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Analysis Method for Steady-State Periodic Solutions in Periodically Driven Nonlinear Circuits using Haar Wavelet Transform

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Abstract—Recently, much attention have been paid to the methods for circuit analysis using wavelet transform. In particular, we have proposed the method which can choose the resolution of the wavelet adaptively. This method can fully bring out the orthogonal and the multiresolution properties of the wavelet, and the efficiency of the calculation can be improved. In this paper, we propose the method to analyze the steady-state periodic solutions of the nonlinear circuits driven by the periodic external input by applying the appropriate boundary conditions, and prove the effectiveness of the proposed method.

1. Introduction

The wavelet transform has been often used in signal processing because of its orthogonality and multiresolution property [1, 2]. Recently, much attention has been paid to the method for circuit analysis using wavelet transform [7]–[9]. In particular, Barmada et al. have proposed the Fourier-like approach for the circuit analysis using the wavelet transform [8]. In this method, the integral and differential operator matrices are introduced to the analysis, and the differential and integral equations are transformed into the algebraic equations like as using Fourier or Laplace transforms. Moreover, the method can treat time varying and nonlinear circuits. Therefore, this method is useful for various circuit analyses.

However, in that method, the use of Daubechies wavelet makes the handling of the operator matrices complicated, especially, in the edges of the interval. Thus, we have proposed the circuit analysis method using Haar wavelet transform [9]. The Haar wavelet is easy to handle itself, and the operator matrices using the Haar wavelets are easily derived by introducing the block pulse functions [5, 6]. Moreover, the proposed method can treat the nonlinear time varying circuits.

In addition, Haar wavelets have the merit to be able to analyze the trajectory near the singular points where the trajectory moves rapidly with high resolution because of the orthogonality and localization property of the wavelet functions. As circuit analysis methods using this merit, some methods were proposed to pick out the ranges automatically where the trajectory moves rapidly near singular points. Thus we have proposed the method for transient circuit analysis using wavelet transform with adaptive resolutions [10]. In this method, the result of the multiresolution analysis is used to choose the range to be analyzed more precisely. It makes the adaptive choice of the wavelet resolution possible, and as a result, the efficient calculation can be achieved.

On the other hand, the wavelet method to analyze the steady-state waveforms for power electronics circuits have proposed by Tam et. al [11]. If we calculate such steady-state waveforms using time-marching methods as in the conventional way, the calculation cost is wasted due to the calculation of the long-term transient response with sufficiently small step size to approximate the discontinuous dynamics typically seen in power electronic circuits. To overcome such disadvantage of the time-marching method, in [11], the Chebyshev polynomials are used as the basis functions for wavelet approach, and the periodic solutions of periodically driven power electronics circuits have been calculated. However it is considered that the calculation should be complicated and the Gibbs-phenomenon-like errors have been seen when the switching is occurred because of the use of the Chebyshev polynomials. In contrast, the Haar wavelet transform will make the calculation simpler, and also the discontinuity of the Haar function will be suitable for the analyses of such discontinuous behavior of the power electronics circuits. Therefore, in this paper, we propose the method to analyze the steady-state periodic solutions of the nonlinear circuits driven by the periodic external input by applying the appropriate boundary conditions. We will show an algorithm for the approximation of the steady-state periodic solution and the better performance for the accuracy using a simple example.

2. Haar Wavelet Matrix and Integral and Differential Operator Matrices

Haar functions are defined on interval [0,1) as follows,

$$h_0 = \frac{1}{\sqrt{m}}, \tag{1}$$

$$h_i = \frac{1}{\sqrt{m}} \times \begin{cases} 2^{\frac{j}{2}}, & \frac{k-1}{2^j} \le t < \frac{k-\frac{1}{2}}{2^j}, \\ -2^{\frac{j}{2}}, & \frac{k-\frac{1}{2}}{2^j} \le t < \frac{k}{2^j}, \\ 0, & \text{otherwise in } [0,1), \end{cases} \tag{2}$$

$$i = 0, 1, \cdots, m-1, m = 2^{\alpha},$$

where α is positive integer, and j and k are nonnegative integers which satisfy $i = 2^j + k$, i.e., $k = 0, 1, \cdots, 2^j - 1$ for $j = 0, 1, 2, \cdots$.

$\vec{y}$ is $m \times 1$-dimensional vector whose elements are the discretized expression of $y(t)$ and $\vec{c}$ is $m \times 1$-dimensional coefficient vector. H is $m \times m$-dimensional Haar wavelet matrix defined as

$$H = [\vec{h}_0^T \quad \vec{h}_1^T \quad \cdots \quad \vec{h}_{m-1}^T] \tag{3}$$

where $\vec{h}_i$ is $1 \times m$-dimensional Haar wavelet basis vector whose elements are the discretized expression of $h_i(t)$. Note that H is an orthonormal matrix. Using these vectors and matrix, Haar wavelet transform and inverse Haar wavelet transform are described as follows, respectively,

$$\vec{c} = H\vec{y}, \tag{4}$$

$$\vec{y} = H^T \vec{c}\,(= H^{-1}\vec{c}\,). \tag{5}$$

The basic idea of the operator matrix has been firstly introduced by using Walsh function [5]. However, in logical way, the matrices introduced by block pulse function are more fundamental [4, 5]. The block pulse function is the set of m rectangular pulses which have $1/m$ width and are shifted $1/m$ each other.

The integral operator matrix of the block pulse function matrix B is defined as the following equation [5, 6].

$$\int_0^i B(\tau)d\tau \equiv Q_B \cdot B(t), \tag{6}$$

$$Q_B = \frac{1}{m}\left[\frac{1}{2} I_{(m\times m)} + \sum_{i=1}^{\infty} P^i_{(m\times m)}\right], \tag{7}$$

where $B(t)$ is $m \times m$-dimensional matrix whose elements are the discretized expression of the block pulse functions $b_i(t),\ i = 0, 1, \cdots, m-1$ and

$$P^i_{(m\times m)} = \left[\begin{array}{c|c} 0 & \mathrm{I}_{(m-i)\times(m-i)} \\ \hline 0_{(i\times i)} & 0 \end{array}\right]$$

for $i < m$,

$$P^i_{(m\times m)} = 0_{(m\times m)}$$

for $i \geq m$.

As the Haar wavelet matrix H is the set of the orthogonal functions, the integral matrix of H is given as follows:

$$Q_H = HQ_B^T H^{-1} = HQ_B^T H^T. \tag{8}$$

Similarly, the differential matrix of H can be written as

$$Q_H^{-1} = H(Q_B^T)^{-1}H^{-1} = H(Q_B^T)^{-1}H^T. \tag{9}$$

3. Haar Wavelet Expression of Branch Characteristics

As the Haar wavelet is defined on interval [0,1), the generic interval $[t_{\min}, t_{\max})$ can be rescaled by a new variable τ on [0,1), where $[t_{\max} - t_{\min})\tau + t_{\min}$. In this paper, $t_{\min} = 0$ without loss of generality, then capacitance c [F] and inductance l [H] are scaled to $C = c/t_{\max}$ and $L = l/t_{\max}$, respectively.

Next, we show the Haar wavelet expression of branch characteristics of nonlinear time varying circuit elements for the expression in wavelet domain. See details in [9].

Capacitor:

$$\begin{aligned} v(t) &= v(0_-) + \frac{1}{C}\int_0^t i(\tau)d\tau, \quad v_0 := v(0_-), \\ V &= V_0 + C_w^{-1}Q_H I, \text{ or } I = C_w Q_H^{-1}[V - V_0], \end{aligned} \tag{10}$$

$$C_w = H\mathrm{diag}[C(i_0, t_0), C(i_1, t_1), \cdots, C(i_{m-1}, t_{m-1})]H^T$$

Inductor:

$$\begin{aligned} i(t) &= i(0_-) + \frac{1}{L}\int_0^t v(\tau)d\tau, \quad i_0 := i(0_-), \\ I &= I_0 + L_w^{-1}Q_H V, \text{ or } V = Q_H^{-1}L_w[I - I_0]. \end{aligned} \tag{11}$$

$$L_w = H\mathrm{diag}[L(i_0, t_0), L(i_1, t_1), \cdots, L(i_{m-1}, t_{m-1})]H^T$$

Resistor:

$$\begin{aligned} v(t) &= Ri(t), \\ V &= R_w I, \quad R_w = \mathrm{diag}[R]. \end{aligned} \tag{12}$$

$$R_w = H\mathrm{diag}[R(i_0, t_0), R(i_1, t_1), \cdots, R(i_{m-1}, t_{m-1})]H^T$$

4. Method to Find Steady-State Periodic Solutions

Consider the following ordinary differential equation,

$$\dot{x} = f(x, t) \triangleq A(x, t)x + u(t) \tag{13}$$

where $x(t) \in R^{n\times 1}$ is an unknown state variable vector, $A(x, t) \in R^{n\times n}$ is a nonlinear time-varying parameter matrix, and $u(t) \in R^{n\times 1}$ is an external force vector. The system is driven by the periodic external force or parameter with period T. Assume that we can find the periodic solution $x_p(t)$ with period T, i.e. $x_p(t) = x_p(t+T)$ for all t. In order to find the steady-state periodic solutions, we should find the solution for the interval $[0, T)$ under the appropriate boundary conditions. For the wavelet expression of the differential equations, we define the discretized expression of $x(t)$ and $u(t)$ as $\vec{x}_i =\in R^{m\times 1}$ and $\vec{u}_i =\in R^{m\times 1}$ for $i = 1, 2, \cdots, m$, respectively. Moreover, $\vec{x}_{i0} = [x_i(0) \quad x_i(0) \quad \cdots \quad x_i(0)]^T \in R^{m\times 1}$ and $\vec{x}_0 = [x_1(0) \quad x_2(0) \quad \cdots \quad x_n(0)]^T \in R^{n\times 1}$ for $i = 1, 2, \cdots, m$ which is the initial value vector.

The wavelet transformed expression of Eq. (13) can be derived as

$$Q_m^{-1}[X - X_0] = A_H X + U \tag{14}$$

where $X = [(H\vec{x}_1)^T \quad (H\vec{x}_2)^T \quad \cdots \quad (H\vec{x}_n)^T]^T \in R^{mn\times 1}$ is an unknown wavelet coefficients vector, X_0 =

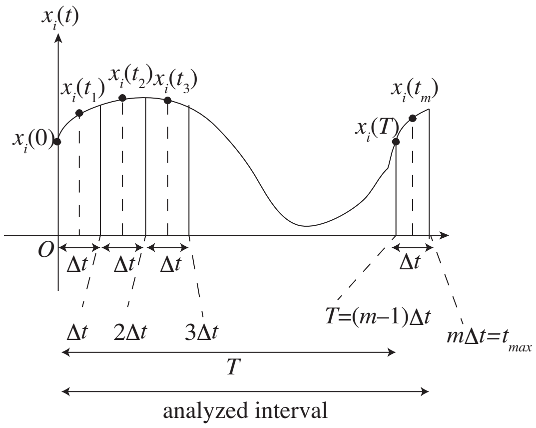

Figure 1: Definition of the analyzed interval and the time step.

$[(H\vec{x}_{10})^T \ (H\vec{x}_{20})^T \ \cdots \ (H\vec{x}_{n0})^T]^T \in R^{mn\times 1}$ and $U = [(H\vec{u}_1)^T \ (H\vec{u}_2)^T \ \cdots \ (H\vec{u}_n)^T]^T \in R^{mn\times 1}$. Note that $\vec{x}_{i0}$ is also unknown for this case. Moreover,

$$Q_m^{-1} = \mathrm{diag}[Q_H^{-1}] \in R^{mn\times mn} \tag{15}$$

and $A_H \in R^{mn\times mn}$ is the wavelet region expression of A derived from Sect. 4. At this moment, as both X and X_0 are unknown, we cannot solve this algebraic equations. On the other hand, for the periodic solutions, $x_p(t) = x_p(t+T)$ can be the boundary condition for Eq. (13). Though $x(0) = x(T)$ is one of the choices of the conditions, these unknown variables cannot be derived from the above equation. Therefore, the other choice of the boundary condition is needed.

To determine the boundary condition, we set the analyzed interval as shown in Fig. 1. The period T is divided by $m-1$ and one more time step included in the next period is added to the analyzed interval. Therefore, the time step $\Delta t = \frac{T}{m-1}$ and $t_{\max} = T + \Delta t$. Because of the feature of the matrix Q_B, time t_i is calculated as $t_i = \frac{\Delta t}{2} + (j-1)\Delta t$ $(j = 1, 2, \cdots, m)$. Due to the periodicity, the relationship $x_i(t_1) = x_i(t_m)$ for all $i = 1, 2, \cdots, n$ is derived. From Eq. (5), this relationship is rewritten as follows.

$$[h_{11} \quad h_{21} \quad \cdots \quad h_{m1}]X_i = [h_{1m} \quad h_{2m} \quad \cdots \quad h_{mm}]X_i, \tag{16}$$

where h_{ij} is an element of Haar wavelet matrix, and then,

$$[h_{11}-h_{1m} \quad h_{21}-h_{2m} \quad \cdots \quad h_{m1}-h_{mm}]X_i = 0. \tag{17}$$

Setting $[h_{11}-h_{1m} \quad h_{21}-h_{2m} \quad \cdots \quad h_{m1}-h_{mm}] \triangleq h_b \in R^{1\times m}$ and $\mathrm{diag}(h_b) \triangleq H_b \in R^{n\times mn}$, the relationship

$$H_b X = 0 \tag{18}$$

is derived.

To derive the unknown vector $\vec{x}_0$, we consider the relationship between X and X_0. From Eq. (14, we see the matrix $Q_H^{-1}X_{i0}$ from $Q_m^{-1}X_0$. From the relationship $X_{i0} = H\vec{x}_{i0}$,

$$Q_H^{-1}X_{i0} = Q_H^{-1}H\vec{x}_{i0} \tag{19}$$

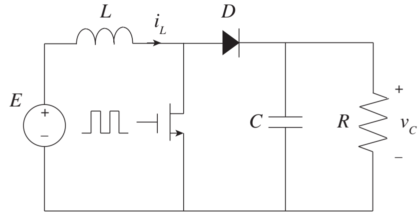

Figure 2: Simple boost converter.

If we set $Q_H^{-1}H \triangleq [q_{ij}] \in R^{m\times m}$,

$$\begin{aligned} Q_H^{-1}H\vec{x}_{i0} &= \begin{bmatrix} q_{11}+q_{12}+\cdots+q_{1m} \\ q_{21}+q_{22}+\cdots+q_{2m} \\ \vdots \\ q_{m1}+q_{m2}+\cdots+q_{mm} \end{bmatrix} x_i(0) \\ &\triangleq q_0 x_i(0) \end{aligned} \tag{20}$$

Then we define $Q_0 = \mathrm{diag}(q_0) \in R^{mn\times n}$ Eq. (14) is rewritten as

$$(Q_m^{-1} - A_H)X - Q_0\vec{x}_0 = U \tag{21}$$

From Eqs. (21) and (18), we can derive $n(m+1)$-dimensional algebraic equations as follows,

$$\left[\begin{array}{c|c} Q_m^{-1} - A_H & -Q_0 \\ \hline H_b & 0 \end{array}\right]\left[\begin{array}{c} X \\ \hline \vec{x}_0 \end{array}\right] = \left[\begin{array}{c} U \\ \hline 0 \end{array}\right]. \tag{22}$$

In this equation, the number of the unknown variables coincides with the dimension of the equation. Therefore, we can derive all the unknown variables to solve it. If the system is nonlinear, Eq. (22) becomes a nonlinear algebraic equation and should be solved by the recursive methods like Newton-Raphson method. Finally, we derive the approximated solution of Eq. (13) from Eq. (5).

5. Example

In this section, we show the simple example to confirm the effectiveness of the proposed method. The simple boost converter circuit shown in Fig. 2 is analyzed in this example. The circuit parameter is set as the same as shown in [11]. The circuit equations written as follows,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} -\frac{R_s(1-s(t))+R_s s(t)}{L} & -\frac{s(t)}{L} \\ \frac{s(t)}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{E-s(t)V_f}{L} \\ 0 \end{bmatrix} \tag{23}$$

where

$$s(t) = \begin{cases} 0, & \text{for } 0 \le t \le T_D \\ 1, & \text{for } T_D \le t \le T \end{cases} \tag{24}$$

An example of the calculated results for the proposed method are shown in Fig. 3. From these figures, we can see good approximation is achieved compared with the exact solutions. To evaluate the accuracy of the proposed method, we calculate mean relative error (MRE) given by

$$\mathrm{MRE} = \frac{1}{m+1}\sum_{i=0}^{m}\left|\frac{x_j(t_i) - \hat{x}_j(t_i)}{\hat{x}_j(t_i)}\right| \tag{25}$$

Table 1: Comparison of MREs for approximation in boost converter.

α	MRE for i_L ([11])	MRE for v_C ([11])	MRE for i_L (proposed)	MRE for v_C (proposed)
4	0.025789	0.025714	0.078896	0.032298
5	0.025745	0.025704	0.011233	0.003098
6	0.025736	0.025703	0.019376	0.010234
7	0.025734	0.025702	0.003816	0.002181
8	0.031353	0.028476	0.004065	0.001844

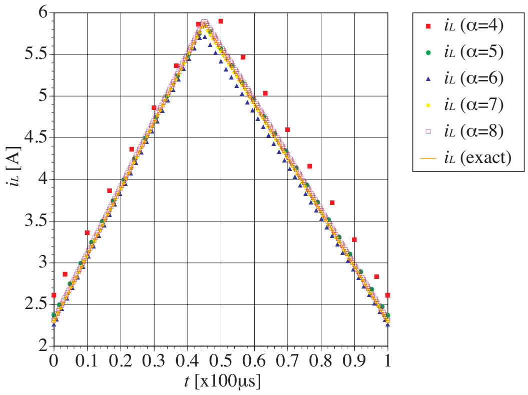

Figure 3: Numerical results of i_L for the proposed method for $\alpha = 4, 5, 6, 7, 8$.

where $\hat{x}$ means the exact solutions. Table 1 shows the MRE for i_L and v_C compare with the errors shown in [11]. From this results, the proposed method can achieve the better approximation than the method shown in [11]. Hence, we can construct the simple and accurate method to calculate the steady-state periodic solution by using Haar wavelet transform. It is considered that the proposed method can play important roles to analyze the power electronic circuits and hybrid dynamical systems.

6. Conclusions

In this paper, we have proposed the method to analyze the steady-state periodic solutions of the nonlinear circuits driven by the periodic external input by applying the boundary conditions that $x_p(t) = x_p(t + T)$. The Haar wavelets make the algorithm simpler and the better accuracy has been achieved compared with the methods previously shown. Therefore, it is considered that the proposed method can play important roles to analyze the power electronic circuits and hybrid dynamical systems. The application of the proposed method to the autonomous systems seems to be the future works.

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