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Abstract—We construct a network from the transport data of particles driven by a flow field and analyze this network using the analytical tools developed in the network science. The aim of this study is to clarify the property which has not been clarified in conventional analytical methods of fluid dynamics. The targets are specification of the important spatial regions, relaxation process to steady state, impact of perturbation, etc. In this paper, we outline the construction method of the network and apply it to a model of the Lagrangian turbulence. We show that the betweenness centrality computed from the network of the transition probability between subregions of the system takes a high value in the boundary region of the roll. This result suggests that those regions are important regions in the sense that they are the bottlenecks of the network. This method has a lot of potential applicability to the local climate analysis, control of transport, etc. Our method serves a new picture to the transport phenomena in turbulent flows.

1. Introduction

After the seminal papers of small world [1] and scale free [2] networks, the network science has been developed in the last decade [3]. Dynamical processes on networks [6] including synchronization [4, 5] are intensively studied in the nonlinear science.

The author has studied synchronization of mobile agents of the phase oscillator which interact with nearby agents in Refs. [7, 8]. This system can be regarded as a temporal network whose topology changes in time. We found that there exist different mechanisms of information propagation in such time dependent networks.

In our previous studies, we regarded mobile agents as nodes of the network, and the connectivity changes in time due to the motion of the agents. In this study, we consider a flow field and transport of particles through this flow. we regard each spatial point as a network node, and transition probability of particles from a node to another node as a link between them. This method provides a coarsening way of looking at flow system. With this approach, it is expected that we can extract the relevant properties of the transport phenomena with much less computational cost compared to the direct simulation of the fluid. Such an approach is called the metapopulation model and has been developed in ecology [9]. Our study applies the metapopulation-like concept to transport phenomena of the fluid, and make the analysis based on the network science.

Transport phenomena by fluid is important not only from scientific but also from engineering viewpoint. There are many potential applicability such as efficient mixing using turbulent diffusion, analysis of transport which is important for environmental problem. Planning the optimal procedure to prevent those particles is an example of the inverse problems and requires a lot of computational cost.

Characteristic of this method is that we construct a network in a spatially continuous system, e.g. fluid field, and analyze this network employing the methodologies of the complex network science. Some examples of the network approach to fluid dynamics can be found in the geophysics (e.g., [10, 11]). They employed correlation-based quantities such as correlation function and mutual information in order to construct networks: The nodes are connected if those quantities are higher than the threshold. On the other hand, the proposed method uses the transition probability of particles from one place to another, thus the physical meaning of the network is clear. It is therefore expected that our method can predict the diffusion process of the transport phenomena. In our study, although node-link correspondence is not clear, we can phenomenologically construct the network from the data. Moreover, we use quantities which are employed in the network science.

This paper is organized as follows. In Sec. 2, we describe the method to construct the transport network from the data. We see how this idea works with the data obtained from the model of particle transport in Sec. 3. Finally, the results are summarized and discussed in the last section.

2. Method

In this section, we describe the method to generate the network from the flow data. The network is constructed as follows. First, we divide the whole space into N subregions S_i $(i = 1, 2, \cdots, N)$. We regard each S_i as a node of the network.

Here, we construct the network as follows. Let the number of particles which starts from the node j at $t = 0$ and are transported to the node k at $t = \tau$ be $n_{kj}(\tau)$. It is judged that there exists a link from node j to k if $n_{kj}(\tau) \neq 0$. Then the transition probability matrix is approximately written as

$$S_{kj}(\tau) \quad = n_{kj}(\tau)/\sum_k n_{kj}(\tau). \tag{1}$$

Devision of the space is arbitrary, but $\sum_k n_{kj}$ must contain enough number for all j.

The physical meaning of the network constructed by Eq. (1) is clear. We assume that the correlation time of the flow is less than τ and the transport after τ is approximated by the Markov process. If we consider the discrete time evolution equation

$$p_k(t+\tau) \quad = \sum_j S_{kj}(\tau) p_j(t), \tag{2}$$

its steady state approximates the steady state distribution of the particles.

Note that this methodology is data-driven. If we know the time evolution equation of the system, we can simulate the evolution of the particle by taking the initial points in S_i. After time t, we examine the point of those particles. However, we can construct the network if we have enough number of flow data, and the network can be constructed even if we do not have enough information of the time evolution equation which governs the dynamics. For example, the connectivity of populations of coral reefs has been studied statistically by analyzing the gene type of corals [12]. We can construct the network with such an indirect transport data and analyze it, and apply the result to e.g., analysis of the population dynamics of coral. This analysis can provide a realistic estimation of the recruitment terms of the coral ecosystems modeling [13]. It would be possible to combine real data and mathematical modeling which can generate more ensembles. Our research procedure is schematically shown in Fig. 1.

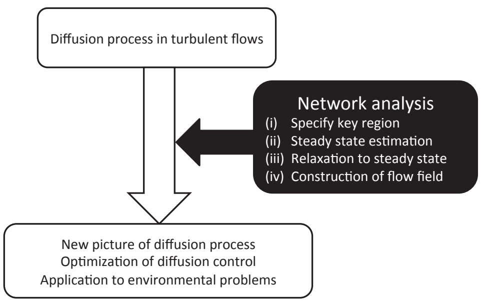

Figure 1: Schematic figure of the proposed analytical framework of network approach of transport phenomena.

3. Example

In this section, we perform the network construction and the analysis of a passive diffusion system governed by a time dependent ordinary differential equation. When the convection flow field oscillates in time, it is know that the dynamics of particles show the so-called Lagrangian turbulence due to jumps of the particles between different streamlines. This is an example of the mixing caused by the fluid. We consider the flow in the x-z plane generated by the time dependent stream function $\Psi(x, z, t)$ [14]. That is

$$\Psi(x, z, t) = \Psi_0(x + B\sin(2\pi t), z), \tag{3}$$

and

$$\dot{x} \quad = \quad -\frac{\partial \Psi(x, z, t)}{\partial z}, \tag{4}$$

$$\dot{z} \quad = \quad \frac{\partial \Psi(x, z, t)}{\partial x}. \tag{5}$$

These equations govern the diffusion process of the particles. Note that the particles can cross the instantaneous stream functions $\Psi(x, z, t)$ due to its time dependent property. The simulation has been performed with the fourth order Runge-Kutta method with $dt = 10^{-3}$. The system size is $L_x = 6$ and $L_z = 1$. Chaotic diffusion of the particles initially distributed in a small region is plotted in Fig. 2. Particles distributed in the small rectangular region near $(1, 0.2)$ at $t = 0$ diffuse following the above equations. The small rectangle is at first stretched along one roll at $t = 2$ and then two ($t = 4$) rolls.

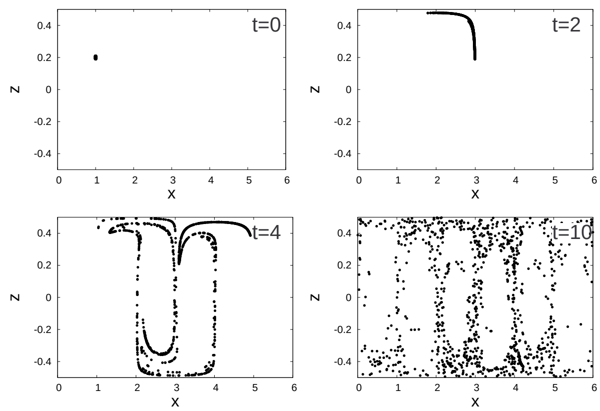

Figure 2: Diffusion process of particles in the x-z plane generated by the Lagrangian turbulence model [14]. Particles initially distributed in a small square at $t = 0$ diffuse to the whole system with Eqs. (4) and (5). The convection roll-like distribution is generated at $t = 10$.

We constructed the network using the method described in the previous section. We divide the whole region into subregions of small squares of 0.2×0.2 size. We take many initial points in each subregion and numerically obtain the time evolution of the particles with Eqs. (4) and (5). We counted the number of particles transported from one node to another and computed the transition probability. As is noted, the constructed network is an approximation of the flow by a Markov process, and the eigenvector corresponding to the largest eigenvalue of the transition matrix is an approximated steady state of the transport of particle caused by the stream function Ψ. The transition matrices of the diffusive particles $S_{kj}(\tau)$ obtained with the numerical simulation at $\tau = 1$ and $\tau = 2$ are depicted in Fig. 3. In the figure, the index k of S is defined as $k = N_z x_k/\Delta x + (z_k + 0.5)/\Delta z$, where (x_k, z_k) are center of kth subregion and $N_z = L_z/\Delta z$. The area of a subregion is $\Delta x \Delta z$, with $\Delta x = \Delta z = 0.2$ in the simulation. As seen from the figure, non-zero elements of the transition matrix are enhanced for larger τ. This represents the diffusion process. The matrix consists of six squares near the diagonal corresponding to six rolls in the x direction.

Once the network is constructed, we can apply many analytical methods developed in the network science. In this paper, we show an example with the weighted betweenness centrality, a quantity which characterizes the bottleneck node of the network. The betweenness centrality is computed numerically and is plotted in Fig. 4. It is a quantity which represents the impact of the number of shortest paths from a node (say, B) to another node (C) when remove a node A. When a node takes a high betweenness centrality, it means that this node is a bottleneck of the network, because the shortest paths decrease when we remove those nodes. In our simulation, we can clearly see that the boundary regions between two rolls take high betweenness centrality. This result suggests that those regions where the streamlines cross under oscillating streaming function are bottlenecks of the flow field.

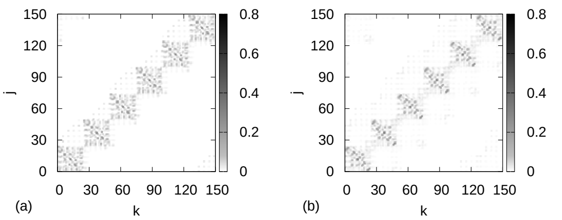

Figure 3: Transition matrix $S_{kj}(\tau)$ for $\tau = 1$ (a) and $\tau = 2$ (b). i (horizontal axis) and j (vertical axis) denote the label of subregions.

4. Summary and discussion

We proposed a method to analyze transport phenomena in flows as diffusion processes on networks. With the data of flow of particles, we construct the transition matrix. We have shown by numerical simulation that the weighted betweenness centrality of the network takes high value near the roll of the original system. This result suggests the applicability of the network approach to uncover the properties of transport phenomena.

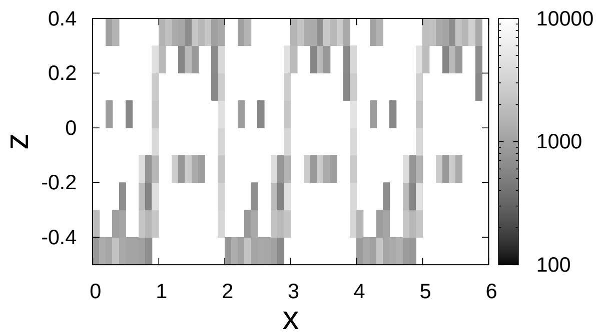

Figure 4: Betweenness centrality distribution of the diffusion network of Eqs. (4) and (5). Subregions where the value exceed 500 are plotted.

We still have many issues regarding to basic of the method and application which must be studied in the future. At first, we have to verify that the obtained transition matrix is consistent. It should give the same steady state with different way of taking subregions for large enough τ. Moreover, S's have to satisfy the semi-group property, i.e., $S_{kj}(\tau_1+\tau_2) = \sum_m S_{km}(\tau_1)S_{mj}(\tau_2)$. It would be necessary to study the range of τ where this approximation works well.

Spectral analysis will provide the information of the relaxation process to the steady state. The second largest eigenvalue of the transition matrix is approximately the inverse of the relaxation time to the steady state distribution. Another important application will be to study the shift of the distribution of the particles under the change of the flow field. If we can predict the response of the distribution, it is useful because the result is obtained without the large scale computer simulation. One possibility would be to use the perturbation expansion developed in quantum mechanics. The perturbation method describes the change of the eigenvalue in terms of the eigenspectrum of the unperturbed matrix. Therefore, if we can specify the unperturbed transition matrix and the perturbed one, the perturbation method can be applied to our problem. Finally, it may be possible to design a flow field which gives the desirable distribution of particles by the above mentioned perturbative method as an inverse problem. This could be an important application of the proposed method.

Acknowledgments

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