on this topic are the Synthetic Aperture Polarimetric Phased Array Interferometer Radar Equipment (SAPPHIRE) and the visibuilding program. The SAPPHIRE is developed by the Netherlands Organization for Applied Scientific (TNO), whose operational principle relies on detecting and identifying dominant scatterers inside a building by exploiting specific phase relations in the measured 3D data cube[3,4]. SAPPHIRE focuses on 2D reconstruction of an empty room on the ground floor. However, the reconstruction method is sensitive to model inaccuracies. Additional filtering step is needed to cluster the detected scatterers. The visibuilding program was funded by the Defense Advanced Research Projects Agency (DARPA) [6]. The program addresses the desire to see inside structure to determine the layout of buildings, using a model-based signal processing method. In addition, the U.S. Army Research Laboratory (ARL) has conducted a field experiment to map an abandoned Army barrack building. The images were obtained from two sides of the building. They also have performed some computer simulations with Xpatch to explain some phenomenon observed in the measured SAR images [7, 8].

In literatures, there is consensus on the fact that a map of a building can be best obtained by detecting and identifying principal scatterers especially trihedrals inside a building [9, 10]. As long as we have got the position and orientation of each trihedral inside a building, the layout of the building can be reconstructed subsequently. Motivated by the desire to estimate the interior layout more accurately, we have proposed an iterative deducing method. Due to compensating the effect of wall mentioned in the literature [11], the estimation results produced by the present iterative loop will be closer to the true layouts of the building than the previous one. Thus, the estimation layout will approach the true structure loop by loop. In order to improve the efficiency of the iterative method and to keep away from mannual intervention, auto-reconstruction method needs to be developed.

The treatment in this paper considers, without the loss of too much generality, that the floorplan of the building is rectangular in shape. Namely, the interior walls are assumed to be either parallel or perpendicular to the front wall. Moreover, any closed structure is not allowed to exist inside the building. The basis for this graph-theoretical approach is drawn from a paper on the subject of layout estimation by Lavely et al. They employ graph theory in its problem formulation [12]. The way that graphs are employed in that paper is that the edges of the graph are used to indicate the walls that are present or not present, and the nodes are used to represent the location and length of walls. In the end of this paper, we have carried some computer simulations to validate the graph method.

II. Graph-Based Models for Building Interior Layout

The building interior layout reconstruction approach considered by us is based on a minimum spanning tree (MST) method. This technique has its roots in graph theory. In this section, we will define the graph elements in advance and make relationship between the layout reconstruction problem and the MST problem.

A. Graph Elements

This paper intends to deduce the interior layout of a building. We treat the interior layouts of a building as a weighted undirected graph. It is called building layout graph in the following text. Usually, a graph is represented as $G(V,E)$, where V is the set of vertices (or nodes) and E is the set of weighted edges connecting them. The vertex set and the edge set of the building layout graph are defined as follows:

1) Vertices (or nodes): The wall-wall-floor trihedrals inside the building are treated as nodes of the building layout graph.

2) Edges: Walls existing between two adjacent nodes, the edges are undirected.

3) Degree: It is used to describe a node. It means the number of walls which relates to current wall-wall-floor trihedral node. For undirected graph, in-degree and out-degree are both treated as uniform degree with no difference.

4) Order: It is used to describe a graph. It means the number of trihedral nodes in the building graph and is denoted as $|V|$.

Positions and orientations of the wall-wall-floor trihedrals are attributes of the vertex set. A weighted undirected graph can be established if we have got all the trihedrals including their attributes through radar measurements. How to obtain these attributes has been mentioned in the literature [13]. From the literature we have known that the pose angles of all the trihedrals in a radar image can be estimated by using a virtual aperture imaging model.

B. MST in Building Layout Graph

In this subsection, we will present the concept of MST and the relationship between MST and the interior layout of a building.

Given a complete weighted undirected graph $G(V,E)$ with $|V|=N$, the number of trees (a subgraph of G without closed loops) that connects all the nodes of the graph is N^{N-2}. The MST is the tree with the minimum total weight, defined as the sum of the weight of each tree's edge. As is mentioned afore, in a radar measurement data set consisting of principal scatterers, we consider the wall-wall-floor trihedrals as the nodes of a graph, the edges being the wall lines joining the nodes. If we have defined reasonable edge weight to translate the building interior layout reconstruction process into searching the MST of the complete weighted undirected graph.

Hereto, the last job we have not finished to construct the weighted undirected graph is the definition of edge weight. In the following sections, we will present the definition process of edge weight so as to establish the equivalence between the MST and the correct building interior layout. The keypoint of solving the MST problem and the reconstruction problem lies in defining the edge weight.

III. Building Layout Reconstruction Using MST Method

A. Edge Weight

Before we define the edge weight, we will give the definition of angle range of each trihedral node in advance. The following figure gives the angle range definition when the degree of a trihedral node equals 2, 3 and 4. θ_s means the starting angle and θ_e means the ending angle. The angle range is denoted as $[\theta_s, \theta_e]$. Due to the rectangular shape of the building. The orientation of trihedrals are divided into four quadrants.

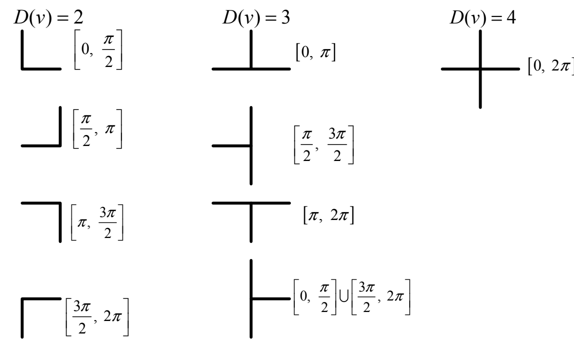


Figure 1. Definition of node angle range.

Next, we will define the edge weight between a couple of trihedral nodes. According to the actual situation, the definition of the edge weight ω_{ij} between a couple of nodes which are denoted as v_i and v_j should subject to the following three conditions:

1) The edge weight that drops within the angle range $[\theta_s, \theta_e]$ should be smaller than that drops out of the angle range. Moreover, the edge weight that drops within the angle range should better be much smaller as it is not reasonable that an edge is outside the angle range.

2) To keep the monotonicity of the Euclid distance in the angle range, the edge whose Euclid distance is smaller should also be smaller in weight.

3) If the weight function is in exponential form, the base should not be equal to 1, and the exponent should not be equal to 0. Otherwise, the weight can not be distinguished from angle range and Euclid distance.

If we have constructed an edge weight that satisfying the above three conditions, the building interior layout reconstruction process can be translated into searching the MST of the complete weighted undirected graph.

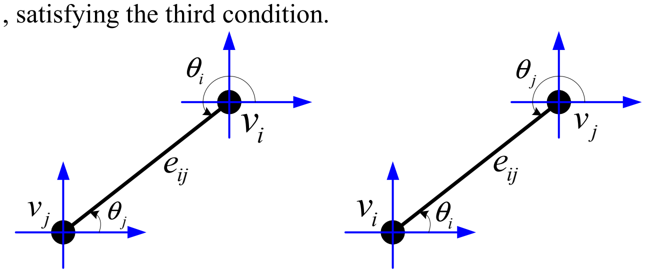
From the above three conditions, we could see that the edge weight between a couple of nodes should consider not only their Euclid distance but also the angle constraint. The edge weight we considered is expressed in exponential form as follows:

$$\omega_{ij} = \left(\frac{d_{ij}}{d_{max}} \right)^{\gamma(\theta_i, \theta_j)} \quad (1)$$

where d_{ij} is the Euclid distance between node v_i and node v_j.

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (2)$$

(x_i, y_i) represents the position attribute of node v_i and (x_j, y_j) has the similar meaning. $\gamma(\theta_i, \theta_j)$ is the exponential term of the edge weight, the definition of θ_i and θ_j is shown in Fig. 2. $d_{max} = \alpha L_{max}$ $(\alpha > 1)$, L_{max} represents the biggest Euclid distance among all possible node pair which is prior known. L_{max} is defined as the diameter of a graph in this paper. The existence of factor α is to keep the base from equaling to 1, satisfying the third condition.



(a) (b)

Figure 2. Definition of node angle.

For the case in Fig. 2(a), θ_i and θ_j is defined as:

$$\begin{cases} \theta_i = \arccos \dfrac{x_i - x_j}{d_{ij}} + \pi & (\theta_i \in [\pi, 2\pi]) \\ \theta_j = \arccos \dfrac{x_i - x_j}{d_{ij}} & (\theta_j \in [0, \pi]) \end{cases} \quad (3)$$

For the other case in Fig. 2(b), θ_i and θ_j is defined as:

$$\begin{cases} \theta_i = \arccos \dfrac{x_j - x_i}{d_{ij}} & (\theta_i \in [0, \pi]) \\ \theta_j = \arccos \dfrac{x_j - x_i}{d_{ij}} + \pi & (\theta_j \in [\pi, 2\pi]) \end{cases} \quad (4)$$

According to the above description and conditions, the exponential term of the edge weight can be expressed as follows when $\theta_i \in [\theta_{is}, \theta_{ie}]$ and $\theta_j \in [\theta_{js}, \theta_{je}]$:

$$\gamma(\theta_i, \theta_j) = -|\sin(2\theta_i)| - |\sin(2\theta_j)| + \beta \quad (5)$$

otherwise, when the edge ω_{ij} drops out of the angle range, the exponential term can be written as:

$$\gamma(\theta_i, \theta_j) = \eta \quad (6)$$

The parameter β is introduced in order to keep the exponential term above zero. Thus, ω_{ij} will increase with the Euclid distance increase when θ_i and θ_j are within the angle range. This is a reply to the second condition. The exponential term will be a negative constant η when either θ_i or θ_j is out of the angle range. The negative property of η will increase the weight dramatically and be greater than the edge weight when both θ_i and θ_j are within the angle range, satisfying the first condition. The greater the absolute value of η, the bigger the edge weight. The parameter η and β adjust the difference between the edge weight that drops in the angle range and the one out of the angle range.

B. *MST Solving Algorithm*

We have give the definition of edge weight of the building layout graph. Now we should focus on how to find the MST of the building graph so as to obtain the interior layout of the building.

The frequently used MST searching algorithms are the Kruskal algorithm and the Prim algorithm. Both of these two algorithms come from greedy ideas. However, proof has been shown that these algorithms would obtain the global optimization results instead of being driven to the local optimization results. The Kruskal method needs to sort the edge weight only one time, while the Prim method needs to sort the edge weight more than one times. As a result, we have used the algorithm of Kruskal to construct our MSTs for computation efficiency. The process of the Kruskal algorithm in building layout reconstruction is described with four steps:

Step 1: Using definition in (3), sort the edge weight of the weighted undirected building graph in ascending order;

Step 2: Set $i = 1$ and let the initial edge be $E_0 = \varnothing$;

Step 3: Select an edge e_i of minimum weight value not in E_{i-1} such that $T_i = < E_{i-1} \cup \{e_i\} >$ is acyclic and define $E_i = E_{i-1} \cup e_i$. If no such edge exists, let $T = < E_i >$ and stop;

Step 4: Replace i by $i+1$. Return to Step 3.

After $N-1$ iterations, where N is the order of the building layout graph, the complete MST is found. Thus, the building interior layout is obtained subsequently.

IV. Simulation Results

In the above sections, we have translate the building layout reconstruction problem into the MST searching problem. With the edge weight defined as (1)~(6) and the Kruskal algorithm, some simulations have been implemented and we have got some useful results. We have simulated two buildings which have different layouts as is shown in the following figure.

We have simulated a simple building consisting of four rooms at first to validate the reconstruction algorithm based on MST. Without loss of generality, the building size is assumed to be $2m \times 2m$ and the size of each room is $1m \times 1m$. The layout is shown in Fig. 3.

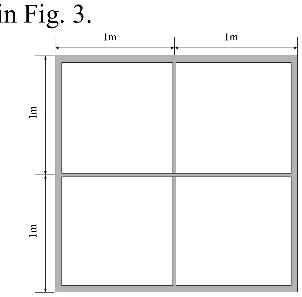
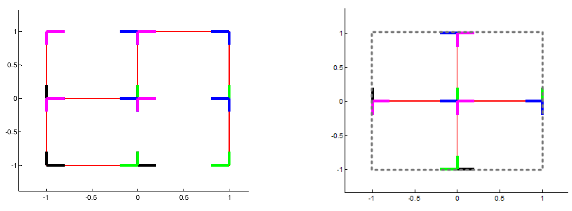


Figure 3. Four room building.

Fig. 4(a) is the simulation results when we consider all the wall-wall-floor trihedrals appearing in the radar image, including those trihedrals located along the building outline corners. The red line represents the generated MST based on the wall-wall-floor trihedral nodes. The other four kind colors denote the quadrant at which the wall-wall-floor trihedral was oriented. In this case, we are unable to obtain the correct layout of the building. Fig. 4(b) is the simulation results after we eliminate the outmost trihedrals of the building. The dashed line represents the outline of the building. It can be seen that the MST based on the remaining trihedrals represent the correct interior layout. It is reasonable to assume that the building outline could be obtained in advance. Thus, combinning the prior outline information, we are able to obtain the whole layout of the building.



(a) (b)

Figure 4. Simulation results of a simple room.

A little more complex residential building layout shown in Fig. 5 is also simulated in this paper. The size of the building is $8m \times 8m$. The outmost trihedrals of the building are eliminated and the final results are presented in Fig. 6. The dashed line represents the outline of the building.

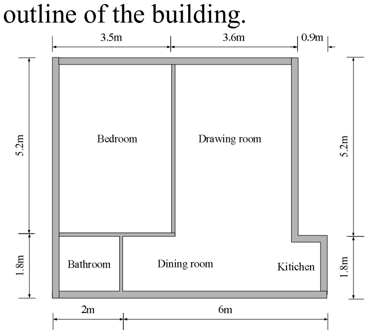


Figure 5. A residential building layout.

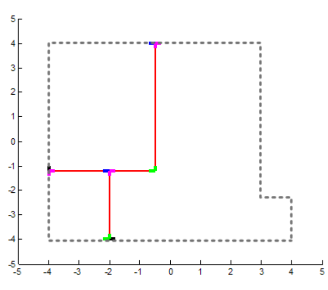


Figure 6. Simulation results of a residential building.

The above simulation results has shown that the MST based reconstruction algorithm is capable of deducing the interior layouts of a rectangular shape building. The whole structure of a building can be obtained if the outline information is known prior. Comparing with other exhaustive attack method, the MST based method will greatly reduce the computation burden. It is more suitable to be applied in the iterative process and enhance the accuracy of the deducing results.

V. Conclusion

We have proposed a building layout reconstruction algorithm assuming that complete position and orientation information has been obtained. It is hoped that based on these preliminary results, we are able to predict the building layouts using radar measurements. However, the following three situations should be further explored in practical radar measurements. The first one is some trihedrals may not be detected in the radar image. The second one is some behind-the-wall targets may be treated as trihedrals by mistake. The last one is the position of the trihedrals will deviate from their original location. These cases will be discussed in our future research work.

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