Radar HRRP Adaptive Denoising via Sparse and Redundant Representations

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***Abstract*-We address the radar high resolution range profile (HRRP) denoising problem for improving the recognition rate of HRRP at low signal-to-noise ratio (SNR). Gaussian white noise in HRRP return is suppressed by an approach based on sparse representation. A Fourier redundant dictionary is established for sparsely representing HRRP returns. An adaptive signal recovering algorithm, Orthogonal Matching Pursuit-Modified Cross Validation (OMP-MCV), is proposed for obtaining denoised HRRP without requiring any knowledge about the noise statistics. As a modification to OMP-CV, OMP-MCV modifies the cross validation iteration condition, which can prevent the iteration procedure from terminating at local minimum impacted by noise. Simulation results show that OMP-MCV achieves better performance than OMP-CV and some other traditional denoising method, like discrete wavelet transform, for HRRP returns denoising.**

I. INTRODUCTION

High resolution range profile (HRRP) automatic target recognition (ATR) has received much attention in recent years [1-3]. In practice, the HRRP signatures are usually distorted due to the presence of noises, such as system noise, environmental noise and complicated battle interference etc. And this results in the recognition performance degradation. Several methods have been proposed to settle this issue [4-6]. The noise-robust bispectrum signatures have been extracted for target recognition [4]. Noise-robust factor analysis models based on multitask learning (noise-robust MTL-FA) has been developed to improve the recognition performance at low signal-to-noise ratio (SNR) [5]. HRRP denoising is another feasible choice, e.g., combine bispectrum-filtering has been used to suppress the noise of HRRP [6]. Nevertheless, there are several limitations of these methods. First, the noise-robust features are limited and maybe not optimal for classification. Secondly, the noise-robust MTL-FA classifier suffers from a large computational burden, impeding its practical application. Lastly, the combine bispectrum-filtering requires a certain number observations to reduce noise, increasing the time cost of HRRP recognition. It is therefore desired to develop new methods to handle the issue.

This research was supported by the National Science Foundation of China (no. 61201311).

Recently, sparse representation (SR) has attracted much attention in signal processing [7, 8]. In sparse and redundant dictionary, the signal energy is concentrated on minority atoms, whereas the noise energy is evenly dispersed over the atoms. This makes the signal part achieve a strong anti-noise ability. Thus in this paper, for extracting HRRP denoised features at low SNR, we seek to suppress the Gaussian white noise in HRRP return by sparse representation with only one observation. A sparse and redundant dictionary is established for sparsely representing the HRRP returns. The OMP-MCV method, a modification to Orthogonal Matching Pursuit-Cross Validation (OMP-CV) [9], is proposed to adaptively denoise the HRRP return without requiring any knowledge about the noise statistics. Simulation results show that the OMP-MCV algorithm outperforms over the original OMP-CV and discrete wavelet transform [10] for HRRP returns denoising.

II. RADAR HIGH RESOLUTION RANGE PROFILING

Usually, radar HRRP is achieved through wideband signal, such as Liner Frequency Modulation (LFM) signal and Stepped-frequency (SF) signal etc. In this paper, LFM signal is taken as an example to discuss HRRP denoising. The principle described here can be extended to other waveforms.

The transmitted wideband LFM signal can be represented as

$$s(t) = \mathrm{rect}\left(\frac{t}{T_p}\right) e^{j2\pi(f_c t + \frac{1}{2}\gamma t^2)}, \tag{1}$$

where

$$\mathrm{rect}(u) = \begin{cases} 1 & |u| \le \frac{1}{2} \\ 0 & |u| > \frac{1}{2} \end{cases}, \tag{2}$$

denotes complex signal envelope, T_p denotes pulse width, f_c is carrier frequency, and γ is frequency modulation slope.

The return from a target located at range R_t is given by

$$s_r(t) = A \cdot \mathrm{rect}\left(\frac{t - 2R_t/c}{T_p}\right) e^{j2\pi\left(f_c\left(t-\frac{2R_t}{c}\right)+\frac{1}{2}\gamma\left(t-\frac{2R_t}{c}\right)^2\right)}, \tag{3}$$

where A is the amplitude of the return. With some straightforward manipulations, the de-chirping output can be written as

$$s_{if}(t) = A\cdot \text{rect}\left(\frac{t-2R_t/c}{T_p}\right)e^{-j\frac{4\pi}{c}\gamma R_\Delta t}e^{j\frac{8\pi R_{ref}}{c^2}\gamma R_\Delta}e^{-j\frac{4\pi}{c}f_c R_\Delta}e^{j\frac{4\pi\gamma}{c^2}R_\Delta^2},(4)$$

where $R_\Delta = R_t - R_{ref}$, and T_{ref} denotes the pulse width of reference signal, usually lager than T_p. The later three phase terms in (4) are constants, with no contribution to HRRP. If the sum of the later three phase terms is denoted as φ, it follows

$$s_{if}(t) = A\cdot \text{rect}\left(\frac{t-2R_t/c}{T_p}\right)e^{-j2\pi f_d t}e^{j\varphi}, \tag{5}$$

where $f_d = 2\gamma R_\Delta/c$. Equation (5) shows that after de-chirping, the return of every scatterer is a complex sinusoidal signal with a frequency proportional to the relative range of the scatterer.

III. HRRP Adaptive Denoising by Sparse Representation

In this section, we start the discussion of HRRP denoising via sparse and redundant representations by first discussing how redundant dictionary is established. Then, the OMP-MCV method is proposed to adaptively denoise the noisy returns without requiring any knowledge about noise statistics.

A. HRRP Return Sparse Representation

In noisy circumstance, assuming that a target contains K scatterers situating at different ranges and a single pulse contains M sampling points. From (5), the time domain sampling sequence of the de-chirping output pulse can be represented as

$$\begin{aligned} y(m) &= s(m) + n(m) \\ &= \sum_{k=1}^{K} A_k \cdot e^{-j2\pi f_k m} e^{j\varphi_k} + n(m), \\ m &= 0,1,\cdots,M-1 \end{aligned} \tag{6}$$

where A_k, f_k, φ_k are the amplitude, relative frequency normalized by sampling rate and constant phase of the return from the k th scatterer respectively. $s(m)$ and $n(m)$ denote signal sequence and Gaussian white noise sampling sequence respectively. Let $\boldsymbol{y} = [y(0), y(1),\cdots, y(M-1)]^T$, $\boldsymbol{s} = [s(0), s(1),\cdots, s(M-1)]^T$, $\boldsymbol{n} = [n(0), n(1),\cdots, n(M-1)]^T$, then (6) can be rewritten as

$$\boldsymbol{y} = \boldsymbol{s} + \boldsymbol{n} = \sum_{k=1}^{K} u_k \cdot \boldsymbol{v}_k + \boldsymbol{n}, \tag{7}$$

where $u_k = A_k e^{j\varphi_k}$ and $\boldsymbol{v}_k = [1, e^{-j2\pi f_k},\cdots, e^{-j2\pi f_k}(M-1)]^T$. Equation (7) indicates that the de-chirping output sequence is superposed of multiple complex sinusoidal signals and noise component. Usually, the number of the main scatterers from a target is much less than that of the range cells in HRRP. Thus $\boldsymbol{s}$ is sparse in frequency domain and can be sparsely represented by complex Fourier redundant dictionary, which is constructed as

$$\boldsymbol{A} = \{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_N\} \in \mathbb{C}^{M\times N} \tag{8}$$

where

$$\boldsymbol{\phi}_i = \exp\{-j2\pi \cdot \boldsymbol{f}_N(i)\cdot \boldsymbol{m}\},\ i = 1,\cdots,N, \tag{9}$$

$\boldsymbol{m} = [0,1,\cdots,M-1]^T$ and $N > M$. In (9), $\boldsymbol{f}_N = [0, 1/N,\cdots,(N-1)/N]$ is the normalized frequency. Then (7) can be represented as

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{n} \tag{10}$$

where $\boldsymbol{x}$ is a sparse vector, composed of the decomposition coefficients of signal $\boldsymbol{s}$ in $\boldsymbol{A}$. Sparse representation theory shows that if $\boldsymbol{x}$ satisfies $\|\boldsymbol{x}\|_0 < (1/2)\mathbf{spark}(\boldsymbol{A})$, where $\mathbf{spark}(\boldsymbol{A})$ denotes the minimum number of columns of $\boldsymbol{A}$ that are linearly dependent and $\|\ \|_0$ denotes the l_0 norm of a vector (i.e., the number of its non-zero components), $\boldsymbol{x}$ can be stably recovered by the following l_0 optimization problem [11]

$$\hat{\boldsymbol{x}} = \arg\min\ \|\boldsymbol{x}\|_0\ \text{ s.t. } \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2 \le \delta, \tag{11}$$

where δ is the representation error boundary dictated by noise level $\|\boldsymbol{n}\|_2$. The solution of (11) is NP hard. Approximated solution can be acquired by greedy algorithms, e.g., OMP [12]. In this paper, OMP is utilized to solve (11) for its simplicity and efficiency. When obtaining $\hat{\boldsymbol{x}}$ from (11), the denoised HRRP return can be acquired by $\hat{\boldsymbol{s}} = \boldsymbol{A}\hat{\boldsymbol{x}}$.

B. Selection of Representation Error Boundary

For denoising by sparse representation, δ is an important parameter. If δ is not selected properly, the denoised HRRP either misses some scatterers information (underfitting) or still contains some noise part (overfitting). δ is determined by noise statistics , which are unknown and need to be estimated in most cases. Cross-validation (CV) is an effective method to recover sparse signal from noisy measurements in compressed sensing (CS) without estimating noise statistics. In this work, we modify OMP-CV to solve (11) for HRRP denoising. The sampling points of HRRP return are randomly separated into two sets: the estimation set and the CV set. The estimation set is used to solve (11) with an selection of δ , and the CV set is employed to verify the solution. The HRRP denoising procedure consists of the following steps:

1) **Initialize:**

Randomly separate the sampling points of HRRP return into two sets, $\boldsymbol{y}_E \in \mathbb{C}^{M_1\times 1}$ and $\boldsymbol{y}_{CV} \in \mathbb{C}^{M_2\times 1}$, where $M_1 + M_2 = M$. And separate the redundant dictionary $\boldsymbol{A}$ into two sub-dictionary $\boldsymbol{A}_E$ and $\boldsymbol{A}_{CV}$ corresponding to $\boldsymbol{y}_E$ and $\boldsymbol{y}_{CV}$ respectively.

Set $\alpha = \sqrt{M_1/M_2}$, $\delta = \|\boldsymbol{y}_E\|_2$, $\varepsilon = \|\boldsymbol{y}_E\|_2$, $\tilde{\boldsymbol{x}} = \boldsymbol{0}$, $i = 1$.

2) **Estimate:**

Estimate the sparse solution $\hat{\boldsymbol{x}}$ of (11) by OMP using $\boldsymbol{A}_E$ and $\boldsymbol{y}_E$.

3) **Cross validate:**

If $\alpha\|\boldsymbol{y}_{CV}-\boldsymbol{A}_{CV}\hat{\boldsymbol{x}}\|_2<\varepsilon$, set $\varepsilon=\alpha\|\boldsymbol{y}_{CV}-\boldsymbol{A}_{CV}\hat{\boldsymbol{x}}\|_2$, $\delta=\alpha\|\boldsymbol{y}_{CV}-\boldsymbol{A}_{CV}\hat{\boldsymbol{x}}\|_2$ and $\tilde{\boldsymbol{x}}=\hat{\boldsymbol{x}}$, else continue to judge: if $\alpha\|\boldsymbol{y}_{CV}-\boldsymbol{A}_{CV}\hat{\boldsymbol{x}}\|_2>\lambda\cdot\delta$, terminate the algorithm, else set $\delta=\beta\cdot\delta$, where λ and β can be selected as $\lambda=1.3$ and $\beta=0.97$ respectively.

4) Iterate:

Increase i by 1 and iterate from Step 2).

In the step 3), we increase the judgment of $\alpha\|\boldsymbol{y}_{CV}-\boldsymbol{A}_{CV}\hat{\boldsymbol{x}}\|_2>\lambda\cdot\delta$ and continuously reduce δ by $\delta=\beta\cdot\delta$. This is because that, as δ gradually decreasing, the CV error is not guaranteed to continuously decrease to the global minimum, but it may present a little fluctuation because of the influence of noise. Step 3) can effectively prevent the algorithm from terminating at local minimum. We refer to this modification as OMP-MCV modification to the OMP-CV algorithm. Finally, the denoised HRRP return is obtained by $\tilde{\boldsymbol{s}}=\boldsymbol{A}\tilde{\boldsymbol{x}}$.

IV. EXPERIMENTAL RESULTS

In this section, the simulated experiments are conducted to investigate the denoising performance of OMP-MCV for HRRP.

A. Experiment Setup

The HRRP returns of a target with 7 scatterers located at different range, 12000m, 12001m, 12002.1m, 12002.3m, 12005.3m, 12006.9m, 12008.7m respectively, are simulated. And the amplitudes of returns from the scatterers are set as 0.2, 0.3, 0.3, 1, 0.3, 0.5, and 0.8 separately. The radar system parameters are set as follows: carrier frequency $f_0=5.52GHz$, bandwidth $B=400MHz$, pulse width $T_p=25us$, reference pulse width $T_{ref}=25.6us$, sampling rate for de-chirped output $f_s=10M$. Under these parameters, every sampling sequence of a single pulse is a 256-length vector. The redundant dictionary is established with $M=256$, $N=1024$ for sparsely representing the HRRP returns. After de-chirping processing, the noise-free HRRP obtained by FFT is illustrated in Fig. 1.

B. CV Error Variation in the Iteration Procedure

To investigate the variation of CV error in the iteration procedure, the representation error δ is gradually decreased with a fixed step, and the CV error is tested in this process. And the SNR is set to 5dB. The sampling sequence is randomly separated two sets: 176 sampling points for estimating the original noise-free signal and the rest 80 sampling points for the CV test. The variation of CV error $\|\boldsymbol{y}_{CV}-\boldsymbol{A}_{CV}\hat{\boldsymbol{x}}\|_2$ and signal reconstruction error $\|\boldsymbol{s}-\boldsymbol{A}\hat{\boldsymbol{x}}\|_2$ in the iteration procedure are shown in Fig. 2.

The results of Fig. 2 indicate that as δ gradually decreasing, the CV error is stepped down to the global minimum. Besides, the CV error presents a little influence nearby the minimum. In other words, CV error may encounter local minimum in the procedure of converging to the global minimum. Compared with OMP-CV, the step 3) in OMP-MCV can effectively prevent the algorithm from terminating at local minimum. Fig. 2 also shows that the signal reconstruction error converging to the rock bottom as the CV error reaches the global minimum.

C. Selection of Number of CV

Usually, the signal pulse width and sampling rate are fixed in radar system, thus the length of the sampling sequence is fastened, i.e., M_1+M_2 is a constant. There is a tradeoff between the size of the estimation set M_1 and the size of the CV set M_2. Increasing M_1 will improve estimation accuracy of the original noise-free signal, whereas, increasing M_2 will enhance CV estimation accuracy, also important for the OMP-MCV algorithm. Thus the influence of the CV size is investigated in this subsection. The SNR is set to 5dB. The root-mean-square error (RMSE) of the denoised signal is surveyed, which is defined as

$$\mathrm{RMSE}=\sqrt{\frac{1}{I}\sum_{i=1}^{I}\|\hat{\boldsymbol{s}}_i-\boldsymbol{s}_0\|_2^2}\,, \tag{12}$$

where $\boldsymbol{s}_0$ is the original noise-free HRRP return, $\hat{\boldsymbol{s}}_i$ is the denoised HRRP return, and I is the times of Monte Carlo experiments which is selected as 100. The RMSE of the denoised returns varying with the number of cross validation is shown in Fig. 3.

The result of Fig. 3 demonstrates that as M_2 increase, the denoising performance improves at first, and then worsens. Thus in practice, a tradeoff should be made for selecting M_1 and M_2. In this experiment, the best balancing point is selected by $M_1=196$ and $M_2=60$.

D. Denoising Performance

In the subsection, the denoising performance of OMP-MCV is verified by compared with other methods. In this experiment, the sampling sequence is randomly separated into two sets with $M_1=196$, $M_2=60$. The RMSE of the denoised returns is investigated as SNR varying from 0dB to 30dB. For reference, we also assume that the exactly noise level $\|\boldsymbol{n}\|_2$ is known and setting $\delta=\|\boldsymbol{n}\|_2$ for solving (11) by OMP, referring to it as OMP-δ for short. Meanwhile, the original OMP-CV [9] and the discrete wavelet transform denoising (DWTDN) [10] are utilized for comparing. In DWTDN, the 'db8' wavelet basis is chosen due to its better denoising performance than other 'db' wavelet basis in our experiments. The returns are decomposed into 10 layers, and then the noise is rejected by Heursure threshold. RMSE of the denoised HRRP returns by various methods are surveyed with 100 times Monte Carlo experiments. The results are exhibited in Fig. 4.

The results of Fig. 4 demonstrate that OMP-MCV achieves much better denoising performance than DWTDN for HRRP. This is because the radar return is composed of complex sinusoidal components after de-chirping processing, and the return energy is more concentrated in Fourier redundant than wavelet basis, leading to a stronger anti-noise performance. The denoising performance of OMP-MCV is also better than that of original OMP-CV, close to that of OMP-δ, because of its capability of preventing the iteration procedure from terminating at local minimum.

V. CONCLUSION

For enhancing the performance of HRRP recognition at noise circumstance, this paper has presented an adaptive denoising method, OMP-MCV, for suppressing Gaussian white noise in HRRP return via sparse representation. This method is a modification to OMP-CV and can effectively prevent iterating estimation from terminating at local minimum. The HRRP returns are denoised by OMP-MCV without needing any knowledge about the noise statistics. Simulation results show that OMP-MCV achieves better denoising performance than original OMP-CV and DWTDN.

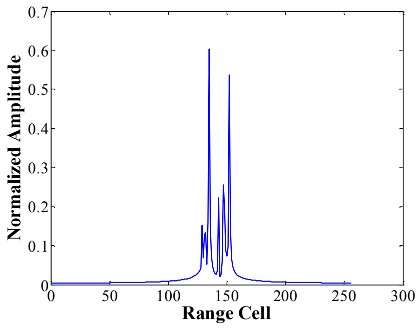

Fig. 1 HRRP of simulated target

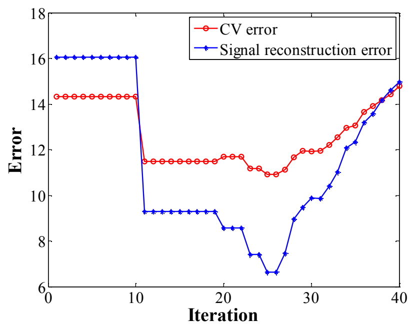

Fig. 2 Evolution of the error with δ gradually decreasing

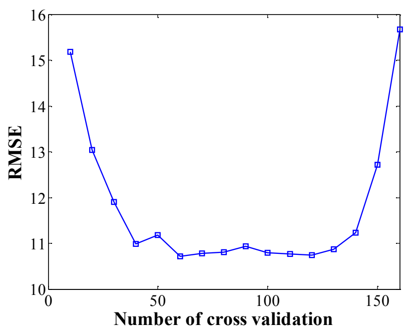

Fig. 3 RMSE of denoised HRRP returns varying with tradeoff of the number for CV set and estimation set.

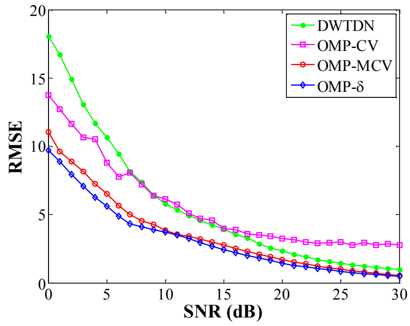

Fig. 4 RMSE of the denoised HRRP by various methods varying with SNR

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