# A Hybrid Method of PEC-TDS and CBF for Electromagnetic Radiation From array 

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## 1. Introduction

The MOM-based integral equation method, because of its universal algorithm with fewer restrictions on the geometric structure and easy to simulate the material features, is widely used in the scattering and radiation problems. In recent years, integral equation methods to handle objects consisting of both dielectric and conducting material are: volume- surface integral equation (VSIE) [1], surface integral equation (SIE) [2], electric- magnetic current combined field integral equation (JMCFIE) [3], and other hybrid methods. However, the above methods using the subsectional basis functions would not be efficient for the analysis of arrays. This is because the dimension of the matrix to solve is too large, and the dimension increases very rapidly with the raise in the number of array elements. Especially for elements including dielectric materials, the matrix dimension will be more, and more memory and computing time will be consumed.

In order to reduce the number of unknowns in dielectric, thin dielectric sheet (TDS) formulation is modified [4], and impedance boundary condition (IBC) [5] is used to account for the dielectric's effect for dielectric-coated PEC surfaces. For large planar arrays, several approaches based on block concept have been studied extensively, including characteristic basis function (CBF) approach [6], macro basis function (MBF) approach [7].

In this paper, a hybrid method of Characteristics Basis Function (CBF) and coupled PEC-TDS surface integral equation is used to solve the radiation from the curved array on the thin dielectric substrate. The coupling between metal element and dielectric substrate is solved by using the PEC-TDS surface integral equation, and three kinds of basis functions are used to represent the currents on the conducting surface, the tangential currents and the normal currents in the substrate in method of moments (MoM). CBF method is used to solve the coupling between mutual elements. The following numerical examples will show the validity of this method.

## 2. The Hybrid method of PEC-TDS and CBF

The proposed approach consists of three basic steps. Firstly, we construct the primary characteristic basis function (PCBF) for each element by PEC-TDS approach. Secondly, we construct the second characteristic basis function (SCBF), ignoring the coupling between the dielectric substrate and other metal elements, only to consider the interaction of metal elements. Finally, we solve the coefficients of these CBFs by using the Galerkin procedure, and get currents on every element and far field patterns.

The PEC-TDS approach [4] has improved the traditional TDS approach which was invented by W.C. Chew's research team, not only considering the impact of the tangential field components, but also adding the role of the normal field components in the dielectric. Brief summaries are as follows:

$$
\begin{equation*}
\left[\boldsymbol{E}^{s}(\boldsymbol{r})+\boldsymbol{E}^{i}(\boldsymbol{r})\right]_{\mathrm{tan}}=0 \quad \boldsymbol{r} \in S_{p e c} \quad \boldsymbol{E}^{s}(\boldsymbol{r})+\boldsymbol{E}^{i}(\boldsymbol{r})=\boldsymbol{E}(\boldsymbol{r}) \quad \boldsymbol{r} \in V_{t d s} \tag{1}
\end{equation*}
$$

In the metal-dielectric structures, electric field satisfies Equation (1), where $\boldsymbol{E}^{\boldsymbol{i}}(\boldsymbol{r})$ is the incident field, and $\boldsymbol{E}(\boldsymbol{r})$ is the total field. The scattered field $\boldsymbol{E}^{s}(\boldsymbol{r})$ is generated by the radiations from the induced current $\boldsymbol{J}_{p}(\boldsymbol{r})$ on the conducting surface and the induced current $\boldsymbol{J}_{v}(\boldsymbol{r})$ in the TDS. The current $\boldsymbol{J}_{v}(\boldsymbol{r})$ can be decomposed into tangential current $\boldsymbol{J}_{t}(\boldsymbol{r})$ and normal current $\boldsymbol{J}_{n}(\boldsymbol{r})$. The coupled surface integral equation can be easily solved by the MoM using CRWG, modified CRWG and pulse bases. The matrix equation is set up as follows:

$$
\left[\begin{array}{lll}
Z_{P P} & Z_{P N} & Z_{P T}  \tag{2}\\
Z_{N P} & Z_{N N} & Z_{N T} \\
Z_{T P} & Z_{T N} & Z_{T T}
\end{array}\right] \cdot\left[\begin{array}{c}
I_{P} \\
I_{N} \\
I_{T}
\end{array}\right]=\left[\begin{array}{c}
V_{P} \\
V_{N} \\
V_{T}
\end{array}\right]
$$

The meanings of matrix elements and the matrix evaluation can be referenced to [5]. We rewrite the formula (2) as follows:

$$
\begin{equation*}
Z_{i i} J_{i}^{P}=V_{i}^{\text {inc }} \quad(i=1,2, \cdots \cdots M) \tag{3}
\end{equation*}
$$

where $J_{i}^{P}$ is the primary characteristic basis function of $i$ th element, including $I_{P}, I_{N}, I_{T}$, and M is the total number of array elements. For the same element with the same amplitude and phase of the excitation, the equation (3) only need to be solved once.

The secondary characteristic basis function only includes the interaction of metal elements, ignoring the impact of the dielectric substrate. The exciting field of the second characteristic basis function of the $i$ th element equals the scattering field which is produced by metal elements of primary characteristic basis function of all the other elements excepting the $i$ th element. The formulas are as follows:

$$
\begin{array}{ll}
Z_{i i}^{p e c} J_{i}^{S 1, p e c}=-\sum_{j=1, j \neq i}^{M} Z_{(i, j)}^{p e c} J_{j}^{P, p e c} & (i=1,2, \cdots \cdots M) \\
Z^{i i} \text { pec } J_{i}^{S 2, p e c}=-\sum_{j=1, j \neq i}^{M} Z_{(i, j)}^{p e c} J_{j}^{S 1, p e c} & (i=1,2, \cdots \cdots M) \tag{5}
\end{array}
$$

The matrix $Z_{i i}^{p e c}$ denotes the self interactions by the PEC current in the $i$ th element, and the matrix $Z_{(i, j)}^{p e c}$ denotes mutual interactions between the $i$ th element and the $j$ th element. The vector $J_{j}^{P, p e c}$ is the metal part of the PCBF in the $i$ th element, and the vector $J_{i}^{S 1, p e c}$ and $J_{i}^{S 2, p e c}$ are the SCBFs in the $i$ th element.

A reduced matrix is generated for coefficients of the CBFs by using the Galerkin method. Because the size of the reduced matrix is relatively small, the direct method can be used to solve the equation, such as LU decomposition. Finally, the metal surface currents can be expressed as

$$
\begin{equation*}
J_{i}^{p e c}=a_{i} J_{i}^{P, p e c}+b_{i} J_{i}^{S 1, p e c}+c_{i} J_{i}^{S 2, p e c}+\cdots \cdots . \quad(i=1,2, \cdots \cdots M) \tag{6}
\end{equation*}
$$

where $a_{i}, b_{i}, c_{i}$ are coefficients of CBFs by using the Galerkin method. Now we can calculate the radiation fields, which are generated by the metal surface currents $J_{i}^{p e c}$ and the dielectric part of the PCBF for each element.

## 3. Numerical Results

In this section, a curved array of 5 bow tie elements on thin dielectric substrates is examined in the method described above, and the results have been compared with those derived by using the HFSS software. Figure 1 shows the array structure, and the dielectric substrate is $\mathrm{a}=34 \mathrm{~mm}, \mathrm{~b}=30 \mathrm{~mm}, \mathrm{~h}=$ 1 mm , and the relative dielectric constant 2.55. The bow tie antenna is $\mathrm{w}=25 \mathrm{~mm}, \mathrm{~L}=15.94 \mathrm{~mm}$, and the middle distance is 1 mm . Antenna elements are placed in a circular curve with a radius of 220 mm , angle of 10 degrees apart. The delta gap voltage source ( 1 Volt) is placed in the middle of the bow tie. The subdivision density is 2 mm , when they are modeled by using triangles. The number of unknowns for one element is 1608, including 161 metal surface currents, 566 dielectric normal currents, 581 dielectric tangential currents. Figure 2 shows the comparison of reflection coefficients of a single element, when the frequency is in the range of $3-7 \mathrm{GHz}$. Comparative results suggest that the PEC-TDS method can correctly express changes of the port impedance and resonance frequency position (around 4 GHz ). Figure 3 shows patterns at 4 GHz , when each element is excited by 1 Volt. These results illustrate a good agreement compared with the results of HFSS software.

## 4. Conclusion

A hybrid method of PEC-TDS and CBF to solve the radiation of array is proposed. Using this method, the pattern of bow tie antenna array with the dielectric substrate is calculated. Since the dielectric volume elements are transferred to the surface elements in the PEC-TDS method, the number of unknowns would be reduced compared with the volume-surface integral equation. The CBF method is employed, so we can use the direct method to solve the matrix equation, avoiding lengthy iterative process. Numerical results show the validity of the method. The method guarantees the accuracy of calculation, while reducing the computation time, so it has good practical and researching value.

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Figure 1: Bow tie antenna array

(a) E-plane


Figure 2: Comparison of reflection coefficients

(b) H-plane

Figure 3: Comparison of the radiation patterns

